GCSE Higher Mathematics
Practice Test 7: Algebra

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Time allowed: 90 minutes

Section A: Linear and Simultaneous Equations

1. Solve these linear equations:

   (a) $9(x - 7) = 5x + 13$
   (b) $\frac{x+13}{6} - \frac{x-11}{4} = 5$
   (c) $\frac{8x-7}{15} = \frac{5x+6}{9} - 3$
   (d) $0.9x + 1.8 = 0.7x - 3.2$

2. Solve these simultaneous equations by elimination:

   (a) $\begin{cases} 8x + 7y = 43 \\ 6x - 5y = 8 \end{cases}$
   (b) $\begin{cases} 5x + 11y = 43 \\ 9x - 6y = 21 \end{cases}$
   (c) $\begin{cases} 8x + 5y = 34 \\ 6x - 9y = 21 \end{cases}$
   (d) $\begin{cases} 10x + 9y = 61 \\ 5x - 7y = 12 \end{cases}$

3. Solve these simultaneous equations by substitution:

   (a) $\begin{cases} y = 8x - 13 \\ 7x + 5y = 43 \end{cases}$
   (b) $\begin{cases} x = 7y - 10 \\ 8x - 6y = 46 \end{cases}$
   (c) $\begin{cases} y = 17 - 7x \\ 6x + 9y = 78 \end{cases}$

4. Find the graphical solution to these simultaneous equations by finding intersection points:

   (a) $y = 8x - 6$ and $y = 18 - 5x$
   (b) $y = x^2 - 13$ and $y = 6x + 3$
   (c) $x^2 + y^2 = 34$ and $y = 6x - 4$

Section B: Quadratic Equations - Factoring

5. Factorize these quadratic expressions:

   (a) $x^2 + 19x + 48$
   (b) $x^2 - 17x - 38$
   (c) $x^2 - 21x + 98$
   (d) $x^2 + 8x - 33$
   (e) $x^2 - 256$
   (f) $x^2 - 32x + 256$

6. Solve these quadratic equations by factorizing:

   (a) $x^2 + 19x + 84 = 0$
   (b) $x^2 - 18x - 19 = 0$
   (c) $x^2 - 15x = 0$
   (d) $x^2 - 289 = 0$
   (e) $x^2 + 34x + 289 = 0$
   (f) $8x^2 - 32x = 0$

7. Factorize these harder quadratics:

   (a) $6x^2 + 13x + 5$
   (b) $8x^2 - 21x + 9$
   (c) $81x^2 - 100$
   (d) $7x^2 + 18x - 32$
   (e) $81x^2 - 144x + 64$
   (f) $11x^2 - 29x - 12$

8. Solve by factorizing:

   (a) $8x^2 + 17x - 9 = 0$
   (b) $9x^2 - 19x + 6 = 0$
   (c) $64x^2 - 49 = 0$
   (d) $6x^2 + 17x - 14 = 0$

Section C: Completing the Square and Quadratic Formula

9. Complete the square for these expressions:

   (a) $x^2 + 18x + 17$
   (b) $x^2 - 22x + 9$
   (c) $x^2 + 14x - 11$
   (d) $x^2 - 22x + 19$
   (e) $8x^2 + 32x + 15$
   (f) $4x^2 - 32x + 13$

