A Level Pure Mathematics
Practice Test 4: Vectors

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Time allowed: 2 hours

Section A: Vector Basics and Notation

1. Given vectors $\mathbf{s} = \begin{pmatrix} 6 \\ -4 \\ 2 \end{pmatrix}$ and $\mathbf{t} = \begin{pmatrix} -3 \\ 2 \\ 5 \end{pmatrix}$, calculate:
   (a) $\mathbf{s} + \mathbf{t}$
   (b) $\mathbf{s} - \mathbf{t}$
   (c) $4\mathbf{s} + 2\mathbf{t}$
   (d) $3\mathbf{s} - 5\mathbf{t}$
   (e) $|\mathbf{s}|$ and $|\mathbf{t}|$
   (f) A unit vector in the direction of $\mathbf{s}$

2. Express these vectors in component form:
   (a) $\overrightarrow{MN}$ where $M(4, 2, -3)$ and $N(1, 5, 2)$
   (b) $\overrightarrow{TU}$ where $T(-2, 1, 3)$ and $U(4, -3, 1)$
   (c) The position vector of point V if $\overrightarrow{OV} = 4\mathbf{i} - 3\mathbf{j} + 7\mathbf{k}$
   (d) $\overrightarrow{NM}$ where $M(5, -1, 4)$ and $N(2, 3, -2)$

3. Given $\mathbf{c} = 4\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ and $\mathbf{d} = 2\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}$:
   (a) Find $|\mathbf{c}|$ and $|\mathbf{d}|$
   (b) Calculate $\mathbf{c} + \mathbf{d}$ and $\mathbf{c} - \mathbf{d}$
   (c) Find scalars p and q such that $p\mathbf{c} + q\mathbf{d} = \begin{pmatrix} 3 \\ -6 \\ 9 \end{pmatrix}$
   (d) Determine if $\mathbf{c}$ and $\mathbf{d}$ are parallel

4. Points U, V, and W have position vectors $\mathbf{u} = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$, $\mathbf{v} = \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix}$, and $\mathbf{w} = \begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix}$.
   (a) Find vectors $\overrightarrow{UV}$ and $\overrightarrow{UW}$
   (b) Calculate the lengths $|UV|$ and $|UW|$

(c) Find the position vector of the midpoint of VW

(d) Determine if triangle UVW is isosceles

5. Find the values of k for which these vectors are perpendicular:

(a) $\mathbf{p} = \begin{pmatrix} 4 \\ k \\ 1 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} k \\ 2 \\ -4 \end{pmatrix}$

(b) $\mathbf{a} = \begin{pmatrix} 2 \\ 3k \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 3 \\ -1 \\ k \end{pmatrix}$

(c) $\mathbf{x} = k\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ and $\mathbf{y} = 2\mathbf{i} + k\mathbf{j} + 3\mathbf{k}$

Section B: Dot Product (Scalar Product)

6. Calculate the dot product of these vectors:

(a) $\mathbf{f} = \begin{pmatrix} 5 \\ -3 \\ 2 \end{pmatrix}$ and $\mathbf{g} = \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix}$

(b) $\mathbf{h} = 4\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$ and $\mathbf{j} = 3\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$

(c) $\mathbf{k} = \begin{pmatrix} 2 \\ -4 \\ 3 \end{pmatrix}$ and $\mathbf{l} = \begin{pmatrix} 5 \\ 1 \\ -2 \end{pmatrix}$

(d) $\mathbf{m} = 3\mathbf{i} + 4\mathbf{j}$ and $\mathbf{n} = 2\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$

7. Find the angle between these pairs of vectors:

(a) $\mathbf{p} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

(b) $\mathbf{r} = \begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix}$ and $\mathbf{s} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$

(c) $\mathbf{t} = 2\mathbf{i} + 5\mathbf{j}$ and $\mathbf{u} = 3\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$

(d) $\mathbf{v} = \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$

8. Use the dot product to verify these properties:

(a) $\mathbf{p} \cdot \mathbf{q} = \mathbf{q} \cdot \mathbf{p}$ (commutative)

(b) $\mathbf{p} \cdot (\mathbf{q} + \mathbf{r}) = \mathbf{p} \cdot \mathbf{q} + \mathbf{p} \cdot \mathbf{r}$ (distributive)

(c) $(k\mathbf{p}) \cdot \mathbf{q} = k(\mathbf{p} \cdot \mathbf{q})$ for scalar k

(d) $\mathbf{p} \cdot \mathbf{p} = |\mathbf{p}|^2$

9. Given vectors $\mathbf{x} = \begin{pmatrix} 4 \\ 3 \\ -2 \end{pmatrix}$, $\mathbf{y} = \begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix}$, and $\mathbf{z} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$:

(a) Show that $\mathbf{x}$ and $\mathbf{y}$ are perpendicular

(b) Find the component of $\mathbf{z}$ in the direction of $\mathbf{x}$

(c) Calculate $|\mathbf{x} + \mathbf{y} + \mathbf{z}|$

(d) Find the angle between $\mathbf{x}+\mathbf{y}$ and $\mathbf{z}$

10. A triangle has vertices at $D(4,1,3)$, $E(2,5,1)$, and $F(3,2,4)$.

 (a) Find the vectors $\overrightarrow{DE}$ and $\overrightarrow{DF}$
 (b) Calculate the angle $\angle EDF$
 (c) Find the area of triangle DEF
 (d) Determine if the triangle is right-angled

