A Level Mechanics
Practice Test 5: Rotational Motion

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Draw diagrams where appropriate to illustrate your solutions.
Time allowed: 3 hours

Section A: Rotational Motion Fundamentals [28 marks]

Question 1 [**10 marks**]

(a) Define angular displacement, angular velocity, and angular acceleration, explaining their vector nature and relationship to their scalar magnitudes. [**4 marks**]

(b) Derive the relationships between tangential velocity, tangential acceleration, and their angular counterparts. [**3 marks**]

(c) State the three kinematic equations for rotational motion with constant angular acceleration. [**3 marks**]

Question 2 [**14 marks**] A wind turbine blade rotates with angular velocity described by $\omega(t) = t^5 - 5t^4 + 8t^3 - 2t + 3$ rad/s.

(a) Determine the angular acceleration function $\alpha(t)$. [**3 marks**]

(b) Calculate the angular velocity and acceleration at $t = 2$ seconds. [**3 marks**]

(c) Find all critical points where the angular acceleration is zero and classify them. [**4 marks**]

(d) Calculate the total angular displacement from $t = 0$ to $t = 2.5$ seconds. [**3 marks**]

(e) Determine the average angular velocity over the interval [1, 3] seconds. [**1 mark**]

Question 3 [**4 marks**] A particle moves in a circle of radius 1.2 m. At a certain instant, it has tangential speed 20 m/s, tangential acceleration 8 m/s^2, and the magnitude of total acceleration is 25 m/s^2.

(a) Verify the given values are consistent and find the centripetal acceleration. [**2 marks**]

(b) Calculate the angular velocity and angular acceleration at this instant. [**2 marks**]

Section B: Moment of Inertia - Point Masses [30 marks]

Question 4 [6 marks]

(a) Define moment of inertia and explain how it depends on both mass distribution and axis of rotation. [**3 marks**]

(b) State and derive the parallel axis theorem, explaining its practical applications. [**3 marks**]

Question 5 [24 marks] Calculate moments of inertia for the following configurations:

(a) Nine point masses arranged as: 2.8 kg at (2,4), 3.5 kg at (-3,1), 1.9 kg at (4,-2), 2.1 kg at (-1,-4), 3.2 kg at (0,3), 2.6 kg at (-4,0), 1.7 kg at (3,3), 2.9 kg at (-2,-3), and 3.8 kg at (1,-1) meters. Find the moment of inertia about the z-axis. [**7 marks**]

(b) Seven masses of 2.5 kg each positioned at the vertices of a regular heptagon with circumradius 2.2 m. Calculate the moment of inertia about the central axis. [**4 marks**]

(c) Five point masses distributed along a line: 5 kg at x = -3 m, 8 kg at x = -1 m, 12 kg at x = 0, 6 kg at x = 2 m, and 9 kg at x = 4 m. Find the moment of inertia about: (i) the origin, (ii) x = 1 m, (iii) the center of mass. [**8 marks**]

(d) A rectangular array has masses at the four corners of a rectangle: 4 kg at (0,0), 6 kg at (3,0), 8 kg at (3,2), and 5 kg at (0,2) meters. Calculate the moment of inertia about an axis through (1.5,1) parallel to the z-axis. [**4 marks**]

(e) Use the parallel axis theorem to verify your calculation in part (d) by first finding the moment of inertia about the center of mass. [**1 mark**]

Section C: Moment of Inertia - Extended Bodies [35 marks]

Question 6 [12 marks]

(a) State the standard moment of inertia formulas for: thin rod (center and end), solid disc, hollow disc (inner and outer radii), solid sphere, hollow sphere, and thick-walled cylinder. [**7 marks**]

(b) Derive the moment of inertia of a hollow sphere of mass M, inner radius R_1, and outer radius R_2 about a diameter. [**4 marks**]

(c) Explain why the moment of inertia of a hollow sphere approaches that of a thin spherical shell as the wall thickness decreases. [**1 mark**]

Question 7 [23 marks] Calculate moments of inertia for composite objects:

(a) A complex disc has three concentric sections: a solid inner disc (mass 8 kg, radius 0.2 m), a hollow middle section (mass 5 kg, inner radius 0.2 m, outer radius 0.35 m), and a hollow outer ring (mass 4 kg, inner radius 0.35 m, outer radius 0.5 m). Find the total moment of inertia. [**6 marks**]

