A Level Mechanics
Practice Test 4: Circular Motion

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Draw diagrams where appropriate to illustrate your solutions.
Time allowed: 3 hours

Section A: Circular Motion Fundamentals [28 marks]

Question 1 [11 marks]

(a) Define angular acceleration and state its SI unit. **[2 marks]**

(b) Derive the relationship between tangential acceleration and angular acceleration: $a_t = r\alpha$. **[3 marks]**

(c) Explain the difference between tangential and centripetal acceleration in circular motion. **[3 marks]**

(d) For non-uniform circular motion, write expressions for the magnitude of total acceleration. **[3 marks]**

Question 2 [12 marks] A grinding wheel of radius 0.15 m starts from rest and accelerates uniformly to 3000 rpm in 8 seconds.

(a) Convert the final angular velocity to rad/s. **[2 marks]**

(b) Calculate the angular acceleration. **[2 marks]**

(c) Find the number of revolutions during acceleration. **[3 marks]**

(d) Calculate the final linear speed of a point on the rim. **[2 marks]**

(e) Find the tangential and centripetal accelerations at the rim when at full speed. **[3 marks]**

Question 3 [5 marks] A particle moves in a circle of radius 3 m such that its angular position is given by $\theta = 2t^3 - 4t + 1$ radians.

(a) Find expressions for angular velocity and angular acceleration. **[3 marks]**

(b) Calculate the angular velocity and acceleration at $t = 2$ seconds. **[2 marks]**

Section B: Centripetal Force [31 marks]

Question 4 [13 marks]

(a) Explain why centripetal force is required for circular motion and why it acts radially inward. **[4 marks]**

(b) Derive the formula $F_c = m\omega^2 r$ starting from $F = ma$. **[4 marks]**

(c) Explain the difference between centripetal force as a requirement and the actual forces that provide it. **[3 marks]**

(d) List five different physical forces that can provide centripetal force in various situations. **[2 marks]**

Question 5 [18 marks] A hammer thrower spins a 7.26 kg hammer on a chain of length 1.2 m before release.

(a) If the hammer moves at 28 m/s just before release, calculate the tension in the chain. **[3 marks]**

(b) Find the angular velocity of the hammer. **[2 marks]**

(c) Calculate the centripetal acceleration. **[2 marks]**

(d) If the chain can withstand maximum tension of 6000 N, find the maximum safe speed. **[3 marks]**

(e) For vertical circular motion at this maximum speed, calculate the minimum speed at the top to keep the chain taut. **[4 marks]**

(f) Find the tension at the bottom of a vertical circle when moving at this critical speed. **[4 marks]**

Section C: Horizontal Circular Motion [34 marks]

Question 6 [19 marks] A Formula 1 racing car of mass 740 kg takes a flat corner of radius 85 m at maximum possible speed.

(a) If the coefficient of static friction between tires and track is 1.2, find the maximum cornering speed. **[4 marks]**

(b) Calculate the maximum centripetal acceleration achievable. **[2 marks]**

(c) Find the maximum lateral force from the tires. **[2 marks]**

(d) If the car corners at 35 m/s, calculate the required centripetal force. **[2 marks]**

(e) Determine whether the car will skid at this speed. **[3 marks]**

(f) Calculate the safety margin (unused friction) as a percentage. **[3 marks]**

(g) Find the minimum coefficient of friction needed to corner at 40 m/s. **[3 marks]**

Question 7 [15 marks] A speed skater of mass 70 kg glides around a curve of radius 25 m on ice where the coefficient of friction is 0.12.

(a) Calculate the maximum centripetal force available from friction. **[2 marks]**

(b) Find the maximum safe cornering speed. **[3 marks]**

(c) If the skater moves at 8 m/s, calculate the required friction force. **[3 marks]**

(d) Find the centripetal acceleration at this speed. **[2 marks]**

(e) To maintain this speed, the skater leans inward. Calculate the lean angle from vertical. **[5 marks]**

Section D: Banked Curves [26 marks]

Question 8 [**10 marks**]

(a) Explain why banking allows higher speeds on curves and reduces tire wear. [**4 marks**]

(b) For a banked curve with friction, derive the maximum speed formula when friction acts up the slope. [**6 marks**]

Question 9 [**16 marks**] A velodrome track has radius 125 m and is banked at 42°.

(a) Calculate the speed for which no friction is needed. [**3 marks**]

(b) A cyclist of mass 65 kg travels at 18 m/s. Calculate the centripetal force required. [**2 marks**]

(c) Find the normal force from the track surface. [**4 marks**]

(d) Calculate the friction force needed and determine its direction. [**4 marks**]

(e) If the coefficient of friction is 0.3, find the maximum and minimum safe speeds. [**3 marks**]

Section E: Vertical Circular Motion [32 marks]

Question 10 [**14 marks**]

(a) Explain the energy changes during vertical circular motion under gravity. [**4 marks**]

(b) For an object on a string, derive the tension equations at any angle θ from the bottom. [**5 marks**]

(c) Show that for minimum speed conditions, the speed at any height h above the bottom is $v = \sqrt{g(5r - 2h)}$. [**5 marks**]

Question 11 [**18 marks**] A stunt pilot flies a vertical loop of radius 200 m. The pilot and aircraft have combined mass 1200 kg.

