# A Level Pure Mathematics
# Practice Test 1: Trigonometry

**Instructions:**
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Time allowed: 2 hours

## Section A: Radian Measure and Basic Functions

1. Convert these angles from degrees to radians (leave answers in terms of $\pi$):
   (a) $30°$
   (b) $45°$
   (c) $60°$
   (d) $90°$
   (e) $135°$
   (f) $240°$
2. Convert these angles from radians to degrees:
   (a) $\frac{\pi}{6}$
   (b) $\frac{\pi}{4}$
   (c) $\frac{2\pi}{3}$
   (d) $\frac{5\pi}{4}$
   (e) $\frac{7\pi}{6}$
   (f) $\frac{3\pi}{2}$
3. Find the exact values of these trigonometric ratios (without calculator):
   (a) $\sin\frac{\pi}{6}$, $\cos\frac{\pi}{6}$, $\tan\frac{\pi}{6}$
   (b) $\sin\frac{\pi}{4}$, $\cos\frac{\pi}{4}$, $\tan\frac{\pi}{4}$
   (c) $\sin\frac{\pi}{3}$, $\cos\frac{\pi}{3}$, $\tan\frac{\pi}{3}$
   (d) $\sin\frac{\pi}{2}$, $\cos\frac{\pi}{2}$, $\tan\frac{\pi}{2}$
4. A circle has radius 8 cm. Find:
   (a) The arc length subtended by an angle of $\frac{2\pi}{3}$ radians
   (b) The area of the sector with angle $\frac{5\pi}{6}$ radians
   (c) The angle (in radians) that subtends an arc of length 12 cm
   (d) The radius of a circle where an angle of $\frac{3\pi}{4}$ radians subtends an arc of length 15 cm
5. Find the exact values:

(a) $\sin \frac{2\pi}{3}$
(b) $\cos \frac{3\pi}{4}$
(c) $\tan \frac{5\pi}{6}$
(d) $\sin \frac{7\pi}{4}$
(e) $\cos \frac{5\pi}{3}$
(f) $\tan \frac{4\pi}{3}$

## Section B: Graphs of Trigonometric Functions

6. For the function $f(x) = \sin x$:
   (a) State the domain and range
   (b) Find the period
   (c) Identify the zeros in the interval $[0, 2\pi]$
   (d) Find the maximum and minimum values and where they occur
   (e) Sketch the graph for $x \in [-2\pi, 2\pi]$
7. For the function $g(x) = \cos x$:
   (a) State the domain and range
   (b) Find the period
   (c) Identify the zeros in the interval $[0, 2\pi]$
   (d) Find the maximum and minimum values and where they occur
   (e) Sketch the graph for $x \in [-2\pi, 2\pi]$
8. For the function $h(x) = \tan x$:
   (a) State the domain and range
   (b) Find the period
   (c) Identify the asymptotes in the interval $[0, 2\pi]$
   (d) Find the zeros in the interval $[0, \pi]$
   (e) Sketch the graph for $x \in [-\pi, \pi]$
9. Sketch the graphs of these transformed functions for $x \in [0, 4\pi]$:
   (a) $y = 2\sin x$
   (b) $y = \sin 2x$
   (c) $y = \sin(x + \frac{\pi}{3})$
   (d) $y = \sin x + 1$
   (e) $y = -\cos x$
   (f) $y = \cos(x - \frac{\pi}{4})$
10. For the function $y = 3\sin(2x - \frac{\pi}{3}) + 1$:
    (a) Identify the amplitude
    (b) Find the period
    (c) Determine the phase shift
    (d) Find the vertical shift
    (e) State the range
    (f) Sketch the graph for $x \in [0, 2\pi]$

