

GCSE Higher Mathematics

Practice Test 8: Number

Instructions:

Answer all questions. Show your working clearly.

Calculators may be used unless stated otherwise.

Time allowed: 90 minutes

Section A: Powers and Roots

1. Evaluate these expressions:

- (a) 11^2
- (b) 6^{-3}
- (c) 23^0
- (d) $(-11)^2$
- (e) $(-6)^3$
- (f) 15^{-2}

2. Write these as single powers:

- (a) $10^7 \times 10^{12}$
- (b) $18^{11} \div 18^5$
- (c) $(7^9)^6$
- (d) $13^{-2} \times 13^{14}$
- (e) $\frac{19^{10}}{19^{-9}}$
- (f) $(11^7)^{-4}$

3. Evaluate these fractional indices:

- (a) $169^{\frac{1}{2}}$
- (b) $729^{\frac{1}{3}}$
- (c) $1048576^{\frac{3}{4}}$
- (d) $128000000^{\frac{2}{7}}$
- (e) $4913^{\frac{2}{3}}$
- (f) $4096^{-\frac{2}{3}}$

4. Simplify these expressions:

- (a) $\sqrt{289}$
- (b) $\sqrt[3]{1331}$

- (c) $\sqrt[4]{6561}$
- (d) $\sqrt{3.24}$
- (e) $\sqrt[3]{-1000}$
- (f) $\sqrt[5]{248832}$

5. Express in index form:

- (a) $\sqrt{31}$
- (b) $\sqrt[3]{23}$
- (c) $\frac{1}{\sqrt{19}}$
- (d) $\sqrt[4]{j^{17}}$
- (e) $\frac{1}{\sqrt[3]{k^{13}}}$
- (f) $\sqrt{j} \times \sqrt[3]{j}$

Section B: Laws of Indices

6. Simplify these expressions (no calculator):

- (a) $9^8 \times 9^{-11} \times 9^{12}$
- (b) $\frac{14^{13} \times 14^{-9}}{14^{-8}}$
- (c) $(11^6)^{-8} \times 11^{22}$
- (d) $\frac{15^{-9} \times 15^{19}}{15^8}$

7. Evaluate these expressions:

- (a) $4913^{\frac{2}{3}}$
- (b) $1048576^{-\frac{3}{4}}$
- (c) $1000000^{\frac{3}{6}}$
- (d) $1048576^{-\frac{5}{20}}$
- (e) $13824^{\frac{1}{3}}$
- (f) $10000000^{-\frac{3}{7}}$

8. Simplify these expressions:

- (a) $x^{\frac{10}{11}} \times x^{\frac{1}{6}}$
- (b) $\frac{a^{\frac{12}{5}}}{a^{\frac{1}{7}}}$
- (c) $(k^{\frac{1}{9}})^{18}$
- (d) $\sqrt{j} \times j^{\frac{1}{10}}$
- (e) $\frac{\sqrt[3]{b^{13}}}{\sqrt{b}}$
- (f) $(z^{-\frac{1}{9}})^{-18}$

9. Write these in the form a^n where a and n are rational:

- (a) $\sqrt{23} \times 23^9$
- (b) $\frac{29^7}{\sqrt[3]{29}}$
- (c) $\sqrt[4]{16^{17}} \times 16^{-\frac{6}{7}}$
- (d) $\frac{\sqrt{31}}{\sqrt[3]{31^{13}}}$

Section C: Surds

10. Simplify these surds:

- (a) $\sqrt{104}$
- (b) $\sqrt{156}$
- (c) $\sqrt{171}$
- (d) $\sqrt{294}$
- (e) $\sqrt{320}$
- (f) $\sqrt{1100}$

11. Simplify these expressions:

- (a) $11\sqrt{23} + 17\sqrt{23}$
- (b) $21\sqrt{9} - 9\sqrt{9}$
- (c) $\sqrt{76} + \sqrt{171}$
- (d) $\sqrt{294} - \sqrt{216}$
- (e) $9\sqrt{32} + 13\sqrt{72}$
- (f) $\sqrt{294} - \sqrt{176} + \sqrt{50}$

12. Multiply and simplify:

- (a) $\sqrt{15} \times \sqrt{60}$
- (b) $\sqrt{32} \times \sqrt{128}$
- (c) $11\sqrt{7} \times 8\sqrt{28}$
- (d) $\sqrt{22} \times \sqrt{88}$
- (e) $\sqrt{10} \times \sqrt{40} \times \sqrt{160}$
- (f) $13\sqrt{9} \times 8\sqrt{36}$

