

GCSE Higher Mathematics

Practice Test 6: Number

Instructions:

Answer all questions. Show your working clearly.

Calculators may be used unless stated otherwise.

Time allowed: 90 minutes

Section A: Powers and Roots

1. Evaluate these expressions:

(a) 8^2

(b) 2^{-5}

(c) 17^0

(d) $(-8)^2$

(e) $(-7)^3$

(f) 10^{-2}

2. Write these as single powers:

(a) $7^5 \times 7^{10}$

(b) $12^9 \div 12^3$

(c) $(5^7)^4$

(d) $9^{-2} \times 9^{12}$

(e) $\frac{13^7}{13^{-8}}$

(f) $(8^5)^{-2}$

3. Evaluate these fractional indices:

(a) $81^{\frac{1}{2}}$

(b) $343^{\frac{1}{3}}$

(c) $16807^{\frac{3}{5}}$

(d) $1000000^{\frac{2}{6}}$

(e) $1331^{\frac{2}{3}}$

(f) $1024^{-\frac{2}{5}}$

4. Simplify these expressions:

(a) $\sqrt{196}$

(b) $\sqrt[3]{729}$

- (c) $\sqrt[4]{2401}$
- (d) $\sqrt{1.44}$
- (e) $\sqrt[3]{-512}$
- (f) $\sqrt[5]{32768}$

5. Express in index form:

- (a) $\sqrt{23}$
- (b) $\sqrt[3]{17}$
- (c) $\frac{1}{\sqrt{13}}$
- (d) $\sqrt[4]{u^{13}}$
- (e) $\frac{1}{\sqrt[3]{g^{10}}}$
- (f) $\sqrt{u} \times \sqrt[3]{u}$

Section B: Laws of Indices

6. Simplify these expressions (no calculator):

- (a) $7^6 \times 7^{-9} \times 7^{10}$
- (b) $\frac{10^{11} \times 10^{-7}}{10^{-6}}$
- (c) $(8^4)^{-6} \times 8^{19}$
- (d) $\frac{11^{-7} \times 11^{15}}{11^6}$

7. Evaluate these expressions:

- (a) $1331^{\frac{2}{3}}$
- (b) $2401^{-\frac{3}{4}}$
- (c) $16807^{\frac{2}{5}}$
- (d) $16384^{-\frac{4}{7}}$
- (e) $4096^{\frac{1}{3}}$
- (f) $100000^{-\frac{3}{5}}$

8. Simplify these expressions:

- (a) $x^{\frac{8}{9}} \times x^{\frac{1}{3}}$
- (b) $\frac{p^{\frac{10}{3}}}{p^{\frac{1}{4}}}$
- (c) $(g^{\frac{1}{7}})^{14}$
- (d) $\sqrt{u} \times u^{\frac{1}{8}}$
- (e) $\frac{\sqrt[3]{r^{10}}}{\sqrt{r}}$
- (f) $(q^{-\frac{1}{7}})^{-14}$

9. Write these in the form a^n where a and n are rational:

- (a) $\sqrt{17} \times 17^7$
- (b) $\frac{19^5}{\sqrt[3]{19}}$
- (c) $\sqrt[4]{12^{13}} \times 12^{-\frac{4}{5}}$
- (d) $\frac{\sqrt{23}}{\sqrt[3]{23^{10}}}$

Section C: Surds

10. Simplify these surds:

- (a) $\sqrt{76}$
- (b) $\sqrt{117}$
- (c) $\sqrt{124}$
- (d) $\sqrt{207}$
- (e) $\sqrt{243}$
- (f) $\sqrt{800}$

11. Simplify these expressions:

- (a) $8\sqrt{17} + 13\sqrt{17}$
- (b) $17\sqrt{7} - 7\sqrt{7}$
- (c) $\sqrt{52} + \sqrt{117}$
- (d) $\sqrt{175} - \sqrt{112}$
- (e) $7\sqrt{24} + 9\sqrt{54}$
- (f) $\sqrt{207} - \sqrt{128} + \sqrt{32}$

12. Multiply and simplify:

- (a) $\sqrt{11} \times \sqrt{44}$
- (b) $\sqrt{24} \times \sqrt{96}$
- (c) $8\sqrt{5} \times 6\sqrt{20}$
- (d) $\sqrt{18} \times \sqrt{72}$
- (e) $\sqrt{8} \times \sqrt{32} \times \sqrt{128}$
- (f) $9\sqrt{7} \times 6\sqrt{28}$