(b) A three-dimensional cross consists of three identical uniform rods (each mass 6 kg, length 2.4 m) intersecting perpendicularly at their centers with one rod along each coordinate axis. Calculate the moment of inertia about the z-axis. [**5 marks**]

(c) A molecular model consists of five solid spheres (each mass 3 kg, radius 0.1 m) connected by four uniform rods (each mass 1.5 kg, length 2 m) in a linear chain with spheres at rod ends and junctions. Find the moment of inertia about an axis through the central sphere perpendicular to the chain. [**7 marks**]

(d) A helicopter rotor has a central hub (solid cylinder, mass 25 kg, radius 0.4 m) with eight uniform blades (each mass 8 kg, length 3 m) extending radially. Each blade can be modeled as a rod with one end at the hub edge. Calculate the total moment of inertia about the central axis. [**5 marks**]

Section D: Torque and Rotational Dynamics [31 marks]

Question 8 [10 marks]

(a) Define torque mathematically using the cross product and explain the physical significance of its vector nature. [**4 marks**]

(b) State and derive the fundamental equation of rotational dynamics from Newton's second law. [**4 marks**]

(c) Explain the relationship between torque and angular momentum, and derive the rotational impulse-momentum theorem. [**2 marks**]

Question 9 [21 marks] A multi-tier rotating platform consists of four coaxial discs of different sizes: bottom disc (mass 30 kg, radius 1.5 m), second disc (mass 20 kg, radius 1.0 m), third disc (mass 15 kg, radius 0.7 m), and top disc (mass 10 kg, radius 0.4 m).

(a) Calculate the total moment of inertia of the platform. [**4 marks**]

(b) Five forces are applied: 50 N tangentially at the bottom disc edge, 35 N at 30° to the radius at 1.2 m from center, 28 N tangentially at the second disc edge in the opposite direction, 20 N at 45° to the radius at 0.5 m from center, and 15 N radially at the top disc edge. Calculate the net torque. [**8 marks**]

(c) Find the angular acceleration of the platform. [**2 marks**]

(d) If the platform starts with angular velocity 1.5 rad/s, find the angular velocity after 6 seconds. [**2 marks**]

(e) Calculate the angular displacement during these 6 seconds. [**2 marks**]

(f) Determine the work done by the net torque and the change in rotational kinetic energy. [**2 marks**]

(g) Find the instantaneous power at t = 6 seconds. [**1 mark**]

Section E: Rotational Energy [32 marks]

Question 10 [10 marks]

(a) Derive the general expression for rotational kinetic energy of a system of particles and extend it to continuous bodies. [**4 marks**]

(b) Establish the work-energy theorem for rotational motion and show that the work done by a torque equals the change in rotational kinetic energy. [**4 marks**]

(c) Derive the expression for instantaneous power in rotational motion and relate it to the rate of change of rotational kinetic energy. [**2 marks**]

Question 11 [22 marks] A thick-walled hollow sphere (mass 22 kg, inner radius 0.15 m, outer radius 0.45 m) rolls without slipping down a helical ramp that descends 4.5 m vertically while the sphere travels 15 m along the ramp surface.

(a) Calculate the moment of inertia of the thick-walled hollow sphere. [**4 marks**]

(b) State the rolling constraint and explain why it must be satisfied. [**3 marks**]

(c) Use energy conservation to find the linear speed at the bottom of the ramp. [**5 marks**]

(d) Calculate the angular velocity at the bottom. [**2 marks**]

(e) If the sphere then rolls onto a horizontal surface and up a 15° incline, find the maximum height it reaches on the incline. [**4 marks**]

(f) Calculate the translational and rotational kinetic energies at the bottom of the helical ramp. [**2 marks**]

(g) Compare the fraction of rotational kinetic energy with that of a solid sphere and a hollow spherical shell. [**2 marks**]

Section F: Angular Momentum [27 marks]

Question 12 [**7 marks**]

(a) Define angular momentum for both point particles and rigid bodies, explaining the vector nature of angular momentum. [**3 marks**]

(b) State and prove the conservation of angular momentum principle from the rotational equation of motion. [**4 marks**]

Question 13 [**20 marks**] A platform diver performs a complex dive, initially in an extended position with moment of inertia 9.5 kg·m^2 and angular velocity 2.8 rad/s about a horizontal axis.

