

GCSE Higher Mathematics

Practice Test 9: Further Algebra

Instructions:

Answer all questions. Show your working clearly.

Calculators may be used unless stated otherwise.

Time allowed: 90 minutes

Section A: Function Notation and Evaluation

1. Given $f(x) = 8x - 9$ and $g(x) = x^2 - 7x$, find:

- (a) $f(5)$
- (b) $g(-4)$
- (c) $f(0)$
- (d) $g(3a)$
- (e) $f(x - 4)$
- (f) $g(x + 7)$

2. For the function $h(x) = 4x^2 - 9x + 6$, calculate:

- (a) $h(3)$
- (b) $h(-1)$
- (c) $h(c - 2)$
- (d) $h(5t)$
- (e) The value(s) of x when $h(x) = 15$
- (f) The value(s) of x when $h(x) = 0$

3. Given $f(x) = \frac{7x-2}{x+5}$ where $x \neq -5$:

- (a) Find $f(3)$
- (b) Find $f(-1)$
- (c) For what value of x is $f(x) = 4$?
- (d) For what value of x is $f(x) = 0$?
- (e) Explain why $x = -5$ is excluded from the domain
- (f) Find the range of values that $f(x)$ cannot take

4. A function is defined as $p(x) = x^3 - 12x + 16$.

- (a) Calculate $p(0)$, $p(1)$, $p(2)$, and $p(-3)$
- (b) Use your results to sketch the graph of $y = p(x)$
- (c) Estimate the roots of $p(x) = 0$
- (d) For what values of k does $p(x) = k$ have three real solutions?

Section B: Composite Functions

5. Given $f(x) = 5x - 8$ and $g(x) = x^2 + 4$, find:

- (a) $f(g(1))$
- (b) $g(f(1))$
- (c) $f(g(x))$
- (d) $g(f(x))$
- (e) $(f \circ g)(x)$
- (f) $(g \circ f)(x)$

6. For $h(x) = 7x + 12$ and $k(x) = \frac{x-12}{7}$:

- (a) Find $h(k(x))$
- (b) Find $k(h(x))$
- (c) What do you notice about your answers?
- (d) Verify that $h(k(26)) = 26$
- (e) Explain the relationship between functions h and k

7. Given $f(x) = x^2 - 7$ and $g(x) = \sqrt{x+7}$ where $x \geq -7$:

- (a) Find the domain of $g(x)$
- (b) Calculate $f(g(9))$
- (c) Calculate $g(f(4))$
- (d) Find $f(g(x))$ and simplify
- (e) Find $g(f(x))$ and state its domain
- (f) Solve $f(g(x)) = 25$

8. If $f(x) = x - 8$, $g(x) = 6x$, and $h(x) = x^2$:

- (a) Find $f(g(h(x)))$
- (b) Find $h(g(f(x)))$
- (c) Find $g(h(f(x)))$
- (d) Calculate $f(g(h(3)))$
- (e) Solve $g(h(f(x))) = 294$

Section C: Inverse Functions

9. Find the inverse function for each of the following:

- (a) $f(x) = 4x + 9$
- (b) $g(x) = \frac{x-7}{3}$
- (c) $h(x) = 8x - 15$
- (d) $k(x) = \frac{9x+1}{6}$

10. For the function $f(x) = \frac{5x+7}{x-3}$ where $x \neq 3$:

- (a) Find $f^{-1}(x)$
- (b) State the domain and range of $f^{-1}(x)$
- (c) Verify that $f(f^{-1}(x)) = x$

- (d) Verify that $f^{-1}(f(x)) = x$
 - (e) Solve $f(x) = f^{-1}(x)$
11. Given $g(x) = x^2 - 8$ for $x \geq 0$:
- (a) Explain why the domain restriction is necessary
 - (b) Find $g^{-1}(x)$
 - (c) State the domain and range of $g^{-1}(x)$
 - (d) Sketch both $g(x)$ and $g^{-1}(x)$ on the same axes
 - (e) Find the point of intersection of $y = g(x)$ and $y = g^{-1}(x)$
12. A function f has the property that $f(4) = 25$, $f(9) = 40$, and $f(x) = 3x + 13$.
- (a) Verify that the given points satisfy $f(x) = 3x + 13$
 - (b) Find $f^{-1}(x)$
 - (c) Calculate $f^{-1}(25)$ and $f^{-1}(40)$
 - (d) What do you notice about these values?
 - (e) If $f(a) = b$, what is $f^{-1}(b)$?

