GCSE Higher Mathematics Practice Test 3: Statistics

Instructions:

Answer all questions. Show your working clearly. Calculators may be used unless stated otherwise.

Time allowed: 90 minutes

Section A: Advanced Averages and Spread

1. The table shows the distribution of phone battery life:

Battery life (hours)	Frequency			
8-11	5			
12-15	9			
16-19	14			
20-23	18			
24-27	21			
28-31	15			
32-35	8			
36-39	4			

Calculate:

- (a) The total number of phones tested
- (b) An estimate of the mean battery life
- (c) The modal class
- (d) An estimate of the median battery life
- (e) An estimate of the range
- 2. For the data set: 16, 19, 22, 25, 28, 31, 34, 37, 40, 46
 - (a) Calculate the mean
 - (b) Find the median and quartiles (Q1 and Q3)
 - (c) Calculate the interquartile range
 - (d) Calculate the standard deviation
 - (e) Identify any outliers using the $1.5 \times IQR$ rule
- 3. Two data sets have the following properties:
 - Set P: mean = 62, standard deviation = 11, n = 30
 - Set Q: mean = 58, standard deviation = 15, n = 20
 - (a) Calculate the combined mean

- (b) Calculate the combined standard deviation
- (c) Which set is more consistent? Explain.
- (d) Calculate the coefficient of variation for each set
- 4. The masses (in grams) of 36 components are summarized:

$$\sum x = 1800, \sum x^2 = 92400$$

- (a) Calculate the mean mass
- (b) Calculate the variance
- (c) Calculate the standard deviation
- (d) If each component is reduced by 5g, find the new mean and standard deviation

Section B: Histograms and Frequency Density

5. The histogram shows the distribution of internet download speeds:

[Imagine a histogram with: 0-20 Mbps (density 1.5), 20-40 Mbps (density 2.8), 40-60 Mbps (density 3.6), 60-80 Mbps (density 2.2), 80-120 Mbps (density 0.9)]

- (a) Complete the frequency table
- (b) Calculate the total number of users
- (c) Estimate the mean download speed
- (d) Find the modal class
- (e) What percentage of users have speeds below 40 Mbps?
- 6. Draw a histogram for this data about apartment rental prices:

Rent (£/month)	Frequency
600-800	18
800-1000	28
1000-1100	12
1100-1300	20
1300-1600	15
1600-2000	8

- (a) Calculate the frequency density for each class
- (b) Draw the histogram
- (c) Estimate the median rental price
- (d) What fraction of apartments rent for more than £1100/month?
- 7. A histogram shows data with unequal class widths. The class 25-30 has frequency density 12 and the class 30-40 has frequency 60.
 - (a) Find the frequency for the 25-30 class
 - (b) Find the frequency density for the 30-40 class
 - (c) If the total frequency is 300, suggest frequencies for other classes

Section C: Cumulative Frequency and Box Plots

8. The table shows the cumulative frequency of running times:

Time (minutes)	Cumulative Frequency
≤ 12	4
≤ 16	14
≤ 20	28
≤ 24	46
≤ 28	62
≤ 32	74
≤ 36	82
≤ 40	85

- (a) Draw the cumulative frequency curve
- (b) Find the median
- (c) Find the quartiles Q1 and Q3
- (d) Calculate the interquartile range
- (e) Draw a box plot
- (f) Estimate the 75th percentile
- 9. Two box plots show the distribution of salaries for two departments:

[Imagine box plots: Department X (min 28000, Q1 35000, median 42000, Q3 48000, max 65000), Department Y (min 32000, Q1 38000, median 45000, Q3 52000, max 68000)]

Compare the distributions by commenting on:

- (a) Central tendency (medians)
- (b) Spread (ranges and IQRs)
- (c) Shape and outliers
- (d) Which department has more variable salaries?
- 10. The cumulative frequency curve for response times (in seconds) passes through these points: (2, 0), (4, 12), (6, 28), (8, 45), (10, 58), (12, 67), (14, 72)
 - (a) Find the median response time
 - (b) Find the quartiles
 - (c) What percentage have response times between 5 seconds and 9 seconds?
 - (d) Draw the corresponding box plot

Section D: Scatter Graphs and Correlation

11. The table shows data for 10 houses:

Area (m ²)	80	95	110	125	140	155	170	185	200	215
Price (£000s)	185	215	245	275	305	335	365	395	425	455

- (a) Plot a scatter graph
- (b) Describe the correlation
- (c) Calculate the equation of the line of best fit
- (d) Use your line to predict the price for a 160m² house

- (e) Estimate the area for a house priced at £320,000
- (f) Calculate the correlation coefficient
- 12. The equation of a regression line is y = -1.5x + 95.
 - (a) Interpret the gradient
 - (b) Interpret the y-intercept
 - (c) If x = 30, predict y
 - (d) If y = 56, estimate x
 - (e) State assumptions made when using this model
- 13. Classify these correlation coefficients and describe the relationships:
 - (a) r = 0.92
 - (b) r = -0.68
 - (c) r = 0.31
 - (d) r = -0.87
 - (e) r = 0.74

