A Level Pure Mathematics
Practice Test 4: Coordinate Geometry in the (x, y) Plane

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Time allowed: 2 hours

Section A: Distance and Midpoint Formulas

1. Find the distance between these pairs of points:
   (a) $A(6, 3)$ and $B(2, 9)$
   (b) $C(-5, 2)$ and $D(1, -6)$
   (c) $E(-2, -4)$ and $F(4, 4)$
   (d) $G(3t, 2t)$ and $H(-2t, 5t)$
2. Find the midpoint of the line segment joining:
   (a) $P(3, 11)$ and $Q(9, 7)$
   (b) $R(-7, 2)$ and $S(1, -10)$
   (c) $T(4s, s)$ and $U(-2s, 7s)$
   (d) The point $(6u, -3u)$ and $(-2u, 5u)$
3. The point $M(2, 5)$ is the midpoint of the line segment GH where $G(-2, 1)$.
   (a) Find the coordinates of point H
   (b) Find the equation of line GH
   (c) Calculate the distance from the origin to line GH
4. Points $A(4, 2)$, $B(8, 5)$, and $C(2, 9)$ form a triangle.
   (a) Show that triangle ABC is a right triangle
   (b) Find the coordinates of the circumcenter
   (c) Calculate the circumradius
   (d) Find the area using the coordinate formula
5. The points $P(0, 4)$, $Q(6, 7)$, $R(3, 13)$, and $S(-3, 10)$ form a quadrilateral.
   (a) Prove that $PQRS$ is a square
   (b) Find the coordinates of the center
   (c) Calculate the length of the side
   (d) Find the area of square $PQRS$

Section B: Equations of Straight Lines

6. Find the equation of the straight line:
   (a) With gradient $-\frac{3}{4}$ passing through $(8, 2)$
   (b) Passing through $(-2, 6)$ and $(3, -4)$
   (c) With x-intercept 5 and y-intercept -4
   (d) Perpendicular to $4x - 3y = 18$ and passing through $(1, 7)$
7. Express these equations in normal form $x \cos\theta + y \sin\theta = p$:
   (a) $3x + 4y = 15$
   (b) $5x - 12y = 39$
   (c) $x - y = 4\sqrt{2}$
   (d) $2x + 3y - 13 = 0$
8. Find the equation of the line that:
   (a) Is parallel to $3x + 2y = 8$ and passes through $(4, -3)$
   (b) Is perpendicular to $x - 4y = 12$ and has x-intercept -2
   (c) Passes through $(2, 5)$ and makes a $120°$ angle with the positive x-axis
   (d) Is equidistant from the points $(1, 3)$ and $(5, -1)$
9. Three lines have equations $L_1 : x + 2y - 5 = 0$, $L_2 : 2x - y + 1 = 0$, and $L_3 : 3x + y - 8 = 0$.
   (a) Find the vertices of the triangle formed by these lines
   (b) Calculate the area of the triangle
   (c) Find the equation of the circumcircle of the triangle
   (d) Determine which triangle this is (acute, right, or obtuse)
10. A quadrilateral has vertices at $A(1, 1)$, $B(4, 2)$, $C(5, 5)$, and $D(2, 4)$.
   (a) Find the equations of all four sides
   (b) Show that opposite sides are parallel
   (c) Calculate the lengths of the diagonals
   (d) Find the point of intersection of the diagonals

Section C: Angle Between Lines

11. Calculate the acute angle between these pairs of lines:
   (a) $y = \frac{3}{4}x - 1$ and $y = -\frac{4}{5}x + 2$
   (b) $4x + 3y = 12$ and $3x - 4y = 16$
   (c) $2x - 5y + 3 = 0$ and $5x + 2y - 7 = 0$
   (d) $y = \tan 15° \cdot x + 3$ and $y = \tan 105° \cdot x - 1$
12. A line passes through $(1, 4)$ and makes an angle of $150°$ with the positive x-axis.
   (a) Find the equation of the line
   (b) Find where this line intersects $3x + y = 10$
   (c) Calculate the angle between the line and $3x + y = 10$
13. Two lines intersect at $(4, 2)$ at an angle of $75°$. If one line has gradient $\frac{1}{3}$:

(a) Find the two possible gradients for the second line
(b) Write the equations of both possible second lines
(c) Determine which line makes the larger angle with the positive x-axis

