# A Level Mechanics
# Practice Test 5: Momentum and Impulse

**Instructions:**
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Draw diagrams where appropriate to illustrate your solutions.
Time allowed: 2 hours 35 minutes

## Section A: Sophisticated Momentum Systems [40 marks]

**Question 1 [25 marks]** A complex system consists of six particles: A (3 kg, velocity $(8, -4, 6)$ m/s), B (5 kg, velocity $(-6, 3, -2)$ m/s), C (2 kg, velocity $(0, 8, -5)$ m/s), D (4 kg, velocity $(7, -1, 4)$ m/s), E (1.5 kg, velocity $(-3, 6, 0)$ m/s), and F (2.5 kg, velocity $(4, -2, 3)$ m/s).

(a) Calculate the total momentum vector of the system. **[5 marks]**

(b) Find the velocity of the center of mass. **[4 marks]**

(c) If particles A and B suddenly stick together while others remain unchanged, find the new total momentum and verify conservation. **[6 marks]**

(d) Calculate the kinetic energy lost when A and B stick together. **[5 marks]**

(e) If an external force $\vec{F} = (15t\hat{i} - 10t^2\hat{j} + 5\hat{k})$ N acts on the system for 3 seconds, find the final momentum of the system. **[5 marks]**

**Question 2 [15 marks]** A satellite of mass 1200 kg orbiting Earth needs to perform a complex maneuver. It ejects a thruster package of mass 300 kg at velocity $(50, -30, 20)$ m/s relative to the satellite. Before ejection, both were moving together at $(4000, 2000, -1000)$ m/s relative to Earth.

(a) Calculate the velocity of the satellite after ejection. **[6 marks]**

(b) Find the velocity of the thruster package relative to Earth. **[4 marks]**

(c) Calculate the change in total kinetic energy of the system. **[5 marks]**

## Section B: Multi-Body Collision Dynamics [45 marks]

**Question 3 [25 marks]** Four balls are arranged in a line on a frictionless surface. Ball A (2 kg, 12 m/s) approaches three stationary balls: B (3 kg), C (4 kg), and D (5 kg). All collisions are elastic and occur sequentially.

(a) Calculate the velocities of A and B after the first collision. **[5 marks]**

(b) Find the velocities of B and C after the second collision. **[5 marks]**

(c) Calculate the velocities of C and D after the third collision. **[5 marks]**

(d) Verify that total momentum is conserved throughout all collisions. **[4 marks]**

(e) Verify that total kinetic energy is conserved throughout all collisions. **[6 marks]**

**Question 4 [20 marks]** A compound pendulum consists of a rod of mass 3 kg and length 2 m with a bob of mass 1 kg attached at the end. A bullet of mass 0.05 kg moving horizontally at 600 m/s embeds in the bob when the pendulum is vertical.

(a) Calculate the velocity of the bob immediately after the bullet embeds. **[4 marks]**

(b) Find the angular velocity of the pendulum system immediately after impact. **[5 marks]**

(c) Calculate the maximum angle the pendulum reaches using energy conservation. **[6 marks]**

(d) Determine the impulse delivered to the pendulum by the bullet. **[3 marks]**

(e) Calculate the percentage of the bullet's kinetic energy lost in the collision. **[2 marks]**

## Section C: Advanced Variable Force Analysis [35 marks]

**Question 5 [20 marks]** A particle of mass 0.8 kg is subjected to a force that varies as $\vec{F}(t) = (12t - 2t^2)\hat{i} + (8 - 4t + t^2)\hat{j} + (6t^2 - 3t^3)\hat{k}$ N for $0 \leq t \leq 4$ seconds. The particle starts from rest.

(a) Calculate the impulse vector delivered during the 4-second interval. **[6 marks]**

(b) Find the final velocity vector of the particle. **[4 marks]**

(c) Determine when each component of force reaches its maximum value. **[6 marks]**

(d) Calculate the magnitude of the resultant impulse and final velocity. **[4 marks]**

**Question 6 [15 marks]** A baseball of mass 0.145 kg is pitched and then hit by a bat. The force during the collision varies as: $F = 0$ for $t < 0$, $F = 4000t$ N for $0 \leq t \leq 0.0015$ s, $F = 6 - 4000t$ N for $0.0015 \leq t \leq 0.003$ s, and $F = 0$ for $t > 0.003$ s. The ball approaches at 30 m/s and leaves in the opposite direction.

(a) Calculate the total impulse delivered to the ball. **[5 marks]**

(b) Find the final velocity of the ball. **[3 marks]**

(c) Determine the maximum force during the collision. **[2 marks]**

(d) Calculate the change in momentum and verify it equals the impulse. **[3 marks]**

(e) Find the average force during the entire contact period. **[2 marks]**

## Section D: Explosion and Fragmentation Analysis [30 marks]

**Question 7 [30 marks]** A military shell of mass 15 kg traveling at $(120, 60, 0)$ m/s explodes into five fragments at the peak of its trajectory. Fragment A (2 kg) flies at $(200, 100, 80)$ m/s, fragment B (3 kg) at $(80, -150, 60)$ m/s, fragment C (4 kg) at $(-50, 200, -40)$ m/s, fragment D (2.5 kg) at $(150, -80, -100)$ m/s.