13. Expand and simplify:

- (a) $(9 + \sqrt{19})(6 - \sqrt{19})$
- (b) $(8 + \sqrt{29})(5 + 9\sqrt{29})$
- (c) $(11 - \sqrt{23})^2$
- (d) $(\sqrt{31} + \sqrt{13})(\sqrt{31} - \sqrt{13})$
- (e) $(9\sqrt{19} + 1)(9\sqrt{19} - 1)$
- (f) $(\sqrt{29} + 9)^2$

14. Rationalize the denominators:

- (a) $\frac{1}{\sqrt{29}}$
- (b) $\frac{19}{\sqrt{31}}$
- (c) $\frac{\sqrt{19}}{\sqrt{76}}$
- (d) $\frac{18}{9\sqrt{2}}$
- (e) $\frac{1}{8+\sqrt{19}}$
- (f) $\frac{13}{1-\sqrt{29}}$

Section D: More Complex Surd Operations

15. Rationalize these denominators:

(a) $\frac{15}{9+\sqrt{23}}$

(b) $\frac{23}{8-\sqrt{37}}$

(c) $\frac{\sqrt{19}}{1+\sqrt{19}}$

(d) $\frac{9\sqrt{29}}{8+\sqrt{29}}$

(e) $\frac{1}{\sqrt{27}-\sqrt{19}}$

(f) $\frac{\sqrt{29+8}}{\sqrt{29-9}}$

16. Simplify these expressions completely:

(a) $\frac{\sqrt{40}+\sqrt{90}}{\sqrt{10}}$

(b) $\frac{\sqrt{108}-\sqrt{80}}{\sqrt{4}}$

(c) $\sqrt{(10+\sqrt{29})(10-\sqrt{29})}$

(d) $\sqrt{171}-9\sqrt{19}+\sqrt{112}$

(e) $(\sqrt{19}+\sqrt{76})^2$

(f) $\frac{\sqrt{156}}{\sqrt{19}}+\frac{\sqrt{114}}{\sqrt{19}}$

17. Prove that:

(a) $(\sqrt{l}+\sqrt{m})(\sqrt{l}-\sqrt{m})=l-m$

(b) $\frac{1}{\sqrt{p}+\sqrt{q}}=\frac{\sqrt{p}-\sqrt{q}}{p-q}$

(c) $(p+q\sqrt{z})^2=p^2+2pq\sqrt{z}+q^2z$

Section E: Standard Form

18. Write these numbers in standard form:

(a) 1034000

(b) 0.000139

(c) 8910000000

(d) 0.0000000628

(e) 1367.2

(f) 0.01395

19. Write these in ordinary form:

(a) 1.19×10^{-1}

(b) 9.13×10^{-11}

(c) 1.3654×10^{-14}

(d) 8.71×10^{17}

(e) 1.39×10^{-9}

(f) 1.034×10^{12}

20. Calculate, giving answers in standard form:

- (a) $(13 \times 10^{11}) \times (15 \times 10^{13})$
- (b) $(9 \times 10^{-9}) \times (17 \times 10^{14})$
- (c) $(27 \times 10^{12}) \div (9 \times 10^{-7})$
- (d) $(24 \times 10^{-11}) \div (16 \times 10^{-14})$
- (e) $(13 \times 10^{10})^2$
- (f) $\sqrt{100 \times 10^{22}}$

21. Calculate these more complex expressions:

- (a) $(9.6 \times 10^{10}) \times (3.75 \times 10^{-12})$
- (b) $\frac{19.2 \times 10^{13}}{4.8 \times 10^{-10}}$
- (c) $(10.3 \times 10^{-9}) + (1.17 \times 10^{-8})$
- (d) $(13.4 \times 10^{12}) - (9.7 \times 10^{11})$
- (e) $\frac{(9.3 \times 10^8) \times (8.1 \times 10^{-9})}{13.95 \times 10^{-7}}$
- (f) $(1.1881 \times 10^{20})^{\frac{1}{2}}$

Section F: Rational Numbers and Operations

22. Calculate these fractions (give answers in simplest form):

- (a) $\frac{9}{17} + \frac{13}{34}$
- (b) $\frac{23}{28} - \frac{15}{42}$
- (c) $\frac{20}{21} \times \frac{42}{30}$
- (d) $\frac{18}{35} \div \frac{27}{45}$
- (e) $\frac{15}{16} - \frac{11}{24} + \frac{19}{48}$
- (f) $(\frac{9}{17})^{-2}$

