

GCSE Higher Mathematics

Practice Test 1: Number

Instructions:

Answer all questions. Show your working clearly.

Calculators may be used unless stated otherwise.

Time allowed: 90 minutes

Section A: Powers and Roots

1. Evaluate these expressions:

(a) 2^5

(b) 3^{-2}

(c) 5^0

(d) $(-2)^4$

(e) $(-3)^3$

(f) 4^{-3}

2. Write these as single powers:

(a) $2^3 \times 2^5$

(b) $3^7 \div 3^4$

(c) $(5^2)^3$

(d) $4^{-2} \times 4^6$

(e) $\frac{6^8}{6^{-2}}$

(f) $(2^3)^{-2}$

3. Evaluate these fractional indices:

(a) $9^{\frac{1}{2}}$

(b) $27^{\frac{1}{3}}$

(c) $16^{\frac{3}{4}}$

(d) $32^{\frac{2}{5}}$

(e) $125^{\frac{2}{3}}$

(f) $8^{-\frac{2}{3}}$

4. Simplify these expressions:

(a) $\sqrt{49}$

(b) $\sqrt[3]{64}$

- (c) $\sqrt[4]{81}$
- (d) $\sqrt{0.25}$
- (e) $\sqrt[3]{-8}$
- (f) $\sqrt[5]{32}$

5. Express in index form:

- (a) $\sqrt{7}$
- (b) $\sqrt[3]{5}$
- (c) $\frac{1}{\sqrt{2}}$
- (d) $\sqrt[4]{x^3}$
- (e) $\frac{1}{\sqrt[3]{a^2}}$
- (f) $\sqrt{x} \times \sqrt[3]{x}$

Section B: Laws of Indices

6. Simplify these expressions (no calculator):

- (a) $2^3 \times 2^{-1} \times 2^4$
- (b) $\frac{3^5 \times 3^{-2}}{3^{-1}}$
- (c) $(4^2)^{-3} \times 4^7$
- (d) $\frac{5^{-3} \times 5^8}{5^2}$

7. Evaluate these expressions:

- (a) $8^{\frac{2}{3}}$
- (b) $9^{-\frac{3}{2}}$
- (c) $16^{\frac{5}{4}}$
- (d) $25^{-\frac{3}{2}}$
- (e) $64^{\frac{1}{6}}$
- (f) $32^{-\frac{4}{5}}$

8. Simplify these expressions:

- (a) $x^{\frac{2}{3}} \times x^{\frac{1}{4}}$
- (b) $\frac{y^{\frac{3}{2}}}{y^{\frac{1}{6}}}$
- (c) $(a^{\frac{1}{2}})^4$
- (d) $\sqrt{x} \times x^{\frac{1}{3}}$
- (e) $\frac{\sqrt[3]{m^2}}{\sqrt{m}}$
- (f) $(p^{-\frac{1}{2}})^{-4}$

9. Write these in the form a^n where a and n are rational:

- (a) $\sqrt{2} \times 2^3$
- (b) $\frac{5^2}{\sqrt[3]{5}}$
- (c) $\sqrt[4]{3^3} \times 3^{-\frac{1}{2}}$
- (d) $\frac{\sqrt{7}}{\sqrt[3]{7^2}}$

Section C: Surds

10. Simplify these surds:

- (a) $\sqrt{18}$
- (b) $\sqrt{50}$
- (c) $\sqrt{72}$
- (d) $\sqrt{98}$
- (e) $\sqrt{200}$
- (f) $\sqrt{300}$

11. Simplify these expressions:

- (a) $3\sqrt{2} + 5\sqrt{2}$
- (b) $7\sqrt{3} - 2\sqrt{3}$
- (c) $\sqrt{8} + \sqrt{18}$
- (d) $\sqrt{50} - \sqrt{32}$
- (e) $2\sqrt{12} + 3\sqrt{27}$
- (f) $\sqrt{75} - \sqrt{48} + \sqrt{12}$

12. Multiply and simplify:

- (a) $\sqrt{3} \times \sqrt{12}$
- (b) $\sqrt{8} \times \sqrt{18}$
- (c) $2\sqrt{5} \times 3\sqrt{10}$
- (d) $\sqrt{6} \times \sqrt{24}$
- (e) $\sqrt{2} \times \sqrt{8} \times \sqrt{32}$
- (f) $3\sqrt{7} \times 2\sqrt{28}$