## Section C: Fundamental Trigonometric Identities

11. Use the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$ to find:
    (a) $\cos\theta$ if $\sin\theta = \frac{3}{5}$ and $\theta$ is acute
    (b) $\sin\theta$ if $\cos\theta = -\frac{4}{5}$ and $\theta$ is obtuse
    (c) $\tan\theta$ if $\sin\theta = \frac{5}{13}$ and $\cos\theta > 0$
    (d) $\cos\theta$ if $\tan\theta = -\frac{7}{24}$ and $\sin\theta < 0$

12. Prove these identities:
    (a) $\tan^2\theta + 1 = \sec^2\theta$
    (b) $1 + \cot^2\theta = \csc^2\theta$
    (c) $\frac{\sin\theta}{\cos\theta} = \tan\theta$
    (d) $\frac{1}{\sin\theta} = \csc\theta$

13. Simplify these expressions:
    (a) $\sin^2\theta + \cos^2\theta + \tan^2\theta$
    (b) $\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$
    (c) $(\sin\theta + \cos\theta)^2$
    (d) $\frac{1-\sin^2\theta}{\cos\theta}$

14. Express in terms of $\sin\theta$ only:
    (a) $\cos^2\theta$
    (b) $\tan^2\theta$
    (c) $\sec^2\theta$
    (d) $\cos^2\theta + \sin^2\theta\tan^2\theta$

15. Prove that:
    (a) $\frac{1-\cos^2\theta}{\sin\theta} = \sin\theta$
    (b) $\cos^4\theta - \sin^4\theta = \cos 2\theta$
    (c) $\frac{\tan\theta+\cot\theta}{\tan\theta-\cot\theta} = \frac{\sin 2\theta}{\cos 2\theta}$
    (d) $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

## Section D: Addition and Double Angle Formulas

16. Use the addition formulas to find the exact values:
    (a) $\sin 75°$ (using $\sin(45° + 30°)$)
    (b) $\cos 15°$ (using $\cos(45° - 30°)$)
    (c) $\tan 105°$ (using $\tan(60° + 45°)$)
    (d) $\sin\frac{5\pi}{12}$ (using $\sin(\frac{\pi}{4} + \frac{\pi}{6})$)

17. Given $\sin A = \frac{3}{5}$ with $A$ acute and $\cos B = \frac{5}{13}$ with $B$ acute:
    (a) Find $\cos A$ and $\sin B$
    (b) Calculate $\sin(A + B)$
    (c) Calculate $\cos(A + B)$
    (d) Find $\tan(A - B)$

18. Use double angle formulas to find:
    (a) $\sin 2\theta$ if $\sin\theta = \frac{4}{5}$ and $\theta$ is acute
    (b) $\cos 2\theta$ if $\cos\theta = \frac{7}{25}$ and $\theta$ is acute
    (c) $\tan 2\theta$ if $\tan\theta = \frac{2}{3}$
    (d) $\cos 2\theta$ if $\sin\theta = -\frac{12}{13}$ and $\theta$ is in the third quadrant

19. Prove these double angle identities:
    (a) $\sin 2\theta = 2\sin\theta\cos\theta$
    (b) $\cos 2\theta = \cos^2\theta - \sin^2\theta$
    (c) $\cos 2\theta = 2\cos^2\theta - 1$
    (d) $\cos 2\theta = 1 - 2\sin^2\theta$

20. Express in terms of the double angle:
    (a) $\cos^2\theta$ in terms of $\cos 2\theta$
    (b) $\sin^2\theta$ in terms of $\cos 2\theta$
    (c) $\sin^4\theta$ in terms of multiple angles
    (d) $\cos^4\theta$ in terms of multiple angles

## Section E: Solving Trigonometric Equations

21. Solve these equations for $0 \le x \le 2\pi$:
    (a) $\sin x = \frac{1}{2}$
    (b) $\cos x = -\frac{\sqrt{3}}{2}$
    (c) $\tan x = 1$
    (d) $\sin x = -\frac{\sqrt{2}}{2}$

22. Solve these equations for $0° \le x \le 360°$:
    (a) $2\sin x - 1 = 0$
    (b) $3\cos x + 2 = 0$
    (c) $\tan x + \sqrt{3} = 0$
    (d) $2\sin^2 x = 1$