13. Expand and simplify:

- (a) $(7 + \sqrt{13})(4 - \sqrt{13})$
- (b) $(6 + \sqrt{19})(3 + 7\sqrt{19})$
- (c) $(9 - \sqrt{17})^2$
- (d) $(\sqrt{23} + \sqrt{7})(\sqrt{23} - \sqrt{7})$
- (e) $(7\sqrt{13} + 1)(7\sqrt{13} - 1)$
- (f) $(\sqrt{19} + 7)^2$

14. Rationalize the denominators:

- (a) $\frac{1}{\sqrt{19}}$
- (b) $\frac{13}{\sqrt{23}}$
- (c) $\frac{\sqrt{13}}{\sqrt{52}}$
- (d) $\frac{14}{7\sqrt{2}}$
- (e) $\frac{1}{6+\sqrt{13}}$
- (f) $\frac{9}{1-\sqrt{19}}$

Section D: More Complex Surd Operations

15. Rationalize these denominators:

(a) $\frac{11}{7+\sqrt{17}}$

(b) $\frac{17}{6-\sqrt{29}}$

(c) $\frac{\sqrt{13}}{1+\sqrt{13}}$

(d) $\frac{7\sqrt{19}}{6+\sqrt{19}}$

(e) $\frac{1}{\sqrt{21}-\sqrt{13}}$

(f) $\frac{\sqrt{19+6}}{\sqrt{19-7}}$

16. Simplify these expressions completely:

(a) $\frac{\sqrt{32}+\sqrt{72}}{\sqrt{8}}$

(b) $\frac{\sqrt{84}-\sqrt{56}}{\sqrt{14}}$

(c) $\sqrt{(8+\sqrt{19})(8-\sqrt{19})}$

(d) $\sqrt{124}-7\sqrt{31}+\sqrt{84}$

(e) $(\sqrt{13}+\sqrt{52})^2$

(f) $\frac{\sqrt{117}}{\sqrt{13}}+\frac{\sqrt{91}}{\sqrt{13}}$

17. Prove that:

(a) $(\sqrt{o}+\sqrt{q})(\sqrt{o}-\sqrt{q})=o-q$

(b) $\frac{1}{\sqrt{r}+\sqrt{t}}=\frac{\sqrt{r}-\sqrt{t}}{r-t}$

(c) $(r+t\sqrt{w})^2=r^2+2rt\sqrt{w}+t^2w$

Section E: Standard Form

18. Write these numbers in standard form:

(a) 891000

(b) 0.000117

(c) 6780000000

(d) 0.0000000429

(e) 1149.5

(f) 0.01169

19. Write these in ordinary form:

(a) 9.7×10^1

(b) 7.89×10^{-9}

(c) 1.0432×10^{-12}

(d) 6.58×10^{15}

(e) 1.17×10^{-7}

(f) 8.91×10^{10}

20. Calculate, giving answers in standard form:

- (a) $(9 \times 10^9) \times (11 \times 10^{11})$
- (b) $(7 \times 10^{-7}) \times (13 \times 10^{12})$
- (c) $(21 \times 10^{10}) \div (7 \times 10^{-5})$
- (d) $(18 \times 10^{-9}) \div (12 \times 10^{-12})$
- (e) $(9 \times 10^8)^2$
- (f) $\sqrt{64 \times 10^{18}}$

21. Calculate these more complex expressions:

- (a) $(7.2 \times 10^8) \times (2.75 \times 10^{-10})$
- (b) $\frac{14.4 \times 10^{11}}{3.6 \times 10^{-8}}$
- (c) $(8.1 \times 10^{-7}) + (9.7 \times 10^{-8})$
- (d) $(10.6 \times 10^{10}) - (7.4 \times 10^9)$
- (e) $\frac{(7.2 \times 10^6) \times (6.3 \times 10^{-7})}{10.8 \times 10^{-5}}$
- (f) $(8.41 \times 10^{16})^{\frac{1}{2}}$

Section F: Rational Numbers and Operations

22. Calculate these fractions (give answers in simplest form):

- (a) $\frac{7}{13} + \frac{9}{26}$
- (b) $\frac{17}{20} - \frac{11}{30}$
- (c) $\frac{16}{17} \times \frac{34}{24}$
- (d) $\frac{14}{27} \div \frac{21}{36}$
- (e) $\frac{11}{12} - \frac{7}{16} + \frac{13}{48}$
- (f) $(\frac{7}{13})^{-2}$