10. Solve by completing the square:

    (a) $x^2 + 18x + 15 = 0$
    (b) $x^2 - 16x - 9 = 0$

(c) $x^2 + 12x - 7 = 0$

(d) $8x^2 + 16x - 7 = 0$

11. Use the quadratic formula to solve (leave in surd form where appropriate):

(a) $x^2 + 15x - 7 = 0$

(b) $8x^2 - 17x + 7 = 0$

(c) $x^2 - 18x + 13 = 0$

(d) $7x^2 + 13x - 8 = 0$

(e) $6x^2 + 13x + 5 = 0$

(f) $10x^2 - 15x - 5 = 0$

12. Find the discriminant and state the nature of the roots:

(a) $x^2 + 17x + 42 = 0$

(b) $x^2 - 16x + 64 = 0$

(c) $x^2 + 8x + 17 = 0$

(d) $7x^2 - 15x + 8 = 0$

Section D: Quadratic Graphs and Applications

13. For the quadratic $y = x^2 - 16x + 55$:

(a) Find the y-intercept

(b) Find the x-intercepts by factorizing

(c) Complete the square to find the vertex

(d) Sketch the graph

(e) State the line of symmetry

14. For the quadratic $y = 7x^2 + 21x - 10$:

(a) Complete the square

(b) Find the coordinates of the vertex

(c) Find the y-intercept

(d) State the line of symmetry

(e) Sketch the graph

15. A basketball is shot upward. Its height h (in meters) after t seconds is given by: $h = -5t^2+60t+7$

(a) What is the initial height?

(b) At what times is the basketball at ground level?

(c) What is the maximum height reached?

(d) At what time does it reach maximum height?

16. The profit P (in thousands of pounds) from producing x thousand items is: $P = -5x^2+25x-24$

(a) How many items should be produced to maximize profit?

(b) What is the maximum profit?

(c) At what production levels does the company break even?

Section E: Linear Inequalities

17. Solve these linear inequalities:

    (a) $9x + 17 > 53$

    (b) $8x - 19 \leq 37$

    (c) $12 - 9x < 3$

    (d) $\frac{x+11}{6} \geq 3$

    (e) $10 - 9x > 8x + 27$

    (f) $\frac{8x-7}{9} < \frac{x+10}{6}$

18. Show these inequalities on number lines:

    (a) $x > -8$

    (b) $x \leq 10$

    (c) $-5 < x \leq 11$

    (d) $x < 7$ or $x > 12$

19. Solve these compound inequalities:

    (a) $-10 < 8x + 6 < 22$

    (b) $15 \leq 7x - 6 \leq 36$

    (c) $-5 \leq \frac{8x+6}{6} < 12$

20. Find the integer solutions to:

    (a) $8x + 13 > 29$ and $x < 11$

    (b) $-6 \leq x + 4 < 10$

    (c) $x^2 < 100$

Section F: Quadratic Inequalities

21. Solve these quadratic inequalities:

    (a) $x^2 - 17x + 42 > 0$

    (b) $x^2 - 18x + 45 \leq 0$

    (c) $x^2 - 169 < 0$

    (d) $x^2 + 8x - 20 \geq 0$

    (e) $x^2 - 18x + 81 > 0$

    (f) $6x^2 - 7x - 10 < 0$

22. Solve and show on number lines:

    (a) $x^2 - 196 \leq 0$

    (b) $x^2 + 7x - 18 > 0$

    (c) $5x^2 - 12x + 7 \geq 0$

23. Find the values of x for which:

    (a) $x^2 < 9x + 10$

    (b) $6x^2 + 13x \geq 15$

    (c) $x^2 + 16x + 64 \leq 0$

Section G: Algebraic Manipulation

24. Expand and simplify:

    (a) $(x + 11)(8x - 6)$
    (b) $(7x - 8)(x + 12)$
    (c) $(6x + 11)^2$
    (d) $(10x - 9)^2$
    (e) $(x + 17)(x - 17)$
    (f) $(9x + 7)(9x - 7)$

25. Expand these expressions:

    (a) $(x + 8)(x^2 - 7x + 9)$
    (b) $(6x - 7)(x^2 + 6x - 5)$
    (c) $(x + 7)^3$
    (d) $(7x - 5)^3$

26. Factorize completely:

    (a) $18x^2 + 27x$
    (b) $64x^2 - 225$
    (c) $x^3 - 49x$
    (d) $8x^3 + 32x^2 + 32x$
    (e) $x^3 - 729$
    (f) $512x^3 + 343$

27. Simplify these algebraic fractions:

    (a) $\frac{x^2-64}{x+8}$
    (b) $\frac{x^2+17x+72}{x+9}$
    (c) $\frac{9x^2-36}{x^2-4}$
    (d) $\frac{x^3-729}{x^2-81}$

