

# GCSE Higher Mathematics

## Practice Test 1: Statistics

### Instructions:

Answer all questions. Show your working clearly.  
Calculators may be used unless stated otherwise.

Time allowed: 90 minutes

### Section A: Advanced Averages and Spread

1. The table shows the distribution of test scores:

Score	Frequency
10-19	3
20-29	8
30-39	15
40-49	22
50-59	18
60-69	12
70-79	7
80-89	3

Calculate:

- The total number of students
  - An estimate of the mean score
  - The modal class
  - An estimate of the median score
  - An estimate of the range
2. For the data set: 12, 15, 18, 20, 22, 25, 28, 30, 35, 40
- Calculate the mean
  - Find the median and quartiles (Q1 and Q3)
  - Calculate the interquartile range
  - Calculate the standard deviation
  - Identify any outliers using the  $1.5 \times \text{IQR}$  rule
3. Two data sets have the following properties:
- Set A: mean = 45, standard deviation = 8,  $n = 20$
  - Set B: mean = 52, standard deviation = 12,  $n = 30$
- Calculate the combined mean

- (b) Calculate the combined standard deviation
  - (c) Which set is more consistent? Explain.
  - (d) Calculate the coefficient of variation for each set
4. The weights (in kg) of 50 people are summarized:

$$\sum x = 3250, \sum x^2 = 218750$$

- (a) Calculate the mean weight
- (b) Calculate the variance
- (c) Calculate the standard deviation
- (d) If each person gains 2 kg, find the new mean and standard deviation

## Section B: Histograms and Frequency Density

5. The histogram shows the distribution of journey times:

*[Imagine a histogram with: 0-10 min (density 2.5), 10-20 min (density 4.0), 20-30 min (density 3.5), 30-50 min (density 1.5), 50-80 min (density 0.8)]*

- (a) Complete the frequency table
  - (b) Calculate the total number of journeys
  - (c) Estimate the mean journey time
  - (d) Find the modal class
  - (e) What percentage of journeys take more than 30 minutes?
6. Draw a histogram for this data about house prices:

Price (£000s)	Frequency
100-150	20
150-200	35
200-220	18
220-250	24
250-300	15
300-400	8

- (a) Calculate the frequency density for each class
  - (b) Draw the histogram
  - (c) Estimate the median house price
  - (d) What fraction of houses cost more than £220,000?
7. A histogram shows data with unequal class widths. The class 20-25 has frequency density 6 and the class 25-35 has frequency 40.
- (a) Find the frequency for the 20-25 class
  - (b) Find the frequency density for the 25-35 class
  - (c) If the total frequency is 200, suggest frequencies for other classes

## Section C: Cumulative Frequency and Box Plots

8. The table shows the cumulative frequency of exam marks:

Mark	Cumulative Frequency
$\leq 20$	5
$\leq 30$	18
$\leq 40$	35
$\leq 50$	58
$\leq 60$	76
$\leq 70$	88
$\leq 80$	95
$\leq 90$	100

- Draw the cumulative frequency curve
- Find the median
- Find the quartiles  $Q_1$  and  $Q_3$
- Calculate the interquartile range
- Draw a box plot
- Estimate the 85th percentile

9. Two box plots show the distribution of heights for boys and girls:

*[Imagine box plots: Boys (min 160,  $Q_1$  168, median 175,  $Q_3$  182, max 195), Girls (min 152,  $Q_1$  160, median 165,  $Q_3$  172, max 185)]*

Compare the distributions by commenting on:

- Central tendency (medians)
  - Spread (ranges and IQRs)
  - Shape and outliers
  - Which group is more variable?
10. The cumulative frequency curve for reaction times (in milliseconds) passes through these points: (200, 0), (250, 12), (300, 28), (350, 45), (400, 58), (450, 67), (500, 70)
- Find the median reaction time
  - Find the quartiles
  - What percentage have reaction times between 280ms and 420ms?
  - Draw the corresponding box plot

## Section D: Scatter Graphs and Correlation

11. The table shows data for 10 students:

Hours study	2	4	6	8	10	12	14	16	18	20
Test score	45	52	58	65	70	75	80	85	88	92

- Plot a scatter graph
- Describe the correlation
- Calculate the equation of the line of best fit
- Use your line to predict the score for 15 hours study

- (e) Estimate how many hours needed to score 95
  - (f) Calculate the correlation coefficient
12. The equation of a regression line is  $y = 1.8x + 12$ .
- (a) Interpret the gradient
  - (b) Interpret the y-intercept
  - (c) If  $x = 25$ , predict  $y$
  - (d) If  $y = 75$ , estimate  $x$
  - (e) State assumptions made when using this model
13. Classify these correlation coefficients and describe the relationships:
- (a)  $r = 0.95$
  - (b)  $r = -0.82$
  - (c)  $r = 0.15$
  - (d)  $r = -0.98$
  - (e)  $r = 0.68$