(a) Calculate the velocity of fragment E using momentum conservation. **[8 marks]**

(b) Find the total kinetic energy before and after explosion. **[6 marks]**

(c) Calculate the energy released in the explosion. **[3 marks]**

(d) If the explosion lasted 0.0003 seconds, find the average force on fragment A during explosion. **[4 marks]**

(e) Determine the impulse experienced by fragment C. **[4 marks]**

(f) Calculate the displacement of the center of mass 5 seconds after explosion. **[5 marks]**

## Section E: Advanced Propulsion Systems [35 marks]

**Question 8 [20 marks]** A three-stage rocket has the following specifications: Stage 1 (mass 12,000 kg including fuel, burns 60 kg/s at 2100 m/s exhaust velocity for 80 seconds), Stage 2 (mass 4,000 kg including fuel, burns 25 kg/s at 2800 m/s exhaust velocity for 90 seconds), Stage 3 (mass 1,500 kg including fuel, burns 8 kg/s at 3500 m/s exhaust velocity for 120 seconds).

(a) Calculate the thrust force for each stage. **[6 marks]**

(b) Find the velocity gained during each stage burn (neglect gravity). **[9 marks]**

(c) Calculate the final velocity of the payload. **[3 marks]**

(d) Determine the total impulse delivered by all three stages. **[2 marks]**

**Question 9 [15 marks]** An ion drive spacecraft of mass 2000 kg continuously ejects ions at 50,000 m/s relative to the spacecraft. The mass ejection rate is 0.01 kg/s.

(a) Calculate the thrust force produced by the ion drive. **[3 marks]**

(b) Find the initial acceleration of the spacecraft. **[2 marks]**

(c) If the drive operates for 1 year (ignore mass change), calculate the final velocity gained. **[5 marks]**

(d) Calculate the actual final velocity accounting for the decreasing mass. **[5 marks]**

## Section F: Fluid Momentum Transfer [25 marks]

**Question 10 [15 marks]** A firefighting aircraft drops water from height 100 m. The water is released at 200 kg/s and hits the ground with velocity components: horizontal 50 m/s, vertical 44.3 m/s.

(a) Calculate the momentum change rate of the water upon hitting the ground. **[5 marks]**

(b) Find the force exerted by the water on the ground. **[3 marks]**

(c) If the water spreads horizontally upon impact, calculate the impulse delivered to the ground in 30 seconds. **[4 marks]**

(d) Determine the power dissipated when the water hits the ground. **[3 marks]**

**Question 11 [10 marks]** A turbine blade is struck by a water jet at 40 m/s. The water flow rate is 80 kg/s, and the blade deflects the water through 120° while reducing its speed to 30 m/s.

(a) Calculate the change in momentum of the water per unit time. **[5 marks]**

(b) Find the force components exerted by the water on the blade. **[3 marks]**

(c) Calculate the power extracted by the turbine. **[2 marks]**

## Section G: Comprehensive System Analysis [30 marks]

**Question 12 [30 marks]** A complex collision occurs between three objects in 2D space. Object A (mass 4 kg) moves at $(6, 8)$ m/s, object B (mass 6 kg) moves at $(-4, 3)$ m/s, and object C (mass 8 kg) moves at $(2, -5)$ m/s. After collision, object A moves at $(1, 4)$ m/s and object B moves at $(-2, 6)$ m/s.

(a) Calculate the total momentum before collision. **[4 marks]**

(b) Find the velocity of object C after collision using momentum conservation. **[6 marks]**

(c) Calculate the kinetic energy before and after collision. **[6 marks]**

(d) Determine the energy lost in the collision. **[3 marks]**

(e) Calculate the impulse experienced by each object. **[6 marks]**

(f) If the collision lasted 0.02 seconds, estimate the average force between the objects. **[3 marks]**

(g) Verify that the center of mass motion is unaffected by the collision. **[2 marks]**

### Physics Data and Formulae

**Momentum and Impulse:**

Momentum: $\vec{p} = m\vec{v}$

Impulse: $\vec{J} = \int \vec{F}\, dt = \Delta\vec{p}$

Conservation: $\sum \vec{p}_{\text{initial}} = \sum \vec{p}_{\text{final}}$

Force on system: $\vec{F}_{ext} = \frac{d\vec{P}_{total}}{dt}$

**Center of Mass:**

Position: $\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$

Velocity: $\vec{v}_{cm} = \frac{\sum m_i \vec{v}_i}{\sum m_i}$

Motion: $\vec{F}_{ext} = M_{total}\vec{a}_{cm}$

**Collision Mechanics:**

Elastic (1D): $v_1' = \frac{(m_1 - m_2)v_1 + 2m_2 v_2}{m_1 + m_2}$

$v_2' = \frac{(m_2 - m_1)v_2 + 2m_1 v_1}{m_1 + m_2}$

Coefficient of restitution: $e = \frac{|\text{separation}|}{|\text{approach}|}$

Perfectly inelastic: $v' = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

**Rocket Propulsion:**

Thrust: $F = \frac{dm}{dt} v_{exhaust}$

Rocket equation: $\Delta v = v_{exhaust} \ln\left(\frac{m_i}{m_f}\right)$

For continuous burn: $v(t) = v_0 + v_{exhaust} \ln\left(\frac{m_0}{m_0 - \dot{m}t}\right)$

**Energy Relations:**

Kinetic energy: $E_k = \frac{1}{2}mv^2$

Rotational KE: $E_r = \frac{1}{2}I\omega^2$

Gravitational PE: $E_p = mgh$

**Vector Mathematics:**

Magnitude: $|\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$

Dot product: $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$
Cross product: $\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\hat{i} + ...$

**Calculus Integration:**
$\int t^n \, dt = \frac{t^{n+1}}{n+1} + C$
$\int f'(t) \, dt = f(t) + C$

**Physical Constants:**
Acceleration due to gravity: $g = 9.8$ m/s$^2$
1 year $= 3.156 \times 10^7$ seconds

**END OF TEST**

Total marks: 240

Grade boundaries: A* 216, A 192, B 168, C 144, D 120, E 96

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