14. Find the equations of the lines through $(3, -1)$ that make an angle of $60°$ with the line $x+2y = 6$.

(a) Express the given line in slope-intercept form
(b) Use the angle between lines formula
(c) Solve for the two possible gradients
(d) Write both equations and verify your answers

Section D: Equation of a Circle

15. Write the equation of the circle with:

(a) Center $(0, 0)$ and radius $2\sqrt{5}$
(b) Center $(6, -2)$ and radius $\sqrt{10}$
(c) Center $(-4, 3)$ and passing through $(2, -1)$
(d) Diameter with endpoints $(4, 1)$ and $(-2, 7)$

16. Express these equations in standard form and find the center and radius:

(a) $x^2 + y^2 - 12x + 10y + 45 = 0$
(b) $x^2 + y^2 + 8x - 6y - 39 = 0$
(c) $x^2 + y^2 - 4x + 8y - 5 = 0$
(d) $5x^2 + 5y^2 - 20x + 30y - 25 = 0$

17. A circle has center $(4, -3)$ and is tangent to the line $3x + 4y = 12$.

(a) Find the radius of the circle
(b) Write the equation of the circle
(c) Find the point of tangency
(d) Find the equation of the tangent at this point

18. Two circles $C_1 : x^2 + y^2 - 6x + 4y - 12 = 0$ and $C_2 : x^2 + y^2 + 2x - 8y + 8 = 0$:

(a) Find the centers and radii
(b) Show that the circles intersect at two points
(c) Find the points of intersection
(d) Find the area of the region common to both circles

19. Find the equation of the circle passing through $(2, 3)$, $(4, 1)$, and $(6, 5)$.

(a) Set up three equations using the general form
(b) Solve the system to find the coefficients
(c) Express in standard form
(d) Verify by checking that all points satisfy the equation

Section E: Parabolas

20. For parabolas with different orientations:
    (a) Find the focus and directrix of $(y-2)^2 = 8(x-1)$
    (b) Find the focus and directrix of $(x+1)^2 = -12(y-3)$
    (c) Find the equation with vertex at $(3,-2)$ and focus at $(3,1)$
    (d) Sketch $(x-2)^2 = 16(y+1)$ showing focus and directrix

21. A parabola has vertex at $(-1,4)$ and directrix $y = 2$.
    (a) Find the focus of the parabola
    (b) Write the equation of the parabola
    (c) Find where the parabola intersects the x-axis
    (d) Calculate the length of the latus rectum

22. The parabola $y = ax^2 + bx + c$ passes through $(1,4)$, $(2,1)$, and $(3,2)$.
    (a) Set up and solve the system for a, b, and c
    (b) Find the vertex coordinates
    (c) Determine the focus and directrix
    (d) Find the equation of the axis of symmetry

23. A parabolic antenna has equation $y^2 = 36x$ where measurements are in centimeters.
    (a) Find the focus of the parabola
    (b) If the antenna is 24 cm wide at the opening, find its depth
    (c) Where should the feed be positioned for optimal reception?
    (d) Find the equation of the tangent at the point $(4,12)$

Section F: Ellipses

24. For the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$:
    (a) When $a = 8$ and $b = 6$, find the foci and eccentricity
    (b) If the eccentricity is $\frac{3}{5}$ and $a = 10$, find b
    (c) Find the equation if vertices are at $(\pm 6,0)$ and foci at $(\pm 4,0)$
    (d) Sketch $\frac{x^2}{36} + \frac{y^2}{16} = 1$

25. An ellipse has center at the origin and passes through $(5,0)$ and $(0,3)$.
    (a) Write the equation of the ellipse
    (b) Find the coordinates of the foci
    (c) Calculate the eccentricity
    (d) Find the length of the major and minor axes

26. The ellipse $\frac{(x-2)^2}{49} + \frac{(y+3)^2}{16} = 1$ has center at $(2,-3)$.
    (a) Find the vertices and co-vertices
    (b) Calculate the foci coordinates
    (c) Find the eccentricity
    (d) Calculate the area of the ellipse

27. An ellipse has center at $(1, 2)$, one focus at $(4, 2)$, and passes through $(1, 7)$.
    (a) Find the distance c from center to focus
    (b) Determine the semi-major axis a
    (c) Calculate the semi-minor axis b
    (d) Write the equation of the ellipse