Section H: Algebraic Fractions and Advanced Topics

28. Add and subtract these algebraic fractions:

    (a) $\frac{9}{x} + \frac{6}{x}$
    (b) $\frac{13}{x} - \frac{8}{x}$
    (c) $\frac{6}{9x} + \frac{1}{18x}$
    (d) $\frac{8}{x+6} + \frac{5}{x-7}$
    (e) $\frac{x}{x+8} - \frac{7}{x-5}$
    (f) $\frac{8x}{x^2-49} + \frac{7}{x+7}$

29. Multiply and divide these algebraic fractions:

    (a) $\frac{x}{9} \times \frac{27}{x^2}$
    (b) $\frac{x+8}{10} \times \frac{20}{x+8}$
    (c) $\frac{x^2-64}{x+7} \div \frac{x-8}{x+7}$

(d) $\frac{8x+24}{x^2-81} \times \frac{x-9}{16}$

30. Solve these equations involving algebraic fractions:

(a) $\frac{x}{9} + \frac{x}{7} = 32$
(b) $\frac{8x+7}{10} = \frac{x-6}{7}$
(c) $\frac{9}{x} = \frac{8}{x-7}$
(d) $\frac{x+7}{x-8} = \frac{6x}{x+7}$

31. Make the subject of these formulae:

(a) $Q = \frac{It}{A}$, make I the subject
(b) $V = \frac{1}{3}\pi r^2 h$, make h the subject
(c) $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$, make R the subject
(d) $K = \frac{1}{2}mv^2$, make v the subject

Section I: Sequences

32. Find the nth term for these arithmetic sequences:

(a) $19, 25, 31, 37, ...$
(b) $11, 18, 25, 32, ...$
(c) $55, 51, 47, 43, ...$
(d) $\frac{2}{5}, \frac{4}{5}, \frac{6}{5}, \frac{8}{5}, ...$

33. For these geometric sequences, find the nth term:

(a) $8, 24, 72, 216, ...$
(b) $11, 44, 176, 704, ...$
(c) $512, 256, 128, 64, ...$
(d) $7, -28, 112, -448, ...$

34. Find the sum of these series:

(a) First 50 terms of $12 + 16 + 20 + 24 + ...$
(b) First 3 terms of $10 + 30 + 90 + 270 + ...$
(c) $7 + 11 + 15 + ... + 103$ (arithmetic series)
(d) $9 + 18 + 36 + ... + 576$ (geometric series)

35. These are quadratic sequences. Find the nth term:

(a) $8, 32, 72, 128, 200, ...$
(b) $7, 19, 37, 61, 91, ...$
(c) $6, 24, 54, 96, 150, ...$
(d) $10, 29, 54, 85, 122, ...$

36. A sequence is defined by $u_1 = 8$ and $u_{n+1} = 6u_n - 7$.

(a) Find the first 5 terms
(b) Find a formula for u_n
(c) Calculate u_3

Section J: Problem Solving

37. The sum of two numbers is 38 and their product is 357. Find the two numbers.

38. A rectangular park has perimeter 88m. If the length is 18m more than the width, find the dimensions.

39. The difference between a positive number and its reciprocal is $\frac{80}{9}$. Find the number.

40. A golf ball's height h (in meters) after t seconds is given by: $h = 180t - 5t^2$

    (a) When does it hit the ground?

    (b) What is its maximum height?

    (c) When is it 1575m high?

41. Prove that the sum of the first n terms of the sequence $5, 10, 15, 20, ...$ is $\frac{5n(n+1)}{2}$.

42. The quadratic $ax^2 + bx + c = 0$ has roots μ and ν.

    (a) Show that $\mu + \nu = -\frac{b}{a}$

    (b) Show that $\mu\nu = \frac{c}{a}$

    (c) If the roots are 9 and -6, find a, b, and c when $a = 6$

43. A function is defined as $n(x) = x^2 + hx + j$. If $n(5) = 28$ and $n(7) = 48$, find h and j.

44. The sum of the first n terms of a sequence is $S_n = 8n^2 - 5n$. Find the nth term of the sequence.

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 100

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