23. Solve these equations for $0 \le x \le 2\pi$:
    (a) $\sin 2x = \frac{\sqrt{3}}{2}$
    (b) $\cos 3x = -\frac{1}{2}$
    (c) $\tan 2x = -1$
    (d) $\sin(x + \frac{\pi}{4}) = \frac{1}{\sqrt{2}}$

24. Solve these quadratic trigonometric equations for $0 \le x \le 2\pi$:
    (a) $2\sin^2 x + \sin x - 1 = 0$
    (b) $\cos^2 x - 3\cos x + 2 = 0$
    (c) $2\tan^2 x - \tan x - 1 = 0$
    (d) $3\sin^2 x + 2\sin x - 1 = 0$

25. Solve these equations involving multiple angles for $0 \le x \le 2\pi$:

(a) $\sin x = \cos x$

(b) $\sin 2x = \cos x$

(c) $\cos 2x = 2\cos^2 x - 1$

(d) $\sin 3x = \sin x$

## Section F: Advanced Trigonometric Identities

26. Prove these sum-to-product identities:

(a) $\sin A + \sin B = 2\sin(\frac{A+B}{2})\cos(\frac{A-B}{2})$

(b) $\cos A + \cos B = 2\cos(\frac{A+B}{2})\cos(\frac{A-B}{2})$

(c) $\sin A - \sin B = 2\cos(\frac{A+B}{2})\sin(\frac{A-B}{2})$

(d) $\cos A - \cos B = -2\sin(\frac{A+B}{2})\sin(\frac{A-B}{2})$

27. Use sum-to-product formulas to simplify:

(a) $\sin 5x + \sin 3x$

(b) $\cos 7x - \cos 3x$

(c) $\sin 75° + \sin 15°$

(d) $\cos 105° + \cos 15°$

28. Prove these product-to-sum identities:

(a) $\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

(b) $\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

(c) $\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$

(d) $\cos A \sin B = \frac{1}{2}[\sin(A+B) - \sin(A-B)]$

29. Express as sums or differences:

(a) $2\sin 4x \cos 2x$

(b) $4\cos 3x \cos x$

(c) $\sin 5x \sin 2x$

(d) $\cos 6x \sin 4x$

30. Prove the triple angle formulas:

(a) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

(b) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

(c) $\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$

## Section G: Complex Trigonometric Problems

31. Solve these equations for $0 \leq x < 2\pi$:

(a) $\sin x + \cos x = 1$

(b) $\sin x + \sin 2x = 0$

(c) $\cos x + \cos 2x + \cos 3x = 0$

(d) $\tan x + \tan 2x = 0$

32. Prove these more complex identities:

(a) $\frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta} = 2$
(b) $\sin^2\theta + \sin^2(\theta + \frac{2\pi}{3}) + \sin^2(\theta + \frac{4\pi}{3}) = \frac{3}{2}$
(c) $\cos^4\theta + \sin^4\theta = 1 - 2\sin^2\theta\cos^2\theta$
(d) $\tan A + \tan B + \tan C = \tan A \tan B \tan C$ when $A + B + C = \pi$

33. Find the general solution to these equations:
   (a) $\sin x = \frac{1}{3}$
   (b) $\cos 2x = 0.7$
   (c) $\tan 3x = -2$
   (d) $\sin(2x + \frac{\pi}{6}) = \frac{\sqrt{3}}{2}$

34. Express these in the form $R\sin(x + \alpha)$ or $R\cos(x + \alpha)$:
   (a) $3\sin x + 4\cos x$
   (b) $5\sin x - 12\cos x$
   (c) $\sin x - \sqrt{3}\cos x$
   (d) $2\cos x + 2\sqrt{3}\sin x$

35. Find the range of these functions:
   (a) $f(x) = 3\sin x + 4\cos x$
   (b) $g(x) = 5\sin 2x - 12\cos 2x + 7$
   (c) $h(x) = \sin^2 x + 2\cos x$
   (d) $k(x) = 2\sin x\cos x + 3$

