

GCSE Higher Mathematics

Practice Test 4: Probability

Instructions:

Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.

Time allowed: 90 minutes

Section A: Conditional Probability Fundamentals

1. A survey of 160 students shows:
 - 105 study History
 - 80 study Geography
 - 50 study both History and Geography
 - (a) Draw a Venn diagram
 - (b) Find $P(\text{studies History} \mid \text{studies Geography})$
 - (c) Find $P(\text{studies Geography} \mid \text{studies History})$
 - (d) Find $P(\text{studies exactly one subject})$
 - (e) Are studying History and Geography independent? Justify your answer
2. A box contains 7 silver balls and 13 gold balls. Two balls are drawn without replacement.
 - (a) Find $P(\text{second ball is silver} \mid \text{first ball is silver})$
 - (b) Find $P(\text{second ball is gold} \mid \text{first ball is silver})$
 - (c) Find $P(\text{both balls are the same color})$
 - (d) Find $P(\text{balls are different colors})$
 - (e) Verify that your probabilities sum to 1
3. Events G and H are such that:
 - $P(G) = 0.65$
 - $P(H) = 0.35$
 - $P(G \cap H) = 0.22$

Calculate:

- (a) $P(G \cap H)$
- (b) $P(G')$
- (c) $P(G \cup H)$
- (d) $P(H \cap G)$

- (e) $P(G' \cap H')$
(f) $P(G \cap H')$
4. A card is drawn from a standard 52-card deck. Let G = "card is a club" and H = "card is a Queen".
- (a) Find $P(G)$, $P(H)$, and $P(G \cap H)$
(b) Calculate $P(G \cap H)$
(c) Calculate $P(H \cap G)$
(d) Are events G and H independent? Show your working
(e) Find $P(G' \cap H')$

Section B: Tree Diagrams and Sequential Events

5. A bag contains 6 white counters and 9 black counters. A counter is drawn, its color noted, and then replaced. This process is repeated twice more.
- (a) Draw a tree diagram for all three draws
(b) Find $P(\text{all three counters are white})$
(c) Find $P(\text{exactly two counters are black})$
(d) Find $P(\text{at least one counter is white})$
(e) Find $P(\text{first counter is black} \cap \text{exactly two are black})$
6. Box P contains 4 red balls and 8 white balls. Box Q contains 9 red balls and 3 white balls. A fair coin is flipped to choose a box, then a ball is drawn.
- (a) Draw a tree diagram
(b) Find $P(\text{red ball})$
(c) Find $P(\text{white ball})$
(d) Given a red ball was drawn, find $P(\text{it came from Box P})$
(e) Given a white ball was drawn, find $P(\text{it came from Box Q})$
7. Three machines produce components with different defect rates:
- Machine S: produces 55% of components, 3% defective
 - Machine T: produces 25% of components, 8% defective
 - Machine U: produces 20% of components, 12% defective
- (a) Find the overall probability of a defective component
(b) If a component is defective, find the probability it came from Machine S
(c) If a component is defective, find the probability it came from Machine U
(d) If a component is not defective, which machine most likely produced it?
8. A student takes three tests. The probability of passing each test is 0.72, and the tests are independent.
- (a) Find $P(\text{passes all three tests})$
(b) Find $P(\text{fails all three tests})$
(c) Find $P(\text{passes exactly two tests})$
(d) Find $P(\text{passes at least one test})$
(e) Given the student passed at least two tests, find $P(\text{passed all three})$

Section C: Bayes' Theorem Applications

9. A medical test for a condition has the following characteristics:

- If a person has the condition, the test is positive 87% of the time
- If a person doesn't have the condition, the test is negative 97% of the time
- 1.5% of the population has the condition

- (a) Find $P(\text{positive test})$
- (b) If someone tests positive, find $P(\text{they have the condition})$
- (c) If someone tests negative, find $P(\text{they don't have the condition})$
- (d) Comment on the reliability of a positive test result
- (e) How would the results change if 15% of the population had the condition?

10. A security system has three sensors. The probability each sensor detects an intruder is:

- Sensor S: 0.89
- Sensor T: 0.94
- Sensor U: 0.84

The sensors operate independently.

- (a) Find $P(\text{all three sensors detect an intruder})$
- (b) Find $P(\text{at least one sensor detects an intruder})$
- (c) Find $P(\text{exactly two sensors detect an intruder})$
- (d) If exactly two sensors detect an intruder, find $P(\text{Sensor U failed})$
- (e) Which single sensor is most reliable for detection?