23. Convert these recurring decimals to fractions:

- (a) $0.\overline{8}$
- (b) $0.\overline{91}$
- (c) $0.6\overline{2}$
- (d) $0.4\overline{28571}$
- (e) $5.3\overline{7}$
- (f) $0.91\overline{8}$

24. Work out these percentage calculations:

- (a) Increase 760 by 65%
- (b) Decrease 980 by 32%
- (c) Find 37.5% of 1120
- (d) What percentage is 147 out of 196?
- (e) If 85% of a number is 221, find the number
- (f) A price increases from £120 to £132. Find the percentage increase

25. Solve these percentage problems:

- (a) After a 70% increase, a price is £289. Find the original price
- (b) After a 65% decrease, a quantity is 126. Find the original quantity
- (c) The value of a car decreases by 45% each year. If it's worth £9075 now, what was it worth 2 years ago?
- (d) An investment grows by 9% per year. After 2 years it's worth £2373.21. Find the initial investment

Section G: Complex Calculations

26. Simplify these mixed expressions:

(a) $7^{-2} + 11^0 - 12^{-1}$

(b) $\sqrt{81} \times 512^{\frac{1}{3}} - 7^{-2}$

(c) $\frac{1728^{\frac{2}{3}} - 16807^{\frac{3}{5}}}{100^{\frac{1}{2}}}$

(d) $49^{-\frac{1}{2}} + 100^{\frac{1}{2}} \times 6^{-1}$

27. Calculate exactly (leave surds in your answer):

(a) $\frac{11}{\sqrt{13}} + \frac{7}{\sqrt{52}}$

(b) $\sqrt{54} \times \sqrt{96} - \sqrt{216}$

(c) $\frac{\sqrt{175} + \sqrt{112}}{\sqrt{7}}$

(d) $(7\sqrt{2} - 6)^2$

28. Work with standard form in context:

(a) The mass of an oxygen atom is 2.66×10^{-26} kg. Find the mass of 6.02×10^{23} oxygen atoms

(b) Infrared radiation travels at 3×10^8 m/s. How far does it travel in one month (use 1 month = 2.628×10^6 seconds)?

(c) The thickness of a sheet of paper is approximately 1.0×10^{-4} m. How many sheets would make a stack 7.5 cm high?

(d) A graphics processor handles 1.44×10^{14} operations per second. How many operations in 25 minutes?

Section H: Problem Solving

29. Prove that $\sqrt{13}$ is irrational. (Use proof by contradiction: assume $\sqrt{13} = \frac{u}{v}$ where u and v are integers with no common factors)

30. The number δ satisfies $\delta^2 = 5\delta - 3$.

(a) Show that $\delta = \frac{5 + \sqrt{13}}{2}$

(b) Calculate δ to 4 decimal places

(c) Find $\frac{1}{\delta}$ in surd form

31. Rationalize the denominator of $\frac{1}{\sqrt{11} + \sqrt{13} + \sqrt{17}}$. (Hint: First rationalize using $(\sqrt{11} + \sqrt{13}) - \sqrt{17}$)

32. A rectangle has sides of length $(7 + \sqrt{13})$ cm and $(7 - \sqrt{13})$ cm.

(a) Find the exact area

(b) Find the exact perimeter

(c) Show that the area is rational but the perimeter is irrational

33. The population of algae septuples every 10 hours. If there are initially 9×10^2 algae:

(a) How many algae after 40 hours?

(b) Express your answer in standard form

(c) After how many hours will there be more than 4×10^{10} algae?

34. Show that $\frac{1}{\sqrt{o} + \sqrt{q}} + \frac{1}{\sqrt{o} - \sqrt{q}} = \frac{2\sqrt{o}}{o - q}$

35. A cylinder has volume $V = \pi r^2 h$. If the volume is $648\pi \text{ cm}^3$ and $h = 18 \text{ cm}$:

- (a) Find the radius in surd form
- (b) Find the surface area (use $A = 2\pi r^2 + 2\pi r h$)
- (c) Express both answers exactly

36. The equation $x^2 - 14x + 1 = 0$ has solutions $x = 7 \pm 4\sqrt{3}$.

- (a) Verify this by substitution
- (b) Find $\frac{1}{7+4\sqrt{3}} + \frac{1}{7-4\sqrt{3}}$ without using a calculator
- (c) Hence find the sum of the reciprocals of the roots

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 100

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