## Section H: Applications of Trigonometry

36. A particle moves in simple harmonic motion with displacement $s = 4\sin(3t + \frac{\pi}{4})$ meters, where $t$ is time in seconds.
   (a) Find the amplitude of the motion
   (b) Determine the period of oscillation
   (c) Find the phase shift
   (d) Calculate the displacement when $t = 0$
   (e) Find when the particle first passes through the equilibrium position

37. The height of a tide is modeled by $h(t) = 3\cos(\frac{\pi t}{6}) + 4$ meters, where $t$ is hours after midnight.
   (a) Find the maximum and minimum heights
   (b) Determine the period of the tide
   (c) Find the height at 6 AM
   (d) Calculate when high tide occurs
   (e) Find when the height is exactly 5.5 meters

38. An alternating current is given by $I = 10\sin(100\pi t + \frac{\pi}{3})$ amperes.
   (a) Find the maximum current
   (b) Determine the frequency (cycles per second)
   (c) Calculate the current when $t = 0.005$ seconds
   (d) Find when the current first equals 5 amperes

(e) Determine the RMS (root mean square) value

39. A Ferris wheel has radius 25 meters and completes one revolution every 8 minutes. The center is 30 meters above ground.

    (a) Write an equation for height as a function of time
    (b) Find the maximum and minimum heights above ground
    (c) Calculate how long a passenger spends above 40 meters height
    (d) Determine the passenger's height after 3 minutes

40. Two waves with equations $y_1 = 3\sin 2x$ and $y_2 = 4\cos 2x$ are superimposed.

    (a) Find the equation of the resultant wave
    (b) Express the result in the form $R\sin(2x + \alpha)$
    (c) Determine the amplitude of the resultant wave
    (d) Find the phase difference between the original waves
    (e) Calculate the points where the waves interfere constructively

## Section I: Advanced Problem Solving

41. In triangle ABC, $a = 7$, $b = 8$, and $\angle C = 60°$.

    (a) Use the cosine rule to find side $c$
    (b) Use the sine rule to find $\angle A$
    (c) Calculate the area of the triangle
    (d) Find the radius of the circumcircle
    (e) Determine the length of the altitude from B to side AC

42. Prove that in any triangle ABC:

    (a) $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$ (sine rule)
    (b) $a^2 = b^2 + c^2 - 2bc\cos A$ (cosine rule)
    (c) $\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$
    (d) Area $= \frac{1}{2}bc\sin A = \frac{abc}{4R}$

43. A regular pentagon is inscribed in a circle of radius $r$.

    (a) Find the central angle for each sector
    (b) Calculate the side length of the pentagon
    (c) Find the area of the pentagon
    (d) Determine the apothem (distance from center to side)
    (e) Calculate the ratio of the pentagon's area to the circle's area

44. The function $f(x) = a\sin x + b\cos x$ has maximum value 5 and minimum value -5.

    (a) Express $f(x)$ in the form $R\sin(x + \alpha)$
    (b) Find the relationship between $a$ and $b$
    (c) If $f(\frac{\pi}{6}) = 3$, find the values of $a$ and $b$
    (d) Solve $f(x) = 4$ for $0 \leq x \leq 2\pi$
    (e) Find the values of $x$ where $f(x)$ achieves its maximum

45. Consider the identity $\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$.

(a) Verify this identity for $\theta = \frac{\pi}{6}$

(b) Use this to solve $\sin 5\theta = \sin\theta$ for $0 \leq \theta \leq 2\pi$

(c) Find the exact values of $\sin\frac{\pi}{10}$ and $\sin\frac{3\pi}{10}$

(d) Express $\cos 5\theta$ in terms of powers of $\cos\theta$

(e) Use these results to construct a regular decagon

**Answer Space**

Use this space for your working and answers.

**END OF TEST**

Total marks: 150

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