11. A factory produces items using two processes. Process P is used 58% of the time and produces 6% defective items. Process Q is used 42% of the time and produces 11% defective items.

- (a) A random item is selected and found to be defective. Use Bayes' theorem to find $P(\text{produced by Process P})$
- (b) If 1000 items are produced, how many would you expect to be defective?
- (c) How many of the defective items would come from each process?
- (d) To reduce overall defect rate to 5%, what should Process Q's defect rate be?

12. Three weather forecasting models predict rain independently:

- Model S: 79% accurate when it will rain, 87% accurate when it won't rain
- Model T: 86% accurate when it will rain, 83% accurate when it won't rain
- Model U: 74% accurate when it will rain, 92% accurate when it won't rain

Historically, it rains 40% of days.

- (a) If all three models predict rain, find $P(\text{it actually rains})$
- (b) If Model S predicts rain but Models T and U predict no rain, find $P(\text{it rains})$
- (c) Which model would you trust most for a "rain" prediction?
- (d) Which model would you trust most for a "no rain" prediction?

Section D: Introduction to Binomial Distribution

13. A fair coin is flipped 14 times.
- (a) Find $P(\text{exactly 8 heads})$
 - (b) Find $P(\text{at most 5 heads})$
 - (c) Find $P(\text{at least 9 heads})$
 - (d) Find the expected number of heads
 - (e) Find the most likely number of heads
 - (f) Calculate the variance of the number of heads
14. A multiple choice test has 20 questions, each with 5 possible answers. A student guesses randomly on all questions.
- (a) State the distribution of the number of correct answers
 - (b) Find $P(\text{exactly 6 correct answers})$
 - (c) Find $P(\text{more than 8 correct answers})$
 - (d) Find the expected number of correct answers
 - (e) Find $P(\text{passes the test})$ if the pass mark is 45%
 - (f) Calculate the standard deviation of correct answers
15. The probability that a seed germinates is 0.82. A packet contains 14 seeds.
- (a) Find $P(\text{all seeds germinate})$
 - (b) Find $P(\text{exactly 11 seeds germinate})$
 - (c) Find $P(\text{fewer than 9 seeds germinate})$
 - (d) How many seeds would you expect to germinate?
 - (e) Find $P(\text{at least 85\% of seeds germinate})$
 - (f) What's the most likely number of seeds to germinate?
16. A manufacturing process produces 7% defective items. Quality control samples 24 items.
- (a) Find $P(\text{no defective items in the sample})$
 - (b) Find $P(\text{exactly 2 defective items})$
 - (c) Find $P(\text{more than 3 defective items})$
 - (d) Calculate the expected number of defective items
 - (e) Find $P(\text{defect rate in sample exceeds 14\%})$
 - (f) Calculate the probability that the sample defect rate is between 4% and 11%

Section E: Advanced Binomial Applications

17. A basketball player has a 73% free throw success rate. In a game, they attempt 18 free throws.
- (a) Model this situation and state any assumptions
 - (b) Find $P(\text{makes at least 14 free throws})$
 - (c) Find $P(\text{makes between 11 and 15 free throws inclusive})$
 - (d) Calculate the expected number of successful free throws
 - (e) Find the probability their success rate in this game is above 80%
 - (f) What's the minimum number of attempts needed for $P(\text{at least 1 success}) = 0.99999$?

18. A quality control inspector checks 28 items per hour. The probability any item is defective is 0.06.
- (a) Find $P(\text{finds exactly 1 defective item in one hour})$
 - (b) Find $P(\text{finds no defective items in one hour})$
 - (c) Over a 9-hour shift, find the expected number of defective items found
 - (d) In what percentage of hours would you expect to find more than 4 defective items?
 - (e) If the inspector finds 6 defective items in one hour, comment on whether this is unusual
19. A pharmaceutical company claims their drug is effective for 83% of patients. A trial involves 45 patients.
- (a) If the claim is true, find $P(\text{drug works for exactly 38 patients})$
 - (b) Find $P(\text{drug works for at least 35 patients})$
 - (c) Calculate the expected number of patients for whom the drug works
 - (d) If the drug works for only 32 patients, test whether this supports the company's claim
 - (e) What's the minimum number of successes that would support the 83% claim at 5% significance?
20. A survey shows 46% of people support a proposal. A random sample of 32 people is surveyed.
- (a) Find $P(\text{exactly 15 people support the proposal})$
 - (b) Find $P(\text{fewer than 12 people support the proposal})$
 - (c) Calculate the expected number of supporters
 - (d) Find $P(\text{between 35\% and 55\% of the sample support the proposal})$
 - (e) If 20 people in the sample support the proposal, is this significantly different from expected?

