A Level Mechanics
Practice Test 6: Gravitational Fields

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Draw diagrams where appropriate to illustrate your solutions.
Time allowed: 3 hours 5 minutes

Section A: Gravitational Force Analysis [35 marks]

Question 1 [20 marks] A spacecraft is positioned along the line connecting Earth and Venus during their closest approach, when they are 4.2×10^1 m apart. Venus has mass 4.87×10^2 kg.

(a) Calculate the gravitational force between Earth and Venus at closest approach. [**4 marks**]

(b) Find the distance from Earth's center where the gravitational forces on the spacecraft due to Earth and Venus are equal in magnitude. [**6 marks**]

(c) At this point, calculate the net gravitational force on a 2000 kg spacecraft. [**3 marks**]

(d) Determine the gravitational field strength at this point due to both planets. [**4 marks**]

(e) Calculate the acceleration that the spacecraft would experience at this point. [**3 marks**]

Question 2 [15 marks] Proxima Centauri, the nearest star to our Sun, has mass 2.4×10^2 kg and is located 4.0×10^1 m away.

(a) Calculate the gravitational force between the Sun and Proxima Centauri. [**4 marks**]

(b) Find the gravitational field strength halfway between the Sun and Proxima Centauri. [**6 marks**]

(c) Compare this field strength with Earth's surface gravity. [**3 marks**]

(d) Calculate the gravitational potential at the midpoint due to both stars. [**2 marks**]

Section B: Planetary Gravitational Fields [40 marks]

Question 3 [22 marks] Uranus has mass 8.68×10^2 kg and radius 2.56×10 m. It has a unique feature of rotating on its side.

(a) Calculate the gravitational field strength on Uranus' surface. [**4 marks**]

(b) Find the gravitational field strength at a height of 5000 km above Uranus' surface. [**4 marks**]

(c) A probe with mass 1200 kg orbits Uranus at this height. Calculate its weight in this orbit. [**3 marks**]

(d) Determine the height above Uranus' surface where the gravitational field strength is 15% of its surface value. [**6 marks**]

(e) Calculate the ratio of gravitational field strengths: Uranus surface to Earth surface. [**3 marks**]

(f) Explain why Uranus' unusual rotation does not significantly affect its gravitational field. [**2 marks**]

Question 4 [**18 marks**]

(a) For a uniform solid sphere, derive the expression for gravitational field strength as a function of distance from the center, both inside and outside the sphere. [**8 marks**]

(b) Earth's core has radius 3.5×10 m and density 12,000 kg/m^3. The mantle extends to Earth's surface with average density 4,500 kg/m^3. Calculate:

  (i) The gravitational field strength at the core-mantle boundary. [**5 marks**]

  (ii) The gravitational field strength at a depth of 1000 km below Earth's surface. [**5 marks**]

Section C: Gravitational Potential and Energy [45 marks]

Question 5 [**20 marks**]

(a) Explain the concept of gravitational potential and its relationship to gravitational potential energy. [**4 marks**]

(b) Derive the expression V = -GM/r for gravitational potential, starting from the definition of potential. [**6 marks**]

(c) Show that the work done in moving a mass m from distance r to distance r in a gravitational field is W = m(V - V). [**4 marks**]

(d) Calculate the gravitational potential at Earth's surface and at the altitude of the International Space Station (408 km). [**6 marks**]

Question 6 [**25 marks**] A Mars mission involves sending a 8000 kg spacecraft from Earth to Mars when the planets are aligned and separated by 5.5×10^1 m.

(a) Calculate the gravitational potential energy of the spacecraft at Earth's surface. [**3 marks**]

(b) Find the gravitational potential energy of the spacecraft at Mars' surface. [**4 marks**]

(c) Determine the change in potential energy for the complete Earth-to-Mars surface journey. [**3 marks**]

(d) Calculate the minimum velocity required to launch the spacecraft from Earth's surface to reach Mars' surface. [**8 marks**]

(e) If the spacecraft actually arrives at Mars with velocity 2.8 km/s, determine its launch velocity from Earth. [**4 marks**]

(f) Calculate the kinetic energy gained by the spacecraft due to Mars' gravitational field during final approach. [**3 marks**]

Section D: Orbital Mechanics and Satellite Systems [50 marks]

Question 7 [25 marks] A commercial space company plans to deploy a constellation of communication satellites.

(a) Calculate the orbital parameters for satellites in three different orbits:

(i) Low Earth Orbit at 550 km altitude: velocity and period. [**4 marks**]

(ii) Medium Earth Orbit at 12,000 km altitude: velocity and period. [**4 marks**]

(iii) Geostationary orbit: altitude, velocity, and confirm 24-hour period. [**5 marks**]

(b) For a 1500 kg satellite, calculate the total energy in each of these orbits. [**6 marks**]

(c) Determine the energy required to transfer the satellite from LEO to GEO. [**4 marks**]

(d) Explain why the geostationary orbit is particularly useful for telecommunications. [**2 marks**]

Question 8 [25 marks]

(a) For circular orbits, derive the relationships between orbital radius, velocity, period, and total energy. [**8 marks**]

(b) Show that for any circular orbit, the kinetic energy is exactly half the magnitude of the potential energy. [**5 marks**]

(c) A satellite experiences orbital decay due to atmospheric drag, causing it to spiral inward. Explain what happens to its:

(i) Orbital velocity as radius decreases. [**3 marks**]

(ii) Total energy during the decay process. [**3 marks**]

(iii) Orbital period as it spirals inward. [**3 marks**]

(d) Calculate how the orbital velocity changes when a satellite's orbital radius decreases from 7000 km to 6800 km due to atmospheric drag. [**3 marks**]

Section E: Planetary Motion and Exoplanets [35 marks]

Question 9 [20 marks] The exoplanet HD 209458b orbits its star at a distance of 7.0×10 m with a period of 3.52 days. The star has a similar mass to our Sun.