Section C: Cross Product (Vector Product)

11. Calculate the cross product of these vectors:

 (a) $\mathbf{r}=\begin{pmatrix}4\\1\\3\end{pmatrix}$ and $\mathbf{s}=\begin{pmatrix}2\\5\\1\end{pmatrix}$
 (b) $\mathbf{t}=3\mathbf{i}+4\mathbf{j}-2\mathbf{k}$ and $\mathbf{u}=2\mathbf{i}+\mathbf{j}+5\mathbf{k}$
 (c) $\mathbf{v}=\begin{pmatrix}1\\-4\\2\end{pmatrix}$ and $\mathbf{w}=\begin{pmatrix}3\\1\\-5\end{pmatrix}$
 (d) $\mathbf{x}=4\mathbf{i}+3\mathbf{j}$ and $\mathbf{y}=3\mathbf{i}+2\mathbf{k}$

12. Verify these properties of the cross product:

 (a) $\mathbf{p}\times\mathbf{q}=-(\mathbf{q}\times\mathbf{p})$ (anti-commutative)
 (b) $\mathbf{p}\times(\mathbf{q}+\mathbf{r})=\mathbf{p}\times\mathbf{q}+\mathbf{p}\times\mathbf{r}$ (distributive)
 (c) $\mathbf{p}\times\mathbf{p}=\mathbf{0}$
 (d) $|\mathbf{p}\times\mathbf{q}|^2=|\mathbf{p}|^2|\mathbf{q}|^2-(\mathbf{p}\cdot\mathbf{q})^2$

13. Find the area of the parallelogram spanned by:

 (a) $\mathbf{c}=\begin{pmatrix}3\\4\\0\end{pmatrix}$ and $\mathbf{d}=\begin{pmatrix}2\\1\\5\end{pmatrix}$
 (b) $\mathbf{e}=5\mathbf{i}+2\mathbf{j}-\mathbf{k}$ and $\mathbf{f}=3\mathbf{i}-2\mathbf{j}+4\mathbf{k}$
 (c) Vectors from origin to points $(3,1,4)$ and $(2,4,1)$
 (d) $\overrightarrow{GH}$ and $\overrightarrow{GI}$ where $G(4,2,1)$, $H(1,5,2)$, $I(3,1,5)$

14. Given $\mathbf{p}=\begin{pmatrix}5\\-2\\3\end{pmatrix}$ and $\mathbf{q}=\begin{pmatrix}2\\4\\-1\end{pmatrix}$:

 (a) Calculate $\mathbf{p}\times\mathbf{q}$
 (b) Verify that $\mathbf{p}\times\mathbf{q}$ is perpendicular to both $\mathbf{p}$ and $\mathbf{q}$
 (c) Find a unit vector perpendicular to both $\mathbf{p}$ and $\mathbf{q}$
 (d) Calculate the area of triangle with sides $\mathbf{p}$ and $\mathbf{q}$

15. Use the scalar triple product $\mathbf{p}\cdot(\mathbf{q}\times\mathbf{r})$ to find:

 (a) The volume of parallelepiped with edges $\mathbf{p}=\begin{pmatrix}4\\2\\1\end{pmatrix}$, $\mathbf{q}=\begin{pmatrix}1\\4\\2\end{pmatrix}$, $\mathbf{r}=\begin{pmatrix}2\\1\\4\end{pmatrix}$
 (b) Whether points $P(4,2,1)$, $Q(1,5,3)$, $R(2,1,5)$, $S(4,4,2)$ are coplanar
 (c) The volume of tetrahedron with vertices at $(0,0,0)$, $(4,2,1)$, $(1,4,2)$, $(2,1,4)$

Section D: Equations of Lines

16. Find the vector equation of the line:

   (a) Passing through $S(4, 2, 1)$ in direction $\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$

   (b) Passing through points $T(3, 1, 5)$ and $U(2, 4, 1)$

   (c) Through origin parallel to vector $3\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$

   (d) Through $(4, 1, 3)$ parallel to the line $\mathbf{r} = \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}$

17. Convert these to parametric form:

   (a) $\mathbf{r} = \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix} + t\begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$

   (b) $\mathbf{r} = \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix} + s\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$

   (c) Line through $(2, 5, 1)$ and $(4, 1, 3)$

   (d) $\mathbf{r} = (2 + 3t)\mathbf{i} + (4 - t)\mathbf{j} + (1 + 2t)\mathbf{k}$

18. Find where these lines intersect the coordinate planes:

   (a) $\mathbf{r} = \begin{pmatrix} 4 \\ 3 \\ 8 \end{pmatrix} + t\begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix}$ and the xy-plane

   (b) $\mathbf{r} = \begin{pmatrix} 3 \\ 6 \\ 2 \end{pmatrix} + s\begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix}$ and the xz-plane