23. Convert these recurring decimals to fractions:

- (a) $0.\overline{1}$
- (b) $0.\overline{73}$
- (c) $0.8\overline{4}$
- (d) $0.\overline{714285}$
- (e) $7.1\overline{6}$
- (f) $0.73\overline{1}$

24. Work out these percentage calculations:

- (a) Increase 960 by 85%
- (b) Decrease 1200 by 42%
- (c) Find 47.5% of 1360
- (d) What percentage is 189 out of 252?
- (e) If 105% of a number is 273, find the number
- (f) A price increases from £150 to £165. Find the percentage increase

25. Solve these percentage problems:

- (a) After a 90% increase, a price is £361. Find the original price
- (b) After a 85% decrease, a quantity is 162. Find the original quantity
- (c) The value of a car decreases by 55% each year. If it's worth £6075 now, what was it worth 2 years ago?
- (d) An investment grows by 13% per year. After 2 years it's worth £2560.07. Find the initial investment

Section G: Complex Calculations

26. Simplify these mixed expressions:

(a) $9^{-2} + 15^0 - 16^{-1}$

(b) $\sqrt{121} \times 1331^{\frac{1}{3}} - 9^{-2}$

(c) $\frac{4913^{\frac{2}{3}} - 1048576^{\frac{3}{4}}}{144^{\frac{1}{2}}}$

(d) $81^{-\frac{1}{2}} + 144^{\frac{1}{2}} \times 8^{-1}$

27. Calculate exactly (leave surds in your answer):

(a) $\frac{15}{\sqrt{19}} + \frac{11}{\sqrt{76}}$

(b) $\sqrt{72} \times \sqrt{128} - \sqrt{288}$

(c) $\frac{\sqrt{245} + \sqrt{176}}{\sqrt{49}}$

(d) $(9\sqrt{2} - 8)^2$

28. Work with standard form in context:

(a) The mass of a helium atom is 6.646×10^{-27} kg. Find the mass of 6.02×10^{23} helium atoms

(b) X-rays travel at 3×10^8 m/s. How far do they travel in one decade (use 1 decade = 3.154×10^8 seconds)?

(c) The width of a DNA strand is approximately 2.5×10^{-9} m. How many DNA strands would fit across a distance of 6 mm?

(d) A quantum processor handles 5.76×10^{16} operations per second. How many operations in 45 minutes?

Section H: Problem Solving

29. Prove that $\sqrt{19}$ is irrational. (Use proof by contradiction: assume $\sqrt{19} = \frac{w}{z}$ where w and z are integers with no common factors)

30. The number ζ satisfies $\zeta^2 = 7\zeta - 5$.

(a) Show that $\zeta = \frac{7+\sqrt{29}}{2}$

(b) Calculate ζ to 4 decimal places

(c) Find $\frac{1}{\zeta}$ in surd form

31. Rationalize the denominator of $\frac{1}{\sqrt{17} + \sqrt{19} + \sqrt{23}}$. (Hint: First rationalize using $(\sqrt{17} + \sqrt{19}) - \sqrt{23}$)

32. A rectangle has sides of length $(9 + \sqrt{19})$ cm and $(9 - \sqrt{19})$ cm.

(a) Find the exact area

(b) Find the exact perimeter

(c) Show that the area is rational but the perimeter is irrational

33. The population of spores nonuples every 14 hours. If there are initially 13×10^2 spores:

(a) How many spores after 56 hours?

(b) Express your answer in standard form

(c) After how many hours will there be more than 1×10^{12} spores?

34. Show that $\frac{1}{\sqrt{l} + \sqrt{m}} + \frac{1}{\sqrt{l} - \sqrt{m}} = \frac{2\sqrt{l}}{l - m}$

35. A cone has volume $V = \frac{1}{3}\pi r^2 h$. If the volume is $800\pi \text{ cm}^3$ and $h = 24 \text{ cm}$:

- (a) Find the radius in surd form
- (b) Find the slant height (use $l^2 = r^2 + h^2$)
- (c) Express both answers exactly

36. The equation $x^2 - 18x + 1 = 0$ has solutions $x = 9 \pm 4\sqrt{5}$.

- (a) Verify this by substitution
- (b) Find $\frac{1}{9+4\sqrt{5}} + \frac{1}{9-4\sqrt{5}}$ without using a calculator
- (c) Hence find the sum of the reciprocals of the roots

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 100

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