Section D: Function Transformations

13. Given the function $f(x) = x^2$, describe the transformation and sketch:
- (a) $y = f(x) + 12$
 - (b) $y = f(x) - 8$
 - (c) $y = f(x + 9)$
 - (d) $y = f(x - 6)$
 - (e) $y = 7f(x)$
 - (f) $y = \frac{1}{4}f(x)$
14. The graph of $y = f(x)$ passes through the points $(0, 9)$, $(4, 12)$, and $(7, 3)$. Find the coordinates of these points on:
- (a) $y = f(x) + 8$
 - (b) $y = f(x - 6)$
 - (c) $y = 4f(x)$
 - (d) $y = f(3x)$
 - (e) $y = -f(x)$
 - (f) $y = f(-x)$
15. Given $f(x) = (x + 4)^2 - 5$:
- (a) Describe the transformations applied to $y = x^2$
 - (b) State the vertex of the parabola
 - (c) Find $f(x - 6)$ and describe its transformation
 - (d) Find $3f(x) + 7$ and describe its transformation
 - (e) Sketch all four graphs on the same axes
16. The function $g(x) = |x|$ is transformed to $h(x) = 6|x - 7| + 3$.
- (a) Describe each transformation step by step
 - (b) State the vertex of $h(x)$
 - (c) Find the range of $h(x)$
 - (d) Solve $h(x) = 21$
 - (e) Sketch both $g(x)$ and $h(x)$

Section E: Exponential Functions - Basics

17. Evaluate these exponential expressions:

- (a) 6^3
- (b) 4^{-3}
- (c) $81^{0.5}$
- (d) $125^{-2/3}$
- (e) $(\frac{1}{6})^{-2}$
- (f) $243^{\frac{2}{5}}$

18. Sketch the graphs of these exponential functions:

- (a) $y = 6^x$
- (b) $y = 4^x$
- (c) $y = (\frac{1}{6})^x$
- (d) $y = (\frac{1}{4})^x$
- (e) $y = 6^x + 5$
- (f) $y = 6^{x-4}$

19. For the function $f(x) = 6^x$:

- (a) Calculate $f(0)$, $f(1)$, $f(2)$, $f(-1)$, $f(-2)$
- (b) State the domain and range of $f(x)$
- (c) Find the y-intercept
- (d) Describe the behavior as $x \rightarrow \infty$ and $x \rightarrow -\infty$
- (e) Solve $6^x = 216$
- (f) Solve $6^x = \frac{1}{36}$

20. Compare the graphs of $y = 6^x$ and $y = (\frac{1}{6})^x$:

- (a) What transformation relates these functions?
- (b) Where do they intersect?
- (c) Which grows faster for $x > 0$?
- (d) Which approaches zero faster as $x \rightarrow \infty$?
- (e) Express $(\frac{1}{6})^x$ in the form $6^{g(x)}$

Section F: Exponential Growth and Decay

21. A virus population quadruples every 40 minutes. Initially, there are 600 viruses.

- (a) Write a function $V(t)$ for the population after t minutes
- (b) Calculate the population after 80 minutes
- (c) Calculate the population after 4 hours
- (d) When will the population reach 153600?
- (e) What is the growth rate per minute?
- (f) How long for the population to increase by 2300%?

22. A radioactive element has a half-life of 15 years. Initially, there are 960g of the element.

- (a) Write a function $A(t)$ for the amount after t years
 - (b) How much remains after 30 years?
 - (c) How much remains after 45 years?
 - (d) When will only 15g remain?
 - (e) What percentage remains after one half-life?
 - (f) Calculate the decay rate per year
23. An investment of £35000 grows at 7.2% per year compound interest.
- (a) Write a function $V(t)$ for the value after t years
 - (b) Calculate the value after 5 years
 - (c) Calculate the value after 10 years
 - (d) When will the investment double?
 - (e) When will it reach £85000?
 - (f) Compare with simple interest of 7.2% per year
24. The temperature of cooling soup follows Newton's law of cooling: $T(t) = 22 + 58e^{-0.22t}$ where T is temperature in °C and t is time in minutes.
- (a) What is the initial temperature?
 - (b) What is the room temperature?
 - (c) Find the temperature after 6 minutes
 - (d) When will the temperature be 32°C?
 - (e) Sketch the graph of $T(t)$
 - (f) What happens as $t \rightarrow \infty$?

