A Level Pure Mathematics
Practice Test 6: Trigonometry

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Time allowed: 2 hours

Section A: Radian Measure and Basic Functions

1. Convert these angles from degrees to radians (leave answers in terms of π):
   (a) $12°$
   (b) $48°$
   (c) $84°$
   (d) $132°$
   (e) $168°$
   (f) $348°$

2. Convert these angles from radians to degrees:
   (a) $\frac{\pi}{30}$
   (b) $\frac{4\pi}{15}$
   (c) $\frac{7\pi}{9}$
   (d) $\frac{11\pi}{4}$
   (e) $\frac{16\pi}{3}$
   (f) $\frac{23\pi}{6}$

3. Find the exact values of these trigonometric ratios (without calculator):
   (a) $\sin(\frac{11\pi}{2})$, $\cos(\frac{11\pi}{2})$, $\tan(\frac{11\pi}{2})$
   (b) $\sin(\frac{13\pi}{2})$, $\cos(\frac{13\pi}{2})$, $\tan(\frac{13\pi}{2})$
   (c) $\sin(-3\pi)$, $\cos(-3\pi)$, $\tan(-3\pi)$
   (d) $\sin(-\frac{7\pi}{2})$, $\cos(-\frac{7\pi}{2})$, $\tan(-\frac{7\pi}{2})$

4. A circle has radius 24 cm. Find:
   (a) The arc length subtended by an angle of $\frac{13\pi}{16}$ radians
   (b) The area of the sector with angle $\frac{11\pi}{18}$ radians
   (c) The angle (in radians) that subtends an arc of length 54 cm
   (d) The radius of a circle where an angle of $\frac{8\pi}{9}$ radians subtends an arc of length 56 cm

5. Find the exact values:

(a) $\sin\frac{16\pi}{3}$
(b) $\cos\frac{17\pi}{4}$
(c) $\tan\frac{19\pi}{6}$
(d) $\sin\frac{22\pi}{3}$
(e) $\cos\frac{19\pi}{4}$
(f) $\tan\frac{25\pi}{6}$

Section B: Graphs of Trigonometric Functions

6. For the function $f(x) = \frac{2}{3}\sin x$:
   (a) State the domain and range
   (b) Find the period
   (c) Identify the zeros in the interval $[0, 2\pi]$
   (d) Find the maximum and minimum values and where they occur
   (e) Sketch the graph for $x \in [-2\pi, 2\pi]$
7. For the function $g(x) = \cos\frac{3x}{2}$:
   (a) State the domain and range
   (b) Find the period
   (c) Identify the zeros in the interval $[0, 4\pi]$
   (d) Find the maximum and minimum values and where they occur
   (e) Sketch the graph for $x \in [0, 4\pi]$
8. For the function $h(x) = \tan 4x$:
   (a) State the domain and range
   (b) Find the period
   (c) Identify the asymptotes in the interval $[0, \frac{\pi}{2}]$
   (d) Find the zeros in the interval $[0, \frac{\pi}{4}]$
   (e) Sketch the graph for $x \in [0, \frac{\pi}{2}]$
9. Sketch the graphs of these transformed functions for $x \in [0, 4\pi]$:
   (a) $y = \frac{2}{3}\cos x$
   (b) $y = \sin\frac{3x}{4}$
   (c) $y = \cos(x + \frac{\pi}{12})$
   (d) $y = \sin x + \frac{5}{2}$
   (e) $y = -\frac{3}{4}\cos x$
   (f) $y = \tan(x - \frac{\pi}{12})$
10. For the function $y = 7\sin(\frac{2x}{3} + \frac{\pi}{3}) + 5$:
   (a) Identify the amplitude
   (b) Find the period
   (c) Determine the phase shift
   (d) Find the vertical shift
   (e) State the range
   (f) Sketch the graph for $x \in [0, 3\pi]$

Section C: Fundamental Trigonometric Identities

11. Use the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$ to find:
    (a) $\sin\theta$ if $\cos\theta = \frac{13}{85}$ and θ is acute
    (b) $\cos\theta$ if $\sin\theta = -\frac{11}{61}$ and θ is in the fourth quadrant
    (c) $\tan\theta$ if $\sin\theta = \frac{35}{37}$ and $\cos\theta > 0$
    (d) $\sin\theta$ if $\tan\theta = -\frac{39}{80}$ and $\cos\theta > 0$