Section G: Hyperbolas

28. For the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:
    (a) When $a = 6$ and $b = 8$, find the foci and eccentricity
    (b) Find the asymptotes when $a = 4$ and $b = 3$
    (c) If the vertices are at $(\pm 5, 0)$ and eccentricity is $\frac{13}{5}$, find b
    (d) Sketch $\frac{x^2}{25} - \frac{y^2}{144} = 1$
29. A hyperbola has equation $\frac{y^2}{36} - \frac{x^2}{64} = 1$.
    (a) Identify the transverse and conjugate axes
    (b) Find the vertices and foci
    (c) Write the equations of the asymptotes
    (d) Calculate the eccentricity
30. For the rectangular hyperbola $xy = k$:
    (a) When $k = 32$, find the intersection with line $2x + y = 12$
    (b) Find the tangent to $xy = 36$ at point $(6, 6)$
    (c) Prove that the tangent at $(a, \frac{k}{a})$ has slope $-\frac{k}{a^2}$
    (d) Find the equation of the chord of contact from external point (h, k)
31. A hyperbola has center at $(3, 1)$, one vertex at $(7, 1)$, and one asymptote with slope $\frac{3}{4}$.
    (a) Find the semi-transverse axis a
    (b) Calculate the semi-conjugate axis b
    (c) Write the equation of the hyperbola
    (d) Find both foci coordinates

Section H: Mixed Conic Sections

32. Identify and analyze these conic sections:
    (a) $25x^2 + 16y^2 = 400$
    (b) $9x^2 - 16y^2 = 144$
    (c) $(y - 3)^2 = 16(x + 2)$
    (d) $x^2 + y^2 + 6x - 8y - 11 = 0$
33. For rotated conics with xy terms:
    (a) Classify: $x^2 + 4xy + 4y^2 - 8x - 16y + 12 = 0$
    (b) Classify: $5x^2 + 6xy + 5y^2 - 8x - 8y - 4 = 0$
    (c) Find the angle of rotation to eliminate the xy term in: $3x^2 + 4xy - 3y^2 + 8 = 0$

(d) Transform $x^2 + 2xy + y^2 - 6x - 6y + 9 = 0$ by rotating axes 45°

34. Find all intersection points:

(a) Line $2x - y = 3$ and circle $x^2 + y^2 = 13$

(b) Line $y = 6$ and parabola $x^2 = 12y$

(c) Ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ and hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$

(d) Circle $x^2 + y^2 = 25$ and rectangular hyperbola $xy = 12$

35. Find equations of tangents and normals:

(a) Tangent to circle $x^2 + y^2 - 4x + 6y - 12 = 0$ at $(4, 0)$

(b) Normal to parabola $(x - 1)^2 = 8(y + 2)$ at $(3, 3)$

(c) Tangent to ellipse $\frac{x^2}{64} + \frac{y^2}{36} = 1$ at $(4\sqrt{2}, 3\sqrt{2})$

(d) Normal to hyperbola $\frac{x^2}{36} - \frac{y^2}{64} = 1$ at $(10, 8)$

Section I: Applications and Problem Solving

36. A semi-elliptical tunnel has width 16 meters and maximum height 6 meters. Design specifications require clearance calculations.

(a) Find the equation of the ellipse

(b) Calculate the height at 2, 4, and 6 meters from the center

(c) A vehicle is 2.8 meters wide and 5.2 meters tall. Determine clearance

(d) Find the cross-sectional area of the tunnel

37. A parabolic reflector telescope has diameter 8 meters and focal length 3 meters.

(a) Find the equation of the parabola

(b) Calculate the depth of the reflector

(c) Where should the secondary mirror be positioned?

(d) If the reflector is shortened to 6 meters diameter, how does this affect the focal properties?

38. A hyperbolic navigation system uses two stations 200 km apart. The time difference between signals gives a distance difference of 60 km.

(a) Set up coordinates with stations at foci

(b) Find the equation of the position hyperbola

(c) Calculate the eccentricity of this hyperbola

(d) If a third station provides another measurement, describe the solution method

39. An asteroid follows a hyperbolic trajectory past Earth. The closest approach is 50,000 km from Earth's center, and the eccentricity is 2.5.

(a) Calculate the semi-major axis of the hyperbola

(b) Find the semi-minor axis

(c) Determine Earth's position relative to the hyperbola

(d) Calculate the asymptotic direction of the asteroid's path

40. A water fountain creates parabolic streams. The highest stream reaches 4 meters height and lands 6 meters away horizontally.

(a) Model the trajectory as a parabola

(b) Find the equation of the water path

(c) Calculate the height at horizontal distances 1, 2, and 3 meters

(d) Determine the angle of launch from the horizontal

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 150

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