A Level Mechanics
Practice Test 5: Newton's Laws of Motion

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Draw diagrams where appropriate to illustrate your solutions.
Time allowed: 2 hours 35 minutes

Section A: Comprehensive Force Theory [32 marks]

Question 1 [18 marks]

(a) Explain the concept of "pseudo-forces" in rotating reference frames and derive the expression for centrifugal force experienced by an object in such a frame. **[6 marks]**

(b) A bus turns a corner at constant speed. Analyze the motion of a pendulum hanging from the bus ceiling from the perspectives of: (i) a passenger on the bus, (ii) an observer on the street. **[6 marks]**

(c) Discuss how Newton's laws relate to the conservation of momentum and explain when momentum is conserved in a system. **[4 marks]**

(d) Explain why Newton's laws are considered universal within their domain of applicability. **[2 marks]**

Question 2 [14 marks] A particle of mass 1.8 kg experiences forces $\vec{F_1} = (2t^2 + 4)\hat{i} + (6t - 3)\hat{j}$ N and $\vec{F_2} = (-t^2 + 7)\hat{i} + (2t^2 - 4t)\hat{j}$ N, where t is time in seconds.

(a) Find the net force and acceleration vectors as functions of time. **[4 marks]**

(b) If the particle starts from rest at position $(0, 1)$ m, calculate its velocity at t = 2.5 s. **[5 marks]**

(c) Determine the position vector at t = 2.5 s. **[5 marks]**

Section B: Advanced Equilibrium Systems [45 marks]

Question 3 [28 marks] A non-uniform beam of mass 24 kg and length 6 m has its center of gravity 2.5 m from the left end. It is supported by four vertical cables: at the left end, 1.8 m from the left end, 4.2 m from the left end, and at the right end. Loads of 12 kg, 16 kg, and 10 kg are placed at 1 m, 3 m, and 5 m from the left end respectively.

(a) Write the complete force and moment equilibrium equations. **[5 marks]**

(b) If the tensions in the second and third cables are 80 N and 100 N respectively, calculate the tensions in the first and fourth cables. **[8 marks]**

(c) Determine where an additional 18 kg load must be placed to make the tension in the first cable zero. **[7 marks]**

(d) Find the maximum mass that can be placed at the left end without the fourth cable becoming slack. **[5 marks]**

(e) Verify your solution using moments about different reference points. **[3 marks]**

Question 4 [17 marks] A complex force system acts on a rigid structure: $\vec{F_1} = 40\hat{i} + 30\hat{j}$ N at $(1, 3)$ m, $\vec{F_2} = -25\hat{i} + 50\hat{j}$ N at $(3, 1)$ m, $\vec{F_3} = -10\hat{i} - 45\hat{j}$ N at $(0, 2)$ m, and $\vec{F_4} = -15\hat{i} - 25\hat{j}$ N at $(4, 0)$ m.

(a) Verify that the force system is in equilibrium. **[4 marks]**

(b) Calculate the resultant moment about the origin and verify moment equilibrium. **[6 marks]**

(c) If forces $\vec{F_2}$ and $\vec{F_3}$ are removed, find the single replacement force and its position to restore equilibrium. **[7 marks]**

Section C: Complex Friction Systems [40 marks]

Question 5 [24 marks] Four blocks with masses 4 kg, 6 kg, 8 kg, and 5 kg are arranged in a line on a horizontal surface with coefficients of friction: between 4 kg and 6 kg $\mu = 0.5$, between 6 kg and 8 kg $\mu = 0.4$, between 8 kg and 5 kg $\mu = 0.6$, and between 5 kg and surface $\mu = 0.3$.

(a) A horizontal force F is applied to the 6 kg block. Determine all critical values of F at which relative motion begins between different interfaces. **[10 marks]**

(b) If F = 60 N, analyze the motion of all four blocks and calculate accelerations. **[8 marks]**

(c) If F = 90 N is applied to the 4 kg block instead, determine the resulting motion pattern. **[6 marks]**

Question 6 [16 marks] A block of mass 12 kg rests on an inclined plane of angle 38°. The coefficients of friction are $\mu_s = 0.8$ and $\mu_k = 0.6$.

(a) Analyze whether the block will remain in equilibrium on the incline. **[4 marks]**

(b) A force F is applied at 30° above the incline surface to push the block up. Find the minimum F to start motion. **[6 marks]**

(c) If F = 100 N is applied horizontally (parallel to the base), calculate the resulting acceleration. **[6 marks]**

Section D: Sophisticated Multi-Body Dynamics [48 marks]

Question 7 [28 marks] A complex system involves five masses: 7 kg and 9 kg connected by a string on a rough horizontal table ($\mu = 0.25$), with the 9 kg mass connected over a pulley to an 11 kg mass hanging vertically, and separately, a 5 kg mass on the table connected over another pulley to a 6 kg hanging mass.

(a) Draw comprehensive free body diagrams for all five masses. **[10 marks]**

(b) Establish the complete set of equations of motion. **[8 marks]**

(c) Calculate the acceleration of each part of the system. **[6 marks]**

(d) Determine the tension in each string segment. **[4 marks]**

Question 8 [20 marks] A cargo train consists of an engine (mass 200,000 kg) pulling five freight cars (masses 40,000 kg, 35,000 kg, 45,000 kg, 30,000 kg, and 50,000 kg). The engine develops 250,000 N thrust, and resistance forces are proportional to mass with constant 0.9 N/kg.

(a) Calculate the acceleration on level track. **[4 marks]**

(b) Find the tension in each coupling between cars. **[10 marks]**

(c) When descending a 2.5° slope, determine the new acceleration and whether any coupling goes into compression. **[6 marks]**

Section E: Advanced Circular Motion Applications [35 marks]

Question 9 [20 marks] A racetrack curve has radius 250 m and is banked at 22°. The coefficient of friction varies with tire condition from 0.2 to 1.1.

