A Level Mechanics
Practice Test 4: Newton's Laws of Motion

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Draw diagrams where appropriate to illustrate your solutions.
Time allowed: 2 hours 30 minutes

Section A: Advanced Force Theory [35 marks]

Question 1 [20 marks]

(a) Explain how Newton's second law can be expressed in terms of momentum and discuss when this form is more useful than $F = ma$. **[5 marks]**

(b) A particle moves in a non-inertial reference frame that accelerates at 2 m/s^2 horizontally. Describe the apparent forces that an observer in this frame would need to include in their analysis. **[6 marks]**

(c) State the principle of superposition for forces and explain why it is fundamental to Newton's laws. **[4 marks]**

(d) Discuss why Newton's laws fail at very high speeds and very small scales, mentioning the theories that replace them. **[5 marks]**

Question 2 [15 marks] A particle of mass 2.5 kg is acted upon by forces $\vec{F_1} = (6t+3)\hat{i}+(4t^2-2)\hat{j}$ N and $\vec{F_2} = (-3t+8)\hat{i}+(5-2t^2)\hat{j}$ N, where t is time in seconds.

(a) Calculate the net force and acceleration as functions of time. **[4 marks]**

(b) If the particle starts from rest at position $(1, 2)$ m, find its velocity at t = 3 s. **[6 marks]**

(c) Determine the position vector at t = 3 s. **[5 marks]**

Section B: Sophisticated Equilibrium Analysis [40 marks]

Question 3 [25 marks] A uniform beam of mass 18 kg and length 5 m is supported by three vertical ropes. Rope A is at the left end, rope B is 2 m from the left end, and rope C is at the right end. Loads of 15 kg and 8 kg are placed 1.5 m and 3.5 m from the left end respectively.

(a) Write the complete equilibrium equations for the system. **[4 marks]**

(b) If the tension in rope B is 120 N, calculate the tensions in ropes A and C. **[8 marks]**

(c) Determine where a 20 kg load must be placed to make the tension in rope A zero. **[6 marks]**

(d) Find the range of positions where the 20 kg load can be placed without any rope becoming slack. **[4 marks]**

(e) Verify your solution by checking equilibrium about different pivot points. **[3 marks]**

Question 4 [15 marks] Three forces act on a rigid body: $\vec{F_1} = 25\hat{i} + 35\hat{j}$ N at $(2, 1)$ m, $\vec{F_2} = -15\hat{i} + 45\hat{j}$ N at $(4, 0)$ m, and $\vec{F_3} = -20\hat{i} - 60\hat{j}$ N at $(0, 3)$ m.

(a) Verify that the forces are in equilibrium. **[3 marks]**

(b) Calculate the resultant moment about the origin. **[5 marks]**

(c) If force $\vec{F_2}$ is removed, find the replacement force and its position to maintain equilibrium. **[7 marks]**

Section C: Advanced Friction Dynamics [38 marks]

Question 5 [22 marks] Three blocks with masses 5 kg, 7 kg, and 6 kg are stacked vertically on a horizontal surface. The coefficients of friction are: between 5 kg and 7 kg blocks $\mu = 0.6$, between 7 kg and 6 kg blocks $\mu = 0.4$, between 6 kg block and surface $\mu = 0.35$.

(a) A horizontal force F is applied to the middle (7 kg) block. Find the critical values of F at which relative motion begins. **[8 marks]**

(b) If F = 45 N, determine the motion of all three blocks. **[8 marks]**

(c) If F = 70 N is applied to the top block instead, analyze the resulting motion. **[6 marks]**

Question 6 [16 marks] A block of mass 10 kg rests on an inclined plane of angle 32°. The coefficients of friction are $\mu_s = 0.75$ and $\mu_k = 0.55$.

(a) Determine whether the block remains stationary on the incline. **[4 marks]**

(b) A force F is applied horizontally (parallel to the base) to push the block up the incline. Find the minimum F to initiate upward motion. **[6 marks]**

(c) If F = 90 N is applied at 25° above the horizontal, calculate the acceleration of the block. **[6 marks]**

Section D: Complex Multi-Body Systems [42 marks]

Question 7 [24 marks] A pulley system involves four masses: 6 kg and 8 kg on a rough horizontal table ($\mu = 0.3$), connected by strings that pass over pulleys to masses of 4 kg and 10 kg hanging vertically.

(a) Draw detailed free body diagrams for all four masses. **[8 marks]**

(b) Establish the equations of motion for the system. **[6 marks]**

(c) Calculate the acceleration of the system and determine the direction of motion. **[6 marks]**

(d) Find the tension in each string segment. **[4 marks]**

Question 8 [18 marks] A freight train consists of an engine (mass 150,000 kg) and four carriages (masses 25,000 kg, 30,000 kg, 35,000 kg, and 20,000 kg). The engine provides 180,000 N thrust, and resistance forces are proportional to mass with constant 1.0 N/kg.

(a) Calculate the acceleration on level track. **[4 marks]**

(b) Determine the tension in each coupling between vehicles. **[8 marks]**

(c) If the train climbs a 3.5° gradient, find the new acceleration and coupling tensions. **[6 marks]**

Section E: Advanced Circular Motion Systems [32 marks]

Question 9 [**18 marks**] A highway curve has radius 180 m and is banked at 18°. The coefficient of friction varies from 0.4 to 0.9 depending on road conditions.