Section E: Advanced Probability

- 14. A jar contains 7 green marbles, 5 blue marbles, and 4 red marbles. Two marbles are drawn without replacement.
 - (a) Draw a tree diagram
 - (b) Find P(both green)
 - (c) Find P(both same color)
 - (d) Find P(at least one blue)
 - (e) Find P(different colors)
- 15. The probability that a light bulb fails within a year is 0.08, independently of other bulbs.
 - (a) Find the probability that exactly 2 out of 6 bulbs fail
 - (b) Find the probability that at least 3 out of 6 bulbs fail
 - (c) Find the expected number of failures in 25 bulbs
 - (d) In a batch of 50 bulbs, find P(more than 6 failures)
- 16. A computer test has 15 multiple choice questions, each with 5 options. A student guesses randomly.
 - (a) Find P(correct answer on one question)
 - (b) Find P(exactly 4 correct answers)
 - (c) Find P(at least 10 correct answers)
 - (d) Find the expected number of correct answers
 - (e) Find the most likely number of correct answers
- 17. Events E and F are such that P(E) = 0.7, P(F) = 0.5, and $P(E \mid F) = 0.28$.
 - (a) Find P(E F)
 - (b) Find P(E')
 - (c) Find P(E F)
 - (d) Find P(F E)
 - (e) Are E and F independent? Justify your answer

Section F: Conditional Probability and Independence

- 18. A survey of 240 teenagers about social media usage gives:
 - 150 use Instagram
 - 96 use TikTok
 - 54 use both platforms
 - (a) Draw a Venn diagram
 - (b) Find P(uses Instagram uses TikTok)
 - (c) Find P(uses TikTok uses Instagram)
 - (d) Find P(uses exactly one platform)
 - (e) Are the platform usages independent? Explain
- 19. In a garment factory, 55% of items are sewn by Machine P and 45% by Machine Q. Machine P produces 4% defective items, Machine Q produces 6% defective items.
 - (a) Draw a tree diagram
 - (b) Find the probability an item is defective
 - (c) If an item is defective, find the probability it was sewn by Machine P
 - (d) If an item is not defective, find the probability it was sewn by Machine Q
- 20. A collection has 12 fiction books numbered 1-12 and 9 non-fiction books numbered 1-9. A book is selected at random.
 - (a) Find P(fiction and prime number)
 - (b) Find P(non-fiction number divisible by 3)
 - (c) Find P(number ; 8)
 - (d) Are book type and number range independent?
- 21. A diagnostic scan is 88% accurate for positive cases and 94% accurate for negative cases. 4% of patients have the condition.
 - (a) Find the probability of a positive scan result
 - (b) If someone has a positive scan, find the probability they have the condition
 - (c) If someone has a negative scan, find the probability they don't have the condition
 - (d) Comment on the effectiveness of the scan

Section G: Hypothesis Testing and Sampling

- 22. A spinner is suspected of being biased towards red. It's spun 25 times and lands on red 9 times.
 - (a) State the null and alternative hypotheses
 - (b) Calculate the probability of getting 9 or more reds if the spinner is fair (assuming 4 equal sections)
 - (c) At the 5% significance level, is there evidence the spinner is biased?
 - (d) What would be a Type I error in this context?
- 23. A sample of 45 smartphone batteries has mean charge time 180 minutes and standard deviation 28 minutes.
 - (a) Calculate a 95% confidence interval for the population mean

- (b) Interpret your confidence interval
- (c) What assumptions are made?
- (d) How would the interval change with a 99% confidence level?
- 24. A website claims 90% customer satisfaction. In a sample of 120 customers, 104 report satisfaction.
 - (a) Test at 5% level whether the claim is justified
 - (b) Calculate the critical value
 - (c) State your conclusion
 - (d) What is the p-value for this test?

Section H: Problem Solving and Integration

- 25. A gym records member workout durations over 150 sessions. The data shows:
 - Mean = 65 minutes
 - Standard deviation = 12 minutes
 - Distribution is approximately normal
 - (a) Find P(workout duration ; 80 minutes)
 - (b) Find the duration exceeded by only 15% of members
 - (c) What percentage of workouts last between 50 and 75 minutes?
 - (d) If the gym has 200 members per day, estimate daily equipment usage over 45 minutes
- 26. A quality inspector samples 10 products every hour. Over 8 hours, the number of defective products found was: 0, 2, 1, 3, 1, 0, 2, 1.
 - (a) Calculate the mean and standard deviation
 - (b) Test whether the defect rate exceeds 12%
 - (c) Create a control chart with warning limits
 - (d) Comment on process stability
- 27. Compare these three mutual fund options over 5 years:
 - Fund A: Mean return 7%, standard deviation 9%
 - Fund B: Mean return 11%, standard deviation 16%
 - Fund C: Mean return 5%, standard deviation 6%
 - (a) Calculate the coefficient of variation for each
 - (b) Which offers the best risk-adjusted return?
 - (c) Using normal distribution, find P(loss) for each fund
 - (d) Recommend an option for a moderate-risk investor
- 28. A university studies the relationship between class attendance and final grades. The correlation is 0.72.
 - (a) What does this correlation suggest?
 - (b) If attendance has mean 85% and standard deviation 12%, and grades have mean 68 and standard deviation 16, find the regression equation
 - (c) Predict the grade for a student with 92% attendance

- (d) Calculate the coefficient of determination and interpret it
- 29. Design a statistical investigation to test whether a new app improves student learning outcomes:
 - (a) State hypotheses
 - (b) Describe the sampling method
 - (c) Identify variables and potential confounding factors
 - (d) Outline the analysis plan
 - (e) Discuss limitations and assumptions

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 100

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