(a) Calculate the initial angular momentum. [**2 marks**]

(b) The diver executes a tight tuck, reducing the moment of inertia to 2.2 kg·m^2. Find the new angular velocity. [**3 marks**]

(c) Calculate the change in rotational kinetic energy and identify the energy source. [**5 marks**]

(d) If the tucking process takes 0.9 seconds, calculate the average power supplied by the diver's muscles during this transition. [**3 marks**]

(e) While in the extended position, the diver is struck by a 1.5 kg object moving horizontally at 12 m/s which embeds in the diver's body at 1.8 m from the rotation axis. Find the resulting angular velocity. [**4 marks**]

(f) Compare the fractional energy losses in parts (b) and (e) and explain the physical reasons for the difference. [**3 marks**]

Section G: Conservation in Rotational Systems [26 marks]

Question 14 [**17 marks**] A massive carousel (uniform disc, mass 500 kg, radius 4 m) rotates at 0.4 rad/s. Five people jump on: 85 kg at 3.5 m from center, 72 kg at 2.8 m, 68 kg at 3.2 m, 79 kg at 2.5 m, and 76 kg at 3.8 m from center.

(a) Calculate the initial angular momentum of the carousel. [**3 marks**]

(b) Find the total moment of inertia after all five people are on the carousel. [**5 marks**]

(c) Determine the final angular velocity using conservation of angular momentum. [**3 marks**]

(d) Calculate the kinetic energy before and after, and explain the mechanism of energy dissipation. [**4 marks**]

(e) If the 85 kg person runs tangentially onto the carousel at 9 m/s while others step on stationary, find the final angular velocity. [**2 marks**]

Question 15 [**9 marks**] Two rotating discs with different characteristics can be coupled together. Disc M (moment of inertia 4.5 kg·m^2) rotates at 18 rad/s, while disc N (moment of inertia 2.8 kg·m^2) rotates at 12 rad/s in the same direction.

(a) Calculate the total angular momentum before coupling. [**2 marks**]

(b) Find the common angular velocity after the discs are connected. [**3 marks**]

(c) Determine the kinetic energy loss and explain the physical processes responsible for this energy dissipation. [**4 marks**]

Section H: Combined Motion [25 marks]

Question 16 [**16 marks**] A uniform solid cylinder (mass 14 kg, radius 0.22 m) with a string wound around it is released from rest and falls vertically while the string unwinds from a fixed support.

(a) Calculate the moment of inertia about the central axis. [**2 marks**]

(b) Apply Newton's second law for both translational and rotational motion to derive the acceleration of the cylinder. [**7 marks**]

(c) Calculate the tension in the string. [**3 marks**]

(d) Express the acceleration as a fraction of gravitational acceleration and explain the physical meaning. [**2 marks**]

(e) If the cylinder falls through 2.5 m, calculate the final linear and angular velocities. [**2 marks**]

Question 17 [**9 marks**] A solid sphere of mass 8 kg and radius 0.3 m rolls down a 22° incline starting with initial linear velocity 6 m/s.

(a) Calculate the initial total kinetic energy. [**3 marks**]

(b) If the sphere rolls 8 m down the incline, find the final linear velocity using energy methods. [**4 marks**]

(c) Calculate the final angular velocity and verify the rolling constraint is satisfied. [**2 marks**]

Section I: Complex Systems [26 marks]

Question 18 [**16 marks**] An advanced pulley system features two coaxial wheels with different radii. The larger wheel (radius 0.9 m) supports a 45 kg mass, while the smaller wheel (radius 0.5 m) supports a 28 kg mass on the opposite side. The combined wheel system has moment of inertia 12 kg·m^2.

(a) Draw detailed free body diagrams for all components of the system. [**4 marks**]

(b) Establish the complete set of dynamic equations for the system. [**5 marks**]

(c) Solve for the angular acceleration of the wheel system. [**4 marks**]

(d) Calculate the tensions in both cables and verify your solution. [**3 marks**]

Question 19 [**10 marks**] A uniform rod of mass 9 kg and length 3.6 m is pivoted at a point 1.5 m from one end. It is released from rest when the longer section makes an angle of 30° below the horizontal.