   (c) Line through $(5, 2, 3)$ and $(1, 4, 0)$ with the yz-plane

19. Determine if these pairs of lines intersect, are parallel, or are skew:

   (a) $L_1 : \mathbf{r} = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix} + t\begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}$ and $L_2 : \mathbf{r} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} + s\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$

   (b) $L_1 : \mathbf{r} = \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix} + t\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$ and $L_2 : \mathbf{r} = \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix} + s\begin{pmatrix} 6 \\ 2 \\ 4 \end{pmatrix}$

   (c) Lines through $(4, 1, 2)$ to $(2, 5, 1)$ and $(3, 2, 4)$ to $(1, 4, 5)$

20. Find the shortest distance between:

   (a) Point $(3, 5, 2)$ and line $\mathbf{r} = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix} + t\begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}$

   (b) Parallel lines $L_1 : \mathbf{r} = \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix} + t\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ and $L_2 : \mathbf{r} = \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix} + s\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$

   (c) Skew lines $L_1 : \mathbf{r} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} + t\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$ and $L_2 : \mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix} + s\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$

Section E: Equations of Planes

21. Find the equation of the plane:

    (a) With normal vector $\begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix}$ passing through $(2, 5, 1)$

    (b) Passing through points $(4, 0, 0)$, $(0, 3, 0)$, and $(0, 0, 2)$

    (c) Containing the lines $\mathbf{r} = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 3 \\ 5 \\ 1 \end{pmatrix} + s \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix}$

    (d) Parallel to vectors $\begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$ through $(2, 1, 4)$

22. Convert between vector and Cartesian forms:

    (a) $\mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} + s \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} + t \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix}$ to Cartesian form

    (b) $4x - 3y + 2z = 8$ to vector form

    (c) $2x + 4y - 3z = 12$ to parametric form

    (d) $\mathbf{r} \cdot \begin{pmatrix} 3 \\ -2 \\ 5 \end{pmatrix} = 8$ to Cartesian form

23. Find where these planes intersect coordinate axes:

    (a) $4x + 2y - 3z = 12$

    (b) $3x - 4y + 2z = 24$

    (c) $2x + 5y + 3z = 30$

    (d) $3x + 4y + 2z = 12$

24. Determine the relationship between these planes:

    (a) $\Pi_1 : 4x + 2y - z = 8$ and $\Pi_2 : 8x + 4y - 2z = 16$

    (b) $\Pi_1 : 3x - 2y + 4z = 6$ and $\Pi_2 : 2x + 3y - z = 8$

    (c) $\Pi_1 : 3x + 4y + 2z = 12$ and $\Pi_2 : 6x + 8y + 4z = 18$

    (d) $\Pi_1 : 4x - 3y + 2z = 10$ and $\Pi_2 : 3x + 4y - z = 7$

25. Find the line of intersection of these planes:

    (a) $4x + 2y + z = 9$ and $2x - 3y + 4z = 6$

    (b) $3x + 4y - 2z = 10$ and $2x - 3y + z = 4$

    (c) $5x - 2y + 3z = 8$ and $2x + 4y - z = 5$

    (d) $3x + 2y + 4z = 12$ and $4x - 3y + 2z = 9$

Section F: Angles and Distances

26. Find the angle between these planes:

    (a) $5x + 2y - 4z = 8$ and $3x - 5y + 2z = 9$

    (b) $4x + 3y - 2z = 10$ and $2x - 4y + 5z = 8$

(c) $\mathbf{r} \cdot \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix} = 6$ and $\mathbf{r} \cdot \begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix} = 7$

(d) $3x + 2y + 5z = 15$ and $2x - 5y + 3z = 9$

27. Calculate the distance from point to plane:

(a) Point $(4, 3, 2)$ to plane $5x + 2y - 3z = 8$

(b) Point $(2, -4, 3)$ to plane $4x - 3y + 2z = 10$

(c) Point $(0, 0, 0)$ to plane $3x + 4y - 2z = 25$

(d) Point $(5, 2, -3)$ to plane $3x - 5y + 4z = 12$

28. Find the angle between line and plane:

(a) Line $\mathbf{r} = \begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$ and plane $3x + 4y + 2z = 10$

(b) Line through $(4, 2, 5)$ and $(1, 6, 2)$ with plane $4x - 3y + 2z = 9$

(c) Line $\mathbf{r} = \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix} + s \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}$ and plane $3x + 5y - 2z = 8$

29. Determine where these lines intersect planes:

(a) $\mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$ and $2x + 4y + 3z = 20$

(b) Line through $(3, 5, 2)$ and $(2, 1, 5)$ with plane $4x - 2y + 3z = 8$

(c) $\mathbf{r} = \begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix} + s \begin{pmatrix} 2 \\ 5 \\ -4 \end{pmatrix}$ and $3x - 4y + 5z = 20$

30. Find the reflection of point in plane:

(a) Point $(5, 3, 2)$ in plane $4x + 3y - 2z = 8$

(b) Point $(2, -4, 5)$ in plane $3x - 2y + 5z = 12$

(c) Point $(3, 5, 0)$ in plane $4x + 2y + 3z = 10$

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 150

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