12. Prove these even-odd identities:
    (a) $\sin(-\theta) = -\sin\theta$ (sine is odd)
    (b) $\cos(-\theta) = \cos\theta$ (cosine is even)
    (c) $\tan(-\theta) = -\tan\theta$ (tangent is odd)
    (d) $\sec(-\theta) = \sec\theta$ (secant is even)

13. Simplify these expressions:
    (a) $\tan^2\theta(1 + \cot^2\theta)$
    (b) $\frac{\tan\theta}{\sec\theta} - \frac{\cot\theta}{\csc\theta}$
    (c) $(\csc\theta + \sin\theta)^2$
    (d) $\frac{\sec^2\theta - \tan^2\theta}{\cos\theta}$

14. Express in terms of $\cot\theta$ only:
    (a) $\tan^2\theta$
    (b) $\sin^2\theta$
    (c) $\cos^2\theta$
    (d) $\tan^2\theta + \sin^2\theta\csc^2\theta$

15. Prove that:
    (a) $\frac{\sin^2\theta}{\cos\theta+1} = \frac{\cos\theta-1}{\cos\theta+1} \cdot \cos\theta$
    (b) $\tan^4\theta - \sin^4\theta = \tan^4\theta\sin^4\theta$
    (c) $\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta} = \frac{\tan\theta-1}{\tan\theta+1}$
    (d) $(1 + \tan\theta)(1 - \tan\theta) = \cos^2\theta - \sin^2\theta$

Section D: Addition and Double Angle Formulas

16. Use the addition formulas to find the exact values:
    (a) $\cos 315°$ (using $\cos(270° + 45°)$)
    (b) $\sin 345°$ (using $\sin(300° + 45°)$)
    (c) $\tan 345°$ (using $\tan(300° + 45°)$)
    (d) $\cos\frac{19\pi}{12}$ (using $\cos(\frac{4\pi}{3} + \frac{\pi}{4})$)

17. Given $\cos A = \frac{35}{37}$ with A acute and $\sin B = \frac{33}{65}$ with B acute:
    (a) Find $\sin A$ and $\cos B$
    (b) Calculate $\sin(A - B)$
    (c) Calculate $\cos(A + B)$
    (d) Find $\tan(2A + B)$

18. Use double angle formulas to find:
    (a) $\cos 2\theta$ if $\sin\theta = \frac{35}{37}$ and θ is acute
    (b) $\sin 2\theta$ if $\cos\theta = \frac{39}{89}$ and θ is acute
    (c) $\tan 2\theta$ if $\tan\theta = \frac{13}{84}$
    (d) $\sin 2\theta$ if $\cos\theta = -\frac{11}{61}$ and θ is in the second quadrant
19. Derive these alternative half-angle formulas:
    (a) $\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$
    (b) $\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$
    (c) $\tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\sin\theta}$
    (d) $\cot\frac{\theta}{2} = \frac{1+\cos\theta}{\sin\theta} = \frac{\sin\theta}{1-\cos\theta}$
20. Express using half-angle formulas:
    (a) $\sin 22.5^\circ$ using $\sin\frac{45^\circ}{2}$
    (b) $\cos 15^\circ$ using $\cos\frac{30^\circ}{2}$
    (c) $\tan 67.5^\circ$ using $\tan\frac{135^\circ}{2}$
    (d) $\cot 37.5^\circ$ using $\cot\frac{75^\circ}{2}$

Section E: Solving Trigonometric Equations

21. Solve these equations for $0 \le x \le 2\pi$:
    (a) $\cos x = \frac{\sqrt{2}}{2}$
    (b) $\sin x = -\frac{\sqrt{3}}{2}$
    (c) $\tan x = \frac{1}{\sqrt{3}}$
    (d) $\cos x = -\frac{\sqrt{2}}{2}$
22. Solve these equations for $0^\circ \le x \le 360^\circ$:
    (a) $5\sin x - 4 = 0$
    (b) $3\cos x + 2 = 0$
    (c) $2\tan x - \sqrt{3} = 0$
    (d) $3\sin^2 x = 2\sin x$
23. Solve these equations for $0 \le x \le 2\pi$:
    (a) $\sin 5x = \frac{\sqrt{2}}{2}$
    (b) $\cos\frac{x}{4} = -\frac{1}{2}$
    (c) $\tan 8x = \sqrt{3}$
    (d) $\cos(x - \frac{\pi}{3}) = -\frac{\sqrt{3}}{2}$
24. Solve these quadratic trigonometric equations for $0 \le x \le 2\pi$:
    (a) $5\sin^2 x - 4\sin x - 1 = 0$
    (b) $4\cos^2 x - 3\cos x - 1 = 0$
    (c) $3\tan^2 x + 4\tan x + 1 = 0$