(a) Calculate the moment of inertia about the pivot point. [**3 marks**]

(b) Find the initial angular acceleration using torque analysis. [**4 marks**]

(c) Use conservation of energy to find the angular velocity when the rod reaches its lowest position. [**3 marks**]

Section J: Advanced Applications [26 marks]

Question 20 [17 marks] A sophisticated satellite system consists of a main cylindrical body (mass 800 kg, radius 2 m, length 6 m) with four solar panel arrays and two communication dishes. Each solar panel (mass 120 kg, dimensions 5 m × 2 m) is located 4 m from the satellite's central axis, and each dish (mass 80 kg, radius 1.5 m) is positioned 3 m from the axis.

(a) Calculate the moment of inertia of the main body about its central axis. **[2 marks]**

(b) Find the moment of inertia contributions from the solar panels and communication dishes. **[6 marks]**

(c) Calculate the total moment of inertia of the satellite system. **[2 marks]**

(d) If the satellite spins at 0.06 rad/s, find its angular momentum. **[2 marks]**

(e) Attitude control thrusters provide a torque of 150 N·m for 50 seconds. Calculate the change in angular velocity. **[2 marks]**

(f) During a collision with space debris, one solar panel and one dish are completely severed. Find the new angular velocity. **[3 marks]**

Question 21 [9 marks] A high-speed lathe chuck (solid disc, mass 35 kg, radius 0.2 m) operates at 450 rpm. When machining a workpiece, the cutting forces create a net friction torque of 25 N·m opposing the rotation.

(a) Convert the operating speed to rad/s and calculate the initial angular momentum. **[3 marks]**

(b) Find the angular deceleration due to the friction torque. **[2 marks]**

(c) Calculate the time required for the chuck speed to decrease to 300 rpm. **[4 marks]**

Physics Data and Formulae

Rotational Kinematics:

$\omega = \frac{d\theta}{dt}$, $\alpha = \frac{d\omega}{dt}$, $v = r\omega$, $a_t = r\alpha$, $a_c = \frac{v^2}{r} = \omega^2 r$

Moment of Inertia:

Point mass: $I = mr^2$

Parallel axis theorem: $I = I_{cm} + md^2$

Uniform rod (center): $I = \frac{1}{12}ML^2$, (end): $I = \frac{1}{3}ML^2$

Solid disc/cylinder: $I = \frac{1}{2}MR^2$, Thin ring: $I = MR^2$

Hollow cylinder: $I = \frac{1}{2}M(R_1^2 + R_2^2)$

Solid sphere: $I = \frac{2}{5}MR^2$, Hollow sphere: $I = \frac{2}{3}MR^2$

Thick hollow sphere: $I = \frac{2}{5}M\frac{R_2^5 - R_1^5}{R_2^3 - R_1^3}$

Rotational Dynamics:

$\tau = I\alpha$ (equation of rotational motion)

$\tau = \vec{r} \times \vec{F}$ (torque from force)

Work: $W = \tau\theta$, Power: $P = \tau\omega$

Rotational Energy:

$KE_{rot} = \frac{1}{2}I\omega^2$

Rolling motion: $KE_{total} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

Rolling constraint: $v = r\omega$ (no slipping)

Angular Momentum:
$L = I\omega$ (rigid body), $L = \vec{r} \times \vec{p}$ (point particle)
Conservation: $L_i = L_f$ (when $\sum \tau_{ext} = 0$)

Constants:
Acceleration due to gravity: $g = 9.8$ m/s²

Trigonometric Values:
$\sin 15 = 0.259$, $\cos 15 = 0.966$, $\tan 15 = 0.268$
$\sin 22 = 0.375$, $\cos 22 = 0.927$, $\tan 22 = 0.404$
$\sin 30 = 0.500$, $\cos 30 = 0.866$, $\tan 30 = 0.577$
$\sin 45 = 0.707$, $\cos 45 = 0.707$, $\tan 45 = 1.000$

END OF TEST

Total marks: 286

Grade boundaries: A* 257, A 229, B 200, C 172, D 143, E 114

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