(a) Use Kepler's third law to determine the mass of the star HD 209458. [**6 marks**]

(b) Calculate the orbital velocity of HD 209458b. [**4 marks**]

(c) Compare this orbital velocity with Earth's orbital velocity around the Sun. [**3 marks**]

(d) If this exoplanet has mass 1.3 times Jupiter's mass (1.90×10^2 kg), calculate its orbital kinetic energy. [**4 marks**]

(e) Determine the gravitational binding energy of this planet to its star. [**3 marks**]

Question 10 [15 marks] The TRAPPIST-1 system contains seven Earth-sized planets. Planet TRAPPIST-1e orbits at 2.9×10 m with a period of 6.1 days.

(a) Calculate the mass of the TRAPPIST-1 star using planet e's orbital data. [**5 marks**]

(b) Planet TRAPPIST-1f orbits at 3.7×10 m. Predict its orbital period using Kepler's third law. [**5 marks**]

(c) Compare the orbital velocities of planets e and f. [**3 marks**]

(d) Explain why this system is of particular interest to astronomers searching for life. [**2 marks**]

Section F: Advanced Gravitational Applications [20 marks]

Question 11 [**12 marks**] Tidal forces arise from gravitational field gradients and are important in many astronomical phenomena.

(a) Explain the physical origin of tidal forces in terms of gravitational field variations. [**3 marks**]

(b) The International Space Station is 109 m long. Calculate the difference in gravitational field strength between its Earth-facing and space-facing ends when orbiting at 408 km altitude. [**5 marks**]

(c) Explain why tidal forces are more significant for larger objects and closer orbits. [**2 marks**]

(d) Describe how tidal forces affect the Moon's orbit around Earth and Earth's rotation. [**2 marks**]

Question 12 [**8 marks**] Binary pulsar systems provide precise tests of gravitational theory and have confirmed the existence of gravitational waves.

(a) Two neutron stars, each with mass 1.4×10^3 kg, orbit their common center of mass at separation 2.0×10 m. Calculate their orbital period. [**5 marks**]

(b) Explain how energy loss through gravitational wave emission affects the orbital period of such a system. [**3 marks**]

Physics Data and Formulae

Gravitational Force and Field:

Newton's Law: $F = \frac{Gm_1m_2}{r^2}$

Field strength: $g = \frac{F}{m} = \frac{GM}{r^2}$

Field inside uniform sphere: $g = \frac{GMr}{R^3}$ (for $r \leq R$)

Field outside uniform sphere: $g = \frac{GM}{r^2}$ (for $r \geq R$)

Gravitational Potential and Energy:

Potential: $V = -\frac{GM}{r}$

Potential energy: $U = mV = -\frac{GMm}{r}$

Work done: $W = m(V_2 - V_1) = \Delta U$

Field-potential relation: $g = -\frac{dV}{dr}$

Orbital Motion:

Circular orbital velocity: $v = \sqrt{\frac{GM}{r}}$

Orbital period: $T = 2\pi\sqrt{\frac{r^3}{GM}}$

Kepler's Third Law: $\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$

Escape velocity: $v_e = \sqrt{\frac{2GM}{r}}$

Energy in Circular Orbits:

Kinetic energy: $E_k = \frac{1}{2}mv^2 = \frac{GMm}{2r}$

Potential energy: $E_p = -\frac{GMm}{r}$

Total energy: $E = E_k + E_p = -\frac{GMm}{2r}$

Binding energy: $E_b = \frac{GMm}{2r}$

Circular Motion:

Centripetal force: $F_c = \frac{mv^2}{r}$

Centripetal acceleration: $a_c = \frac{v^2}{r}$

Angular velocity: $\omega = \frac{2\pi}{T} = \frac{v}{r}$

Physical Constants:

Gravitational constant: $G = 6.67 \times 10^{-11}$ Nm²/kg²
Earth's mass: $M_E = 5.97 \times 10^{24}$ kg
Earth's radius: $R_E = 6.37 \times 10^6$ m
Mars' mass: $M_M = 6.39 \times 10^{23}$ kg
Mars' radius: $R_M = 3.39 \times 10^6$ m
Jupiter's mass: $M_J = 1.90 \times 10^{27}$ kg
Sun's mass: $M_S = 1.99 \times 10^{30}$ kg
Earth-Sun distance: 1.50×10^{11} m (1 AU)
Standard gravity: $g = 9.81$ m/s²

END OF TEST

Total marks: 225

Grade boundaries: A* 203, A 180, B 158, C 135, D 113, E 90

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