(a) Calculate the ideal speed for the banking (no friction needed). [**4 marks**]

(b) Determine the maximum safe speed in poor conditions ($\mu = 0.4$). [**6 marks**]

(c) Find the minimum speed in excellent conditions ($\mu = 0.9$) below which vehicles would slide down. [**5 marks**]

(d) Calculate the forces on a 1200 kg car traveling at 30 m/s in good conditions ($\mu = 0.7$). [**3 marks**]

Question 10 [**14 marks**] A mass of 0.8 kg is suspended by two strings of lengths 2.0 m and 1.4 m attached to points 2.2 m apart in the same horizontal plane. The mass moves in a horizontal circle 1.2 m below the attachment points.

(a) Calculate the radius of circular motion and the angles of both strings with the vertical. [**5 marks**]

(b) Find the tension in each string. [**6 marks**]

(c) Determine the speed and period of the circular motion. [**3 marks**]

Section F: Variable Force Analysis [30 marks]

Question 11 [**18 marks**] A particle of mass 3.5 kg moves along a straight line under force $F = 18\cos(1.5t) + 12\sin(0.8t) - 6$ N, where t is in seconds.

(a) Starting from rest at the origin, derive expressions for velocity and displacement. [**8 marks**]

(b) Calculate the velocity and displacement at t = π seconds. [**4 marks**]

(c) Find the times when the particle is momentarily at rest during the first 2π seconds. [**6 marks**]

Question 12 [**12 marks**] An elevator accelerates according to $a(t) = 2.5 - 0.3t$ m/s² for 0 t 6 s, then moves at constant velocity for 8 seconds, then decelerates at -1.8 m/s² until it stops.

(a) Calculate the maximum velocity reached and total displacement. [**6 marks**]

(b) If an 80 kg person is in the elevator, sketch the variation of apparent weight throughout the journey. [**4 marks**]

(c) Determine the maximum and minimum scale readings during the motion. [**2 marks**]

Section G: Advanced Constraint Systems [25 marks]

Question 13 [**25 marks**] A particle of mass 0.4 kg slides on a smooth track shaped like $y = 0.08x^2$ (where x and y are in meters) in a vertical plane. The particle starts from rest at point (5, 2).

(a) Find the speed when the particle reaches the vertex (0, 0). [**4 marks**]

(b) Calculate the normal force from the track at the vertex. [**5 marks**]

(c) Determine the normal force when the particle is at position (3, 0.72). [**6 marks**]

(d) If the track can provide a maximum normal force of 25 N, find where the particle leaves the track. [**6 marks**]

(e) Calculate the trajectory of the particle after leaving the track. **[4 marks]**

Physics Data and Formulae

Newton's Laws:
First Law: $\sum \vec{F} = 0 \Leftrightarrow \vec{a} = 0$
Second Law: $\vec{F}_{net} = m\vec{a} = \frac{d\vec{p}}{dt}$
Third Law: $\vec{F}_{AB} = -\vec{F}_{BA}$

Force Analysis:
Friction: $f_s \leq \mu_s N$, $f_k = \mu_k N$
Inclined plane: $F_{\parallel} = mg\sin\theta$, $N = mg\cos\theta$
Circular motion: $F_c = \frac{mv^2}{r}$, $a_c = \frac{v^2}{r}$

Equilibrium Analysis:
Force equilibrium: $\sum F_x = 0$, $\sum F_y = 0$
Moment equilibrium: $\sum M = 0$ (about any point)
Moment calculation: $M = \vec{r} \times \vec{F}$

Circular Motion with Banking:
No friction: $\tan\theta = \frac{v^2}{rg}$
With friction: $\frac{mv^2}{r} = mg\tan\theta \pm \mu mg\sec\theta$
Apparent force: $F_{apparent} = m\vec{a}_{frame}$ (non-inertial)

Variable Motion:
$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$
$v = \int a\,dt$, $s = \int v\,dt$
For projectile: $s = ut + \frac{1}{2}gt^2$

Constraint Forces:
Normal force: perpendicular to surface
Tension: along string direction
Curvature: $\kappa = \frac{d^2y/dx^2}{(1+(dy/dx)^2)^{3/2}}$

Vector Operations:
Magnitude: $|\vec{F}| = \sqrt{F_x^2 + F_y^2 + F_z^2}$
Cross product: $\vec{A} \times \vec{B} = (A_yB_z - A_zB_y)\hat{i} + ...$
Integration: $\int \sin(at)\,dt = -\frac{1}{a}\cos(at) + C$

Constants:
Acceleration due to gravity: $g = 9.8$ m/s^2

Trigonometric Values:
$\sin 18 = 0.309$, $\cos 18 = 0.951$, $\tan 18 = 0.325$
$\sin 25 = 0.423$, $\cos 25 = 0.906$, $\tan 25 = 0.466$
$\sin 32 = 0.530$, $\cos 32 = 0.848$, $\tan 32 = 0.625$
$\sin 3.5 = 0.061$, $\cos 3.5 = 0.998$, $\tan 3.5 = 0.061$

END OF TEST

Total marks: 242

Grade boundaries: A* 218, A 194, B 169, C 145, D 121, E 97

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