GCSE Higher Mathematics
Practice Test 3: Ratio, Proportion, and Rates of Change

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Time allowed: 90 minutes

Section A: Advanced Ratios

1. Express these ratios in their simplest form:

   (a) $3.6 : 1.8 : 4.5$

   (b) $\frac{5}{6} : \frac{1}{3} : \frac{3}{4}$

   (c) $0.35 : 0.175 : 0.525$

   (d) $1\frac{3}{4} : 2\frac{1}{4} : 3\frac{1}{2}$

2. Five departments in a company receive budget allocations in the ratio $6 : 4 : 8 : 3 : 5$. If the total budget is £130,000:

   (a) How much does each department receive?

   (b) If there's a bonus pool of £39,000 distributed in the same ratio, how much bonus does each department get?

   (c) What percentage of the total budget does the largest department receive?

3. The internal angles of a pentagon are in the ratio $4 : 5 : 6 : 7 : 8$. Find each angle.

4. A garden fertilizer mix uses nitrogen, phosphorus, and potassium in the ratio $12 : 5 : 7$ by mass.

   (a) How much of each nutrient is needed for 120kg of fertilizer?

   (b) If 25kg of phosphorus is used, find the mass of nitrogen and potassium needed

   (c) What fraction of the mixture is nitrogen?

5. Share £1260 among three siblings so that the eldest gets £80 more than the middle child, and the middle child gets £60 more than the youngest.

Section B: Direct Proportion

6. y is directly proportional to x. When $x = 18$, $y = 12$.

   (a) Find the equation connecting y and x

   (b) Calculate y when $x = 27$

   (c) Calculate x when $y = 16$

   (d) Sketch the graph of y against x

7. Q varies directly as the square of R. When $R = 6$, $Q = 72$.

   (a) Express Q in terms of R
   (b) Find Q when $R = 8$
   (c) Find R when $Q = 162$
   (d) What happens to Q when R is halved?

8. The cost of gold jewelry varies directly as its mass. 25 grams cost £875.

   (a) Find the cost per gram
   (b) How much does 42 grams cost?
   (c) What mass can be bought for £1225?
   (d) Express the cost C in terms of mass M

9. The volume of water displaced by a sphere varies directly as the cube of its radius. A sphere with radius 4 cm displaces 268.8 cm^3.

   (a) Find the volume displaced by a sphere with radius 6 cm
   (b) What radius displaces 1728 cm^3?
   (c) Write the relationship as an equation

Section C: Inverse Proportion

10. y is inversely proportional to x. When $x = 6$, $y = 20$.

    (a) Find the equation connecting y and x
    (b) Calculate y when $x = 15$
    (c) Calculate x when $y = 24$
    (d) Sketch the graph of y against x

11. The time taken to lay a path is inversely proportional to the number of workers. With 6 workers, it takes 8 hours.

    (a) How long does it take with 12 workers?
    (b) How many workers are needed to complete it in 3 hours?
    (c) Express time T in terms of number of workers W
    (d) If 2 workers are absent, how much longer will the job take?

12. P is inversely proportional to the square of Q. When $Q = 3$, $P = 16$.

    (a) Express P in terms of Q
    (b) Find P when $Q = 4$
    (c) Find Q when $P = 4$
    (d) What happens to P when Q is tripled?

13. The brightness of a lamp is inversely proportional to the square of the distance from it. At 2 meters, the brightness is 400 lux.

    (a) Find the brightness at 5 meters
    (b) At what distance is the brightness 100 lux?
    (c) Write the relationship as an equation

Section D: Combined Proportion

14. z varies directly as p and inversely as q. When $p = 12$ and $q = 8$, $z = 15$.

    (a) Express z in terms of p and q

    (b) Find z when $p = 16$ and $q = 5$

    (c) Find q when $p = 20$ and $z = 12$

    (d) What happens to z if p is doubled and q is halved?

15. The kinetic energy E of an object varies directly as its mass m and as the square of its velocity v. When $m = 5$ kg and $v = 10$ m/s, $E = 250$ joules.

    (a) Express E in terms of m and v

    (b) Find E when $m = 8$ kg and $v = 15$ m/s

    (c) Find v when $m = 12$ kg and $E = 1200$ joules

    (d) What happens to E if m is tripled and v is doubled?

16. The electrical power P in a circuit varies directly as the square of the current I and as the resistance R. When $I = 3$ amps and $R = 8$ ohms, $P = 72$ watts.