(a) Calculate the minimum speed at the top to maintain circular flight. [**3 marks**]

(b) Find the minimum speed at the bottom using energy conservation. [**4 marks**]

(c) If the aircraft enters the loop at 120 m/s, calculate the speed at the top. [**3 marks**]

(d) Find the apparent weight of the pilot at the top for this speed. [**3 marks**]

(e) Calculate the apparent weight at the bottom. [**3 marks**]

(f) Compare these with the pilot's actual weight and comment on the g-forces experienced. [**2 marks**]

Section F: Conical Pendulums [24 marks]

Question 12 [**10 marks**]

(a) For a conical pendulum, derive the relationship $\tan\theta = \frac{\omega^2 r}{g}$. [**4 marks**]

(b) Show that the period is independent of the mass: $T = 2\pi\sqrt{\frac{l\cos\theta}{g}}$. [**6 marks**]

Question 13 [**14 marks**] A conical pendulum consists of a 1.2 kg mass on a 2.4 m string rotating such that the string makes 28° with the vertical.

(a) Calculate the radius of the horizontal circular path. [**2 marks**]

(b) Find the vertical distance below the suspension point. **[2 marks]**

(c) Calculate the tension in the string. **[3 marks]**

(d) Find the angular velocity and linear speed. **[3 marks]**

(e) Calculate the period and frequency of revolution. **[3 marks]**

(f) Find the centripetal force acting on the mass. **[1 mark]**

Section G: Motion in a Vertical Circle - Loops [30 marks]

Question 14 [18 marks] A spherical ball rolls down a track and enters a vertical circular loop of radius 1.5 m. The ball has mass 0.5 kg and radius 0.1 m.

(a) Find the minimum speed at the top for the ball to maintain contact. **[3 marks]**

(b) Calculate the minimum speed at the bottom considering rolling motion (moment of inertia $I = \frac{2}{5}mr^2$). **[5 marks]**

(c) If the ball starts from rest at height 4 m above the bottom, find its speed entering the loop. **[3 marks]**

(d) Calculate the speed at the top of the loop. **[3 marks]**

(e) Find the normal force at the top. **[2 marks]**

(f) Calculate the normal force at the bottom. **[2 marks]**

Question 15 [12 marks] A water bucket is swung in a vertical circle of radius 0.8 m. The bucket has mass 2 kg and contains 5 kg of water.

(a) Find the minimum speed at the top to prevent water spilling. **[3 marks]**

(b) Calculate the minimum speed at the bottom. **[3 marks]**

(c) If swung at this minimum speed, find the apparent weight of water at the bottom. **[3 marks]**

(d) Calculate the force exerted by the bucket on the water at the top. **[3 marks]**

Section H: Applications and Problem Solving [30 marks]

Question 16 [16 marks] A space station rotates to provide artificial gravity. The station has radius 150 m and rotates to simulate Earth gravity at the outer rim.

(a) Calculate the required angular velocity. **[3 marks]**

(b) Find the period of rotation. **[2 marks]**

(c) Calculate the linear speed of the outer rim. **[2 marks]**

(d) Find the artificial gravity at radius 100 m from the center. **[3 marks]**

(e) Calculate the artificial gravity at radius 75 m. **[2 marks]**

(f) If an astronaut of mass 80 kg stands at the outer rim, find the normal force from the floor. **[2 marks]**

(g) Compare the artificial gravity gradient across a 2 m tall astronaut. **[2 marks]**

Question 17 [14 marks] A theme park ride consists of a giant swing with 8 m chains. During operation, riders swing out to make 50° with the vertical.

(a) Calculate the radius of the circular path. **[2 marks]**

(b) Find the height of riders above the lowest point. **[3 marks]**

(c) Calculate the angular velocity of the ride. **[4 marks]**

(d) Find the linear speed of the riders. **[2 marks]**

(e) If a rider has mass 70 kg, calculate the tension in the chain. **[3 marks]**

Section I: Advanced Circular Motion [28 marks]

Question 18 [15 marks] A banked circular racetrack has two lanes. The inner lane has radius 180 m and the outer lane has radius 200 m. Both are banked at 15°.

(a) Calculate the design speed for each lane (no friction required). **[4 marks]**

(b) A car travels at 35 m/s in the inner lane. Find the friction force required. **[5 marks]**

(c) The same car moves to the outer lane at the same speed. Compare the friction requirements. **[4 marks]**

(d) Which lane allows higher maximum speeds? Justify your answer. **[2 marks]**

Question 19 [13 marks] A particle slides on the inside of a smooth hemispherical bowl of radius 0.6 m. It is given an initial horizontal speed at the bottom.