Section G: Advanced Exponential Applications

25. A laptop depreciates in value according to $V(t) = 24000 \times 0.68^t$ where V is value in pounds and t is age in years.
- (a) What was the original value?
 - (b) What is the annual depreciation rate?
 - (c) Calculate the value after 4 years
 - (d) When will the laptop be worth £6000?
 - (e) After how many years will it lose half its value?
 - (f) What percentage of value is retained each year?
26. The spread of a meme follows $N(t) = 380 \times 2.3^t$ where N is shares (in thousands) and t is days since posting.
- (a) How many shares after 1 day?
 - (b) How many shares after 4 days?
 - (c) When will it reach 32 million shares?
 - (d) What is the daily growth rate?
 - (e) If the growth rate drops to 60% per day after 2 days, model the new function
27. A lake's fish population decreases due to pollution. The population P (in thousands) after t years is $P(t) = 15000 \times 0.85^t$.

- (a) What is the initial fish population?
 - (b) What percentage is lost each year?
 - (c) Calculate the population after 8 years
 - (d) When will half the fish population be gone?
 - (e) If cleanup efforts reduce the loss to 6% per year, how does this change the model?
 - (f) Compare the populations after 20 years under both scenarios
28. An antibiotic concentration in bloodstream follows $C(t) = 180e^{-0.32t}$ where C is concentration (mg/L) and t is hours after injection.
- (a) What is the initial concentration?
 - (b) Find the concentration after 1.5 hours
 - (c) When will the concentration drop to 40 mg/L?
 - (d) What is the half-life of the antibiotic?
 - (e) A second injection should be given when concentration drops to 18 mg/L. When should this be?
 - (f) Sketch the concentration curve

Section H: Problem Solving and Integration

29. A function f is defined by $f(x) = ax + b$ where a and b are constants. Given that $f(7) = 31$ and $f(-2) = 4$:
- (a) Find the values of a and b
 - (b) Write down $f(x)$
 - (c) Find $f^{-1}(x)$
 - (d) Solve $f(x) = f^{-1}(x)$
 - (e) If $g(x) = x^2$, find $f(g(x))$ and $g(f(x))$
30. Two exponential functions $p(x) = 6^x$ and $q(x) = 4^x$ intersect at the point where $x = 0$.
- (a) Verify this intersection point
 - (b) For what values of x is $p(x) > q(x)$?
 - (c) Find the function $r(x) = \frac{q(x)}{p(x)}$
 - (d) Simplify $r(x)$ and identify what type of function it is
 - (e) Sketch all three functions on the same axes
31. A population model combines growth and limiting factors: $P(t) = \frac{3200}{1+31e^{-0.8t}}$ where P is population and t is time in years.
- (a) Find the initial population $P(0)$
 - (b) Calculate $P(1)$ and $P(2)$
 - (c) What happens to $P(t)$ as $t \rightarrow \infty$?
 - (d) When will the population reach 1600?
 - (e) Sketch the graph and describe its shape
 - (f) How does this differ from unlimited exponential growth?
32. A transformation maps the function $f(x) = 6^x$ to $g(x) = 9 \times 6^{x-2} + 5$.
- (a) Identify each transformation in the correct order

- (b) Find the y-intercept of $g(x)$
 - (c) Find the horizontal asymptote of $g(x)$
 - (d) Solve $g(x) = 59$
 - (e) Find $g^{-1}(x)$
 - (f) Verify that $g(g^{-1}(59)) = 59$
33. A savings account earns compound interest. After 1 year, £8000 becomes £8560. After 2 years, it becomes £9159.20.
- (a) Verify this follows exponential growth
 - (b) Find the annual interest rate
 - (c) Write the exponential function $A(t)$ for any initial amount P
 - (d) How long to triple an investment?
 - (e) Compare with quarterly compounding at the same annual rate
 - (f) What continuous compound rate gives the same result?
34. Design a real-world scenario that can be modeled by an exponential function:
- (a) Describe your scenario clearly
 - (b) Define variables and state assumptions
 - (c) Write the exponential function
 - (d) Calculate specific values and time periods
 - (e) Discuss limitations of the model
 - (f) Suggest modifications for greater realism

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 100

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