13. Expand and simplify:

- (a) $(2 + \sqrt{3})(5 - \sqrt{3})$
- (b) $(1 + \sqrt{5})(3 + 2\sqrt{5})$
- (c) $(4 - \sqrt{2})^2$
- (d) $(\sqrt{7} + \sqrt{3})(\sqrt{7} - \sqrt{3})$
- (e) $(2\sqrt{3} + 1)(2\sqrt{3} - 1)$
- (f) $(\sqrt{5} + 2)^2$

14. Rationalize the denominators:

- (a) $\frac{1}{\sqrt{5}}$
- (b) $\frac{3}{\sqrt{7}}$
- (c) $\frac{\sqrt{2}}{\sqrt{8}}$
- (d) $\frac{4}{2\sqrt{3}}$
- (e) $\frac{1}{3+\sqrt{2}}$
- (f) $\frac{2}{1-\sqrt{5}}$

Section D: More Complex Surd Operations

15. Rationalize these denominators:

(a) $\frac{3}{2+\sqrt{7}}$

(b) $\frac{5}{3-\sqrt{11}}$

(c) $\frac{\sqrt{3}}{1+\sqrt{3}}$

(d) $\frac{2\sqrt{5}}{3+\sqrt{5}}$

(e) $\frac{1}{\sqrt{6}-\sqrt{2}}$

(f) $\frac{\sqrt{7}+1}{\sqrt{7}-2}$

16. Simplify these expressions completely:

(a) $\frac{\sqrt{12}+\sqrt{27}}{\sqrt{3}}$

(b) $\frac{\sqrt{32}-\sqrt{8}}{\sqrt{2}}$

(c) $\sqrt{(3+\sqrt{5})(3-\sqrt{5})}$

(d) $\sqrt{50} - 2\sqrt{8} + \sqrt{18}$

(e) $(\sqrt{2} + \sqrt{8})^2$

(f) $\frac{\sqrt{48}}{\sqrt{3}} + \frac{\sqrt{75}}{\sqrt{3}}$

17. Prove that:

(a) $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$

(b) $\frac{1}{\sqrt{x}+\sqrt{y}} = \frac{\sqrt{x}-\sqrt{y}}{x-y}$

(c) $(a + b\sqrt{c})^2 = a^2 + 2ab\sqrt{c} + b^2c$

Section E: Standard Form

18. Write these numbers in standard form:

(a) 347000

(b) 0.000052

(c) 1890000000

(d) 0.0000000034

(e) 567.8

(f) 0.00456

19. Write these in ordinary form:

(a) 4.7×10^6

(b) 2.34×10^{-4}

(c) 9.876×10^{-7}

(d) 1.23×10^{10}

(e) 5.6×10^{-2}

(f) 7.89×10^5

20. Calculate, giving answers in standard form:

- (a) $(3 \times 10^4) \times (2 \times 10^6)$
- (b) $(8 \times 10^{-3}) \times (5 \times 10^7)$
- (c) $(6 \times 10^5) \div (2 \times 10^{-3})$
- (d) $(9 \times 10^{-4}) \div (3 \times 10^{-7})$
- (e) $(4 \times 10^3)^2$
- (f) $\sqrt{9 \times 10^8}$

21. Calculate these more complex expressions:

- (a) $(2.5 \times 10^3) \times (1.6 \times 10^{-5})$
- (b) $\frac{7.2 \times 10^8}{1.8 \times 10^{-4}}$
- (c) $(3.6 \times 10^{-2}) + (4.8 \times 10^{-3})$
- (d) $(5.4 \times 10^5) - (2.7 \times 10^4)$
- (e) $\frac{(2.4 \times 10^3) \times (1.5 \times 10^{-2})}{6 \times 10^{-1}}$
- (f) $(1.44 \times 10^6)^{\frac{1}{2}}$

Section F: Rational Numbers and Operations

22. Calculate these fractions (give answers in simplest form):

- (a) $\frac{2}{3} + \frac{5}{8}$
- (b) $\frac{7}{12} - \frac{3}{16}$
- (c) $\frac{4}{9} \times \frac{15}{8}$
- (d) $\frac{7}{10} \div \frac{21}{25}$
- (e) $\frac{3}{4} - \frac{5}{6} + \frac{7}{12}$
- (f) $(\frac{2}{3})^{-2}$