(d) $7\sin^2 x - 8\sin x + 1 = 0$

25. Solve these equations involving multiple angles for $0 \leq x \leq 2\pi$:

(a) $\cot x = \tan x$

(b) $\sin 3x = \cos x$

(c) $\cos 4x = \sin 2x$

(d) $\tan 8x = \tan 3x$

Section F: Advanced Trigonometric Identities

26. Prove these Newton's identities for triangle ABC:

(a) $a\cos A + b\cos B + c\cos C = a + b + c - 4R\sin A\sin B\sin C$

(b) $a\sin A + b\sin B + c\sin C = 2(a\cos A + b\cos B + c\cos C)$

(c) $\cos A + \cos B + \cos C = 1 + \frac{r}{R}$ where r is inradius, R is circumradius

(d) $\sin A + \sin B + \sin C = \frac{s}{R}$ where s is semiperimeter

27. Use trigonometric identities to simplify:

(a) $\sin 12x - \sin 4x$

(b) $\cos 13x + \cos 7x$

(c) $\sin 112.5^\circ - \sin 67.5^\circ$

(d) $\cos 157.5^\circ + \cos 112.5^\circ$

28. Prove these hyperbolic-trigonometric relationships:

(a) $\cos(ix) = \cosh(x)$ where $i = \sqrt{-1}$

(b) $\sin(ix) = i\sinh(x)$

(c) $\cosh^2(x) - \sinh^2(x) = 1$

(d) $\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$

29. Transform using complex analysis:

(a) $\sin^6\theta$ as a sum of cosines

(b) $\cos^6\theta$ as a sum of cosines

(c) $\sin^2\theta\cos^4\theta$ as a sum

(d) $\sin^4\theta\cos^2\theta$ as a sum

30. Prove the septuple angle formulas:

(a) $\sin 7\theta = 64\sin^7\theta - 112\sin^5\theta + 56\sin^3\theta - 7\sin\theta$

(b) $\cos 7\theta = 64\cos^7\theta - 112\cos^5\theta + 56\cos^3\theta - 7\cos\theta$

(c) Verify for $\theta = \frac{\pi}{7}$

Section G: Complex Trigonometric Problems

31. Solve these equations for $0 \leq x < 2\pi$:
    (a) $3\sin x - 2\cos x = 2$
    (b) $\cos x + \cos 2x = 0$
    (c) $\sin x + \sin 7x + \sin 9x = 0$
    (d) $\tan x + \tan 3x = 0$

32. Prove these sophisticated identities:
    (a) $\frac{\sin 7\theta}{\sin\theta} - \frac{\cos 7\theta}{\cos\theta} = 2\sin 6\theta$
    (b) $\sin^2\theta + \sin^2(\theta + \frac{2\pi}{5}) + \sin^2(\theta + \frac{4\pi}{5}) + \sin^2(\theta + \frac{6\pi}{5}) + \sin^2(\theta + \frac{8\pi}{5}) = \frac{5}{2}$
    (c) $\sin^{10}\theta + \cos^{10}\theta = (\sin^2\theta + \cos^2\theta)(\sin^8\theta - \sin^6\theta\cos^2\theta + \sin^4\theta\cos^4\theta - \sin^2\theta\cos^6\theta + \cos^8\theta)$
    (d) $\tan\frac{A}{2}\tan\frac{B}{2} + \tan\frac{B}{2}\tan\frac{C}{2} + \tan\frac{C}{2}\tan\frac{A}{2} = 1$ when $A + B + C = \pi$

33. Find the general solution to these equations:
    (a) $\sin x = \frac{6}{7}$
    (b) $\cos 7x = 0.9$
    (c) $\tan\frac{3x}{4} = 7$
    (d) $\sin(7x - \frac{\pi}{3}) = -\frac{1}{2}$

34. Express these in the form $R\sin(x + \alpha)$ or $R\cos(x + \alpha)$:
    (a) $33\sin x + 56\cos x$
    (b) $39\sin x - 52\cos x$
    (c) $7\sin x + 7\cos x$
    (d) $12\cos x + 12\sqrt{3}\sin x$

35. Find the range of these functions:
    (a) $f(x) = 33\sin x + 56\cos x$
    (b) $g(x) = 39\sin 6x - 52\cos 6x - 20$
    (c) $h(x) = \sin^2 x + 8\cos x$
    (d) $k(x) = 12\sin x\cos x - 5$