(a) Calculate the optimal design speed requiring no friction. **[4 marks]**

(b) Determine the maximum safe speed in wet conditions ($\mu = 0.2$). **[6 marks]**

(c) Find the minimum speed in racing conditions ($\mu = 1.1$) below which cars would slide down. **[6 marks]**

(d) For a 800 kg race car at 40 m/s with $\mu = 0.8$, calculate all forces and verify safe operation. **[4 marks]**

Question 10 [15 marks] A particle of mass 1.2 kg is suspended by three strings from three points forming an equilateral triangle with side length 2.4 m in a horizontal plane. The particle moves in a horizontal circle 1.8 m below the plane of the triangle.

(a) Calculate the radius of the circular motion and the angle each string makes with the vertical. **[5 marks]**

(b) Find the tension in each string. **[6 marks]**

(c) Determine the speed and angular velocity of the circular motion. **[4 marks]**

Section F: Advanced Variable Force Analysis [32 marks]

Question 11 [20 marks] A particle of mass 4.5 kg moves in a straight line under the action of force $F = 24\cos(0.5t) - 16\sin(1.2t) + 8e^{-0.3t} - 12$ N, where t is in seconds.

(a) Starting from rest at the origin, derive expressions for velocity and displacement as functions of time. **[10 marks]**

(b) Calculate the velocity and displacement at t = 2π seconds. **[5 marks]**

(c) Determine when the particle first comes to rest after t = 0. **[5 marks]**

Question 12 [12 marks] A sophisticated elevator system has three phases: acceleration $a_1(t) = 4 - 0.4t$ m/s² for 0 t 5 s, constant velocity for 10 s, then deceleration $a_3(t) = -2 + 0.2(t - 15)$ m/s² until stopping.

(a) Calculate the velocity profile and total displacement for the complete journey. **[7 marks]**

(b) For a 90 kg passenger, determine the maximum and minimum apparent weights experienced. **[3 marks]**

(c) Plot the variation of apparent weight throughout the entire journey. **[2 marks]**

Section G: Complex Constraint Analysis [28 marks]

Question 13 [28 marks] A particle of mass 0.5 kg slides on a smooth three-dimensional track described by the curve $\vec{r}(s) = s\hat{i} + 0.1s^2\hat{j} + 0.05s^3\hat{k}$ meters, where s is the arc length parameter. The particle starts from rest at s = 0.

(a) Find the speed of the particle when s = 2 m using energy methods. **[5 marks]**

(b) Calculate the tangential and normal components of acceleration at s = 2 m. **[7 marks]**

(c) Determine the magnitude of the constraint force from the track at s = 2 m. **[6 marks]**

(d) If the track can provide a maximum constraint force of 40 N, find the value of s where the particle would leave the track. **[6 marks]**

(e) Calculate the trajectory of the particle after leaving the track in terms of time. **[4 marks]**

Physics Data and Formulae

Newton's Laws:
First Law: $\sum \vec{F} = 0 \Leftrightarrow \vec{a} = 0$
Second Law: $\vec{F}_{net} = m\vec{a} = \frac{d\vec{p}}{dt}$
Third Law: $\vec{F}_{AB} = -\vec{F}_{BA}$
Momentum: $\vec{p} = m\vec{v}$

Force Analysis:
Friction: $f_s \leq \mu_s N$, $f_k = \mu_k N$
Inclined plane: $mg\sin\theta$ (parallel), $mg\cos\theta$ (normal)
Circular motion: $F_c = \frac{mv^2}{r} = m\omega^2 r$
Centrifugal force: $F_{cf} = m\omega^2 r$ (rotating frame)

Equilibrium Analysis:
Force equilibrium: $\sum F_x = 0$, $\sum F_y = 0$, $\sum F_z = 0$
Moment equilibrium: $\sum \vec{M} = 0$ (about any point)
Moment: $\vec{M} = \vec{r} \times \vec{F}$

Circular Motion:
Banking: $\tan\theta = \frac{v^2}{rg}$ (no friction)
With friction: $\frac{mv^2}{r} = mg\tan\theta \pm \mu mg\sec\theta$
Conical pendulum: $T\cos\theta = mg$, $T\sin\theta = \frac{mv^2}{r}$

Variable Motion:
$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$
$v = \int a\,dt$, $s = \int v\,dt$
Parametric motion: $\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$

Constraint Forces:
Normal force: perpendicular to surface
Tangential acceleration: $a_t = \frac{d|\vec{v}|}{dt}$
Normal acceleration: $a_n = \frac{v^2}{\rho}$ (= radius of curvature)

Vector Calculus:
$\frac{d}{dt}(e^{at}) = ae^{at}$
$\int e^{at}dt = \frac{1}{a}e^{at} + C$

$$\int \cos(at)dt = \frac{1}{a}\sin(at) + C$$
$$\int \sin(at)dt = -\frac{1}{a}\cos(at) + C$$

Constants:

Acceleration due to gravity: $g = 9.8$ m/s^2

Trigonometric Values:

$\sin 22 = 0.375$, $\cos 22 = 0.927$, $\tan 22 = 0.404$
$\sin 30 = 0.500$, $\cos 30 = 0.866$, $\tan 30 = 0.577$
$\sin 38 = 0.616$, $\cos 38 = 0.788$, $\tan 38 = 0.781$
$\sin 2.5 = 0.044$, $\cos 2.5 = 0.999$, $\tan 2.5 = 0.044$

END OF TEST

Total marks: 260

Grade boundaries: A* 234, A 208, B 182, C 156, D 130, E 104

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