A Level Statistics
Practice Test 6: Advanced Topics

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Draw diagrams where appropriate to illustrate your solutions.
Time allowed: 3 hours

Section A: Fundamental Concepts [25 marks]

1. **[12 marks]** Define and explain fundamental concepts:

(a) Define sampling and explain its importance in statistical analysis.

(b) Explain what is meant by "sampling distribution" and "sampling error."

(c) State the difference between probability sampling and non-probability sampling.

(d) Define bias in sampling and describe common sources of bias.

(e) Distinguish between sampling with and without replacement.

(f) Explain how sample size affects the precision of statistical estimates.

2. **[8 marks]** Explain the importance of these concepts:

(a) Why is random sampling crucial for statistical inference?

(b) Explain how proper sampling design ensures representative samples.

(c) Describe the trade-off between cost and precision in sampling.

(d) Explain the relationship between population parameters and sample statistics.

3. **[5 marks]** Practical and theoretical context:

(a) Explain why census data is not always practical or necessary.

(b) Describe the role of sampling in market research and opinion polling.

(c) Explain how sampling applies to quality control in manufacturing.

Section B: Sampling Methods and Distributions [30 marks]

4. **[15 marks]** Types of sampling methods:

(a) Describe simple random sampling and explain how to implement it.

(b) Explain systematic sampling and when it's appropriate to use.

(c) Describe stratified sampling and its advantages over simple random sampling.

(d) Explain cluster sampling and when it's more practical than other methods.

(e) Describe convenience sampling and explain its limitations.

(f) Explain quota sampling and how it differs from stratified sampling.

5. **[15 marks]** Sampling distributions and their properties:

(a) Define the sampling distribution of the sample mean.

(b) Explain how the Central Limit Theorem applies to sampling distributions.

(c) Describe the sampling distribution of the sample proportion.

(d) Explain the standard error and how it relates to sample size.

(e) Describe finite population correction and when to apply it.

(f) Explain the sampling distribution of the difference between two means.

(g) Describe bootstrap sampling and its applications.

(h) Explain the concept of sampling variability.

(i) Describe how sampling distributions form the basis for confidence intervals.

Section C: Sampling Applications [35 marks]

6. **[18 marks]** A population of 5000 students has mean GPA = 3.2 and standard deviation = 0.6. A random sample of 100 students is selected:

(a) Describe the sampling distribution of the sample mean $\bar{X}$.

(b) Calculate the mean and standard error of $\bar{X}$.

(c) Find the probability that $\bar{X}$ is between 3.1 and 3.3.

(d) Calculate the probability that $\bar{X}$ exceeds 3.25.

(e) Find the value of $\bar{x}$ that is exceeded by only 5

(f) If the sample size were 400 instead of 100, recalculate the standard error.

(g) Compare the probabilities in part (c) for n = 100 and n = 400.

(h) Calculate the finite population correction factor.

(i) Determine if the finite population correction significantly affects the standard error.

7. **[17 marks]** A survey aims to estimate the proportion of voters supporting a candidate. Previous polls suggest p 0.45:

(a) For a sample of n = 800, describe the sampling distribution of $\hat{p}$.

(b) Calculate the mean and standard error of $\hat{p}$.

(c) Find the probability that $\hat{p}$ is within 0.03 of the true proportion.

(d) Calculate the probability that $\hat{p}$ exceeds 0.50.

(e) Determine the sample size needed so that P(—$\hat{p}$ - p— ¡ 0.02) = 0.95.

(f) If the true proportion is actually 0.48, find P($\hat{p}$ ¡ 0.45) for n = 800.

(g) Compare stratified sampling by age groups versus simple random sampling.

(h) Calculate the margin of error for 95

(i) Explain how non-response bias might affect the poll results.

Answer Space

Use this space for your working and answers.

Formulae and Key Concepts

Sampling Distribution of Sample Mean:

$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ when population is normal

By CLT: $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ for large n (any population)

Standard Error: $SE(\bar{X}) = \frac{\sigma}{\sqrt{n}}$

Sampling Distribution of Sample Proportion:

$\hat{p} \sim N\left(p, \frac{p(1-p)}{n}\right)$ for large n

Standard Error: $SE(\hat{p}) = \sqrt{\frac{p(1-p)}{n}}$

Conditions: $np \geq 5$ and $n(1-p) \geq 5$

Finite Population Correction:

When sampling without replacement from finite population N:

$SE(\bar{X}) = \frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}$

$SE(\hat{p}) = \sqrt{\frac{p(1-p)}{n}}\sqrt{\frac{N-n}{N-1}}$

Use when $\frac{n}{N} > 0.05$

Sample Size for Proportions:

Margin of error E: $n = \frac{z^2_{\alpha/2}p(1-p)}{E^2}$

Conservative estimate: use p = 0.5

For specified precision: $n = \frac{z^2_{\alpha/2}p(1-p)}{(\text{desired margin})^2}$

Sample Size for Means:

Margin of error E: $n = \frac{z_{\alpha/2}^2 \sigma^2}{E^2}$

For t-distribution: $n = \frac{t_{\alpha/2}^2 s^2}{E^2}$ (iterative process)

Two-Sample Differences:

$$\bar{X_1} - \bar{X_2} \sim N\left(\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)$$

$$\hat{p_1} - \hat{p_2} \sim N\left(p_1 - p_2, \frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}\right)$$

Stratified Sampling:

Overall mean: $\bar{x}_{st} = \sum_{i=1}^{L} W_i \bar{x}_i$ where $W_i = \frac{N_i}{N}$

Variance: $\text{Var}(\bar{x}_{st}) = \sum_{i=1}^{L} W_i^2 \frac{\sigma_i^2}{n_i}$

Central Limit Theorem:

For any population with mean and variance ²:

$\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \xrightarrow{d} N(0,1)$ as $n \to \infty$

Generally adequate for n 30

Confidence Intervals from Sampling:

Mean: $\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$ or $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}}$

Proportion: $\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

Common Sampling Errors:

Selection bias, Non-response bias, Measurement bias
Coverage bias, Voluntary response bias, Undercoverage

END OF TEST

Total marks: 90

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