## Section E: Advanced Probability

14. A bag contains 5 red balls, 3 blue balls, and 2 green balls. Two balls are drawn without replacement.
- (a) Draw a tree diagram
  - (b) Find  $P(\text{both red})$
  - (c) Find  $P(\text{both same color})$
  - (d) Find  $P(\text{at least one blue})$
  - (e) Find  $P(\text{different colors})$
15. The probability that it rains on any day is 0.3, independently of other days.
- (a) Find the probability it rains on exactly 2 out of 5 days
  - (b) Find the probability it rains on at least 3 out of 5 days
  - (c) Find the expected number of rainy days in a week
  - (d) In a 30-day month, find  $P(\text{more than 12 rainy days})$
16. A multiple choice test has 10 questions, each with 4 options. A student guesses randomly.
- (a) Find  $P(\text{correct answer on one question})$
  - (b) Find  $P(\text{exactly 3 correct answers})$
  - (c) Find  $P(\text{at least 7 correct answers})$
  - (d) Find the expected number of correct answers
  - (e) Find the most likely number of correct answers
17. Events A and B are such that  $P(A) = 0.6$ ,  $P(B) = 0.4$ , and  $P(A \cap B) = 0.24$ .
- (a) Find  $P(A \cap B)$
  - (b) Find  $P(A')$
  - (c) Find  $P(A \cup B)$
  - (d) Find  $P(B \cap A)$
  - (e) Are A and B independent? Justify your answer

## Section F: Conditional Probability and Independence

18. A survey of 200 people about pet ownership gives:

- 120 own a dog
- 80 own a cat
- 50 own both a dog and a cat

- (a) Draw a Venn diagram
- (b) Find  $P(\text{owns a dog} \mid \text{owns a cat})$
- (c) Find  $P(\text{owns a cat} \mid \text{owns a dog})$
- (d) Find  $P(\text{owns exactly one type of pet})$
- (e) Are pet ownership independent? Explain

19. In a factory, 70% of items are produced by Machine A and 30% by Machine B. Machine A produces 5% defective items, Machine B produces 8% defective items.

- (a) Draw a tree diagram
- (b) Find the probability an item is defective
- (c) If an item is defective, find the probability it came from Machine A
- (d) If an item is not defective, find the probability it came from Machine B

20. A box contains 8 red cards numbered 1-8 and 6 blue cards numbered 1-6. A card is drawn at random.

- (a) Find  $P(\text{red and even number})$
- (b) Find  $P(\text{blue} \mid \text{odd number})$
- (c) Find  $P(\text{number} < 5)$
- (d) Are color and number type independent?

21. A medical test is 95% accurate for positive cases and 98% accurate for negative cases. 2% of the population has the condition.

- (a) Find the probability of testing positive
- (b) If someone tests positive, find the probability they have the condition
- (c) If someone tests negative, find the probability they don't have the condition
- (d) Comment on the effectiveness of the test

## Section G: Hypothesis Testing and Sampling

22. A coin is suspected of being biased towards heads. It's flipped 20 times and lands heads 15 times.

- (a) State the null and alternative hypotheses
- (b) Calculate the probability of getting 15 or more heads if the coin is fair
- (c) At the 5% significance level, is there evidence the coin is biased?
- (d) What would be a Type I error in this context?

23. A sample of 50 light bulbs has mean lifetime 980 hours and standard deviation 120 hours.

- (a) Calculate a 95% confidence interval for the population mean
- (b) Interpret your confidence interval

- (c) What assumptions are made?
- (d) How would the interval change with a larger sample size?
24. A manufacturer claims 95% of items pass quality control. In a sample of 100 items, 92 pass.
- (a) Test at 5% level whether the claim is justified
- (b) Calculate the critical value
- (c) State your conclusion
- (d) What is the power of this test?

## Section H: Problem Solving and Integration

25. A company records daily sales over 100 days. The data shows:
- Mean = £2450
  - Standard deviation = £380
  - Distribution is approximately normal
- (a) Find  $P(\text{sales} < £3000)$
- (b) Find the sales value exceeded on 10% of days
- (c) What percentage of days have sales within one standard deviation of the mean?
- (d) If the company operates 250 days per year, estimate annual sales
26. A quality control process samples 20 items every hour. Over 8 hours, the number of defective items found was: 2, 1, 3, 0, 2, 1, 4, 2.
- (a) Calculate the mean and standard deviation
- (b) Test whether the defect rate exceeds 5%
- (c) Create a control chart with warning limits
- (d) Comment on process stability
27. Compare these three investment options over 5 years:
- Option A: Mean return 8%, standard deviation 12%
  - Option B: Mean return 6%, standard deviation 8%
  - Option C: Mean return 10%, standard deviation 18%
- (a) Calculate the coefficient of variation for each
- (b) Which offers the best risk-adjusted return?
- (c) Using normal distribution, find  $P(\text{negative return})$  for each
- (d) Recommend an option for a risk-averse investor
28. A school monitors student performance using two tests. The correlation between Test 1 and Test 2 scores is 0.75.
- (a) What does this correlation suggest?
- (b) If Test 1 has mean 65 and standard deviation 10, and Test 2 has mean 70 and standard deviation 12, find the regression equation
- (c) Predict Test 2 score for a student scoring 80 on Test 1
- (d) Calculate the coefficient of determination and interpret it

29. Design a statistical investigation to determine if a new teaching method improves student performance:
- (a) State hypotheses
  - (b) Describe the sampling method
  - (c) Identify variables and potential confounding factors
  - (d) Outline the analysis plan
  - (e) Discuss limitations and assumptions

**Answer Space**

Use this space for your working and answers.

**END OF TEST**

Total marks: 100

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