(a) If the initial speed is 4 m/s, find the maximum height reached. **[4 marks]**

(b) Calculate the speed when the particle is at height 0.3 m. **[3 marks]**

(c) Find the normal force from the bowl at this height if the particle has mass 0.2 kg. **[3 marks]**

(d) Determine the minimum initial speed needed to reach the top of the bowl. **[3 marks]**

Section J: Comprehensive Applications [30 marks]

Question 20 [18 marks] A centrifuge used for astronaut training has radius 8 m and can subject trainees to up to 10g acceleration.

(a) Calculate the maximum angular velocity. **[3 marks]**

(b) Find the maximum linear speed. **[2 marks]**

(c) Calculate the period at maximum speed. **[2 marks]**

(d) If a trainee of mass 75 kg experiences 8g, find the centripetal force. **[2 marks]**

(e) Calculate the angular velocity for 8g acceleration. **[3 marks]**

(f) Find the apparent weight of the trainee at 8g. **[2 marks]**

(g) If the centrifuge starts from rest and reaches 8g in 30 seconds, calculate the angular acceleration. **[2 marks]**

(h) Find the number of revolutions during this acceleration. **[2 marks]**

Question 21 [12 marks] Two particles of masses 2 kg and 3 kg are connected by a rigid rod of length 1.5 m and negligible mass. The system rotates about an axis perpendicular to the rod.

(a) If the axis passes through the 2 kg mass, find the centripetal force on the 3 kg mass when rotating at 4 rad/s. **[3 marks]**

(b) Calculate the moment of inertia about this axis. **[2 marks]**

(c) Find the kinetic energy of the system. **[2 marks]**

(d) If the axis is moved to pass through the center of mass, find the new moment of inertia. **[3 marks]**

(e) Calculate the kinetic energy about the center of mass axis at the same angular velocity. **[2 marks]**

Physics Data and Formulae

Circular Motion:

Angular velocity: $\omega = \frac{v}{r} = \frac{2\pi}{T} = 2\pi f$

Angular acceleration: $\alpha = \frac{d\omega}{dt}$

Tangential acceleration: $a_t = r\alpha$

Centripetal acceleration: $a_c = \frac{v^2}{r} = \omega^2 r$

Total acceleration: $a = \sqrt{a_t^2 + a_c^2}$

Centripetal force: $F_c = ma_c = \frac{mv^2}{r} = m\omega^2 r$

Rotational Kinematics:

$\omega = \omega_0 + \alpha t$

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\omega^2 = \omega_0^2 + 2\alpha\theta$

Vertical Circular Motion:

At top: $T + mg = \frac{mv^2}{r}$ or $N + mg = \frac{mv^2}{r}$

At bottom: $T - mg = \frac{mv^2}{r}$ or $N - mg = \frac{mv^2}{r}$

At angle θ from bottom: $T - mg\cos\theta = \frac{mv^2}{r}$

Minimum speed at top: $v_{min} = \sqrt{gr}$

Banking:

No friction: $\tan\theta = \frac{v^2}{rg}$

Maximum speed with friction: $\tan\theta = \frac{v^2/rg - \mu}{1 + \mu v^2/rg}$

Minimum speed with friction: $\tan\theta = \frac{v^2/rg + \mu}{1 - \mu v^2/rg}$

Conical Pendulum:

$\cos\theta = \frac{g}{\omega^2 l}$, $\tan\theta = \frac{\omega^2 r}{g}$

$T\cos\theta = mg$, $T\sin\theta = m\omega^2 r$

Period: $T = 2\pi\sqrt{\frac{l\cos\theta}{g}}$

Energy Conservation:

$\frac{1}{2}mv_1^2 + mgh_1 = \frac{1}{2}mv_2^2 + mgh_2$

Rolling motion: $KE = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

Moment of Inertia:

Point mass: $I = mr^2$

Solid sphere: $I = \frac{2}{5}mr^2$

Two masses on rod: $I = m_1 r_1^2 + m_2 r_2^2$

Constants:

Acceleration due to gravity: $g = 9.8$ m/s^2

Trigonometric Values:

$\sin 15 = 0.259$, $\cos 15 = 0.966$, $\tan 15 = 0.268$
$\sin 28 = 0.469$, $\cos 28 = 0.883$, $\tan 28 = 0.532$
$\sin 42 = 0.669$, $\cos 42 = 0.743$, $\tan 42 = 0.900$
$\sin 50 = 0.766$, $\cos 50 = 0.643$, $\tan 50 = 1.192$

END OF TEST

Total marks: 283

Grade boundaries: A* 255, A 227, B 198, C 170, D 142, E 113

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