Section F: Combined Probability Scenarios

21. An online retailer has two suppliers. Supplier P provides 68% of goods with 5% defect rate. Supplier Q provides 32% of goods with 12% defect rate.
- (a) A customer receives 18 items. Find $P(\text{exactly 1 is defective})$
 - (b) If a customer complains about a defective item, find $P(\text{it came from Supplier Q})$
 - (c) A batch of 200 items arrives. Find the expected number from each supplier
 - (d) Calculate the overall defect rate
 - (e) If the company wants to reduce defects to 4%, what should Supplier Q's rate be?
22. A casino game involves drawing 6 cards from a standard deck without replacement. The player wins if all 6 cards are spades.
- (a) Calculate $P(\text{all 6 cards are spades})$
 - (b) Calculate $P(\text{all 6 cards are the same suit})$
 - (c) If 1500 people play this game, how many would you expect to win?
 - (d) What should be the payout ratio for this to be a fair game?
 - (e) How does the probability change if cards are replaced after each draw?
23. A communication system sends signals through 6 independent channels. Each channel has probability 0.88 of successful transmission.
- (a) Find $P(\text{message received successfully through all channels})$

- (b) Find $P(\text{message fails on exactly one channel})$
 - (c) The system works if at least 5 channels succeed. Find $P(\text{system works})$
 - (d) If the system sends 80 messages, find $P(\text{fewer than 70 are received successfully})$
 - (e) What should be the individual channel reliability for 99.7% system reliability?
24. A hospital emergency department sees an average of 18% critical cases. On a particular shift, 30 patients arrive.
- (a) Model the number of critical cases and state assumptions
 - (b) Find $P(\text{exactly 5 critical cases})$
 - (c) Find $P(\text{no critical cases})$
 - (d) Find $P(\text{more than 8 critical cases})$
 - (e) Calculate the expected number of critical cases
 - (f) If there are 10 critical cases in one shift, is this unusually high?

Section G: Advanced Problem Solving

25. A genetic disorder affects 1 in 750 births. A screening test is 91% accurate for positive cases and 99.1% accurate for negative cases.
- (a) Calculate the probability of testing positive
 - (b) If a baby tests positive, what's the probability they have the disorder?
 - (c) How many false positives occur per 75,000 births?
 - (d) Design a two-stage testing procedure to reduce false positives
 - (e) Comment on the ethical implications of these probabilities
26. A software company releases updates with bugs 25% of the time. They use a testing protocol that catches 82% of buggy updates but also flags 7% of good updates as potentially buggy.
- (a) If an update is flagged, find $P(\text{it actually has bugs})$
 - (b) If an update passes testing, find $P(\text{it's actually bug-free})$
 - (c) In 250 updates, how many false alarms would you expect?
 - (d) Suggest improvements to the testing protocol
 - (e) Calculate the overall accuracy of the testing system
27. A lottery has the following structure: pick 6 numbers from 1-42. You win the jackpot if all 6 match.
- (a) Calculate $P(\text{winning the jackpot})$
 - (b) Find $P(\text{matching exactly 5 numbers})$
 - (c) Find $P(\text{matching exactly 4 numbers})$
 - (d) If 12 million tickets are sold, find $P(\text{no one wins the jackpot})$
 - (e) Model the number of jackpot winners as a binomial distribution
28. A cybersecurity system monitors network traffic. It correctly identifies 88% of malicious attacks and incorrectly flags 5% of normal traffic. On average, 0.4% of traffic is malicious.
- (a) Find the probability of an alert
 - (b) If there's an alert, find $P(\text{it's a real attack})$
 - (c) In monitoring 600,000 data packets, how many false alarms occur?

- (d) Design a cost-benefit analysis for this system
 - (e) How would increasing the detection rate to 92% affect false alarms?
29. Design and analyze a probability model for a real-world scenario of your choice:
- (a) Define the scenario and identify random variables
 - (b) State all assumptions clearly
 - (c) Choose appropriate probability distributions
 - (d) Calculate relevant probabilities
 - (e) Discuss limitations and potential improvements
 - (f) Consider practical applications of your analysis

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 100

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