23. Convert these recurring decimals to fractions:

- (a) $0.\overline{3}$
- (b) $0.\overline{27}$
- (c) $0.1\overline{6}$
- (d) $0.\overline{142857}$
- (e) $2.3\overline{6}$
- (f) $0.58\overline{3}$

24. Work out these percentage calculations:

- (a) Increase 240 by 15%
- (b) Decrease 350 by 8%
- (c) Find 12.5% of 480
- (d) What percentage is 72 out of 160?
- (e) If 35% of a number is 91, find the number
- (f) A price increases from £80 to £92. Find the percentage increase

25. Solve these percentage problems:

- (a) After a 20% increase, a price is £144. Find the original price
- (b) After a 15% decrease, a quantity is 68. Find the original quantity
- (c) The value of a car decreases by 25% each year. If it's worth £12000 now, what was it worth 2 years ago?
- (d) An investment grows by 8% per year. After 2 years it's worth £2333.28. Find the initial investment

Section G: Complex Calculations

26. Simplify these mixed expressions:

- (a) $2^{-3} + 3^0 - 5^{-1}$
- (b) $\sqrt{16} \times 8^{\frac{1}{3}} - 2^{-2}$
- (c) $\frac{27^{\frac{2}{3}} - 16^{\frac{3}{4}}}{9^{\frac{1}{2}}}$
- (d) $8^{-\frac{2}{3}} + 25^{\frac{1}{2}} \times 4^{-1}$

27. Calculate exactly (leave surds in your answer):

- (a) $\frac{3}{\sqrt{2}} + \frac{2}{\sqrt{8}}$
- (b) $\sqrt{12} \times \sqrt{27} - \sqrt{48}$
- (c) $\frac{\sqrt{50} + \sqrt{32}}{\sqrt{2}}$
- (d) $(2\sqrt{3} - 1)^2$

28. Work with standard form in context:

- (a) The mass of an electron is 9.11×10^{-31} kg. Find the mass of 6.02×10^{23} electrons
- (b) Light travels at 3×10^8 m/s. How far does it travel in one year (use 1 year = 3.15×10^7 seconds)?
- (c) The radius of an atom is approximately 1×10^{-10} m. How many atoms would fit in a line 1 cm long?
- (d) A computer processes 2.4×10^9 operations per second. How many operations in 5 minutes?

Section H: Problem Solving

29. Prove that $\sqrt{2}$ is irrational. (Use proof by contradiction: assume $\sqrt{2} = \frac{p}{q}$ where p and q are integers with no common factors)

30. The golden ratio ϕ satisfies $\phi^2 = \phi + 1$.

- (a) Show that $\phi = \frac{1+\sqrt{5}}{2}$
- (b) Calculate ϕ to 4 decimal places
- (c) Find $\frac{1}{\phi}$ in surd form

31. Rationalize the denominator of $\frac{1}{\sqrt{2}+\sqrt{3}+\sqrt{5}}$. (Hint: First rationalize using $(\sqrt{2} + \sqrt{3}) - \sqrt{5}$)

32. A rectangle has sides of length $(2 + \sqrt{3})$ cm and $(2 - \sqrt{3})$ cm.

- (a) Find the exact area
- (b) Find the exact perimeter
- (c) Show that the area is rational but the perimeter is irrational

33. The population of bacteria doubles every 3 hours. If there are initially 5×10^4 bacteria:

- (a) How many bacteria after 12 hours?
- (b) Express your answer in standard form
- (c) After how many hours will there be more than 10^7 bacteria?

34. Show that $\frac{1}{\sqrt{a}+\sqrt{b}} + \frac{1}{\sqrt{a}-\sqrt{b}} = \frac{2\sqrt{a}}{a-b}$

35. A sphere has volume $V = \frac{4}{3}\pi r^3$. If the volume is $288\pi \text{ cm}^3$:

- (a) Find the radius in surd form
- (b) Find the surface area (use $A = 4\pi r^2$)
- (c) Express both answers exactly

36. The equation $x^2 - 4x + 1 = 0$ has solutions $x = 2 \pm \sqrt{3}$.

- (a) Verify this by substitution
- (b) Find $\frac{1}{2+\sqrt{3}} + \frac{1}{2-\sqrt{3}}$ without using a calculator
- (c) Hence find the sum of the reciprocals of the roots

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 100

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