Section H: Applications of Trigonometry

36. A rotating wheel has angular position $\phi = 0.6\sin(8t - \frac{\pi}{4})$ radians, where t is time in seconds.
    (a) Find the maximum angular displacement
    (b) Determine the period of rotation
    (c) Find the phase shift
    (d) Calculate the angular position when $t = 0$
    (e) Find when the wheel first reaches maximum negative displacement

37. The height of a platform on a Ferris wheel is modeled by $h(t) = 25\sin(\frac{\pi t}{10} - \frac{\pi}{2}) + 30$ meters, where t is time in minutes.
    (a) Find the maximum and minimum heights
    (b) Determine the period of one complete revolution

(c) Find the height after 7 minutes
(d) Calculate when the platform reaches its lowest point
(e) Find when the height is exactly 42.5 meters

38. A laser beam oscillates with angular displacement $\beta = 0.05\cos(120\pi t + \frac{\pi}{3})$ radians.
(a) Find the maximum angular displacement
(b) Determine the frequency of oscillation
(c) Calculate the angular displacement when $t = \frac{1}{240}$ seconds
(d) Find when the beam first crosses the zero position
(e) Determine the angular velocity amplitude

39. A vibrating string has displacement $y = 0.8\sin(100\pi t - \frac{\pi}{4})$ millimeters, where t is time in seconds.
(a) Find the maximum displacement amplitude
(b) Determine the frequency of vibration
(c) Calculate the displacement at $t = 0$
(d) Find when the string first reaches maximum displacement
(e) Determine the period of one complete oscillation

40. Two electromagnetic waves with equations $E_1 = 9\sin 12x$ and $E_2 = 40\cos 12x$ interfere.
(a) Find the equation of the resultant field
(b) Express the result in the form $R\cos(12x - \alpha)$
(c) Determine the amplitude of the resultant field
(d) Find the phase relationship between the original fields
(e) Calculate the power ratio of the combined field to the individual fields

Section I: Advanced Problem Solving

41. In triangle JKL, $j = 29$, $k = 35$, and $\angle L = 150°$.
(a) Use the cosine rule to find side l
(b) Use the sine rule to find $\angle J$
(c) Calculate the area of the triangle
(d) Find the length of the median from L to side JK
(e) Determine the radius of the excircle opposite vertex L

42. Prove that in any triangle JKL:
(a) $\frac{j}{\sin J} = \frac{k}{\sin K} = \frac{l}{\sin L} = 2R$ (sine rule)
(b) $j^2 = k^2 + l^2 - 2kl\cos J$ (cosine rule)
(c) $r_a + r_b + r_c = 4R + r$ where r_a, r_b, r_c are exradii, R is circumradius, r is inradius
(d) Area $= \sqrt{s(s-j)(s-k)(s-l)}$ where $s = \frac{j+k+l}{2}$ (Heron's formula)

43. A regular icosagon (20-sided polygon) is inscribed in a circle of radius r.
(a) Find the central angle for each sector
(b) Calculate the side length of the icosagon
(c) Find the area of the icosagon
(d) Determine the apothem length

(e) Calculate the perimeter of the icosagon

44. The function $m(x) = w \sin 6x + z \cos 6x$ has maximum value 65 and minimum value -65.

(a) Express $m(x)$ in the form $R \sin(6x + \zeta)$
(b) Find the relationship between w and z
(c) If $m(\frac{\pi}{36}) = 52$, find the values of w and z
(d) Solve $m(x) = 39$ for $0 \leq x \leq \frac{\pi}{3}$
(e) Find the critical points of $m(x)$ in the interval $[0, \frac{\pi}{3}]$

45. Consider the Chebyshev polynomial identity: $\cos(n \arccos x) = T_n(x)$ where T_n is the n-th Chebyshev polynomial.

(a) Find $T_6(x)$ such that $T_6(\cos\theta) = \cos 6\theta$
(b) Verify this identity for $x = \frac{1}{2}$ (i.e., $\theta = \frac{\pi}{3}$)
(c) Use this to solve $\cos 6\theta = \cos 2\theta$ for $0 \leq \theta \leq 2\pi$
(d) Find the exact values of $\cos\frac{\pi}{9}$, $\cos\frac{2\pi}{9}$, and $\cos\frac{4\pi}{9}$
(e) Apply these results to construct a regular enneagon (9-sided polygon)

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 150

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