    (a) Express P in terms of I and R

    (b) Find P when $I = 4$ amps and $R = 12$ ohms

    (c) Find R when $P = 200$ watts and $I = 5$ amps

17. s varies directly as t^2 and inversely as u^3. When $t = 3$ and $u = 2$, $s = 18$.

    (a) Find the constant of proportionality

    (b) Express s in terms of t and u

    (c) Calculate s when $t = 4$ and $u = 3$

Section E: Speed, Distance, and Time

18. A bus travels 168 km in 2 hours 48 minutes. Calculate:

    (a) The average speed in km/h

    (b) The average speed in m/s

    (c) How far it travels in 3 hours 30 minutes at this speed

    (d) How long it takes to travel 126 km at this speed

19. Convert these speeds:

    (a) 22 m/s to km/h

    (b) 90 km/h to m/s

    (c) 55 mph to km/h (use 1 mile = 1.6 km)

    (d) 120 km/h to mph

20. A delivery van travels from warehouse to shop at 50 km/h and returns at 30 km/h. The total journey takes 3.2 hours.

    (a) Find the distance from warehouse to shop

    (b) Calculate the average speed for the whole journey

    (c) How much time was lost due to slower return speed?

21. Two motorcycles start simultaneously from points 240 km apart and ride towards each other. One travels at 75 km/h and the other at 45 km/h.

    (a) When do they meet?

    (b) How far from each starting point do they meet?

    (c) What is their relative speed of approach?

22. A marathon runner covers 42.2 km. For the first 25 km, they run at 15 km/h. For the remaining distance, they slow to 10 km/h.

    (a) Calculate the total time taken

    (b) Find the average speed for the whole marathon

    (c) How much faster would they need to run the second part to finish in exactly 3 hours?

Section F: Density and Flow Rates

23. A stone block has volume 450 cm^3 and mass 1.215 kg.

    (a) Calculate its density in g/cm^3

    (b) Calculate its density in kg/m^3

    (c) What mass of this stone would have volume 600 cm^3?

    (d) What volume would 2.43 kg of this stone occupy?

24. Different metals have these densities:

    - Copper: 8.9 g/cm^3
    - Silver: 10.5 g/cm^3
    - Lead: 11.3 g/cm^3

    (a) Compare the masses of 50 cm^3 of each metal

    (b) What volume of lead has the same mass as 75 cm^3 of copper?

    (c) A medallion weighs 315 g and has volume 30 cm^3. Which metal is it most likely made from?

25. Gas flows through a pipe at a rate of 3.6 m^3 per minute.

    (a) How much gas flows in 12 minutes?

    (b) How long to fill a 540 m^3 storage tank?

    (c) Express the flow rate in liters per second

    (d) If the pipe cross-sectional area is halved, what happens to the flow velocity?

26. A packaging machine operates at different rates throughout the day:

    - Morning (5 hours): 480 packages/hour
    - Afternoon (4 hours): 360 packages/hour
    - Evening (3 hours): 320 packages/hour

    (a) Calculate total daily output

    (b) Find the average production rate

    (c) How long to package 2000 items at the average rate?

Section G: Scale Factors and Similar Shapes

27. Two similar hexagons have corresponding sides in the ratio 5:8.

    (a) If the smaller hexagon has area 75 cm^2, find the area of the larger hexagon
    (b) If the larger hexagon has perimeter 96 cm, find the perimeter of the smaller hexagon
    (c) Find the ratio of their areas

28. A scale model of a ship is built to scale 1:75. The real ship is 225 m long and 30 m wide.

    (a) Find the dimensions of the model
    (b) If the model uses 1.2 m^2 of paint, how much paint does the real ship need?
    (c) The model weighs 8 kg. Estimate the mass of the real ship if made from the same material

29. A tourist map has scale 1:20000. Two landmarks are 15.6 cm apart on the map.

    (a) Calculate the actual distance in km
    (b) What map distance represents 2.4 km?
    (c) A park covers 8.4 cm^2 on the map. Find its actual area in hectares

30. Two similar pyramids have base edge ratio 2:5.

    (a) Find the ratio of their base areas
    (b) Find the ratio of their volumes
    (c) If the smaller pyramid has volume 64 cm^3, what is the volume of the larger pyramid?
    (d) If the larger pyramid has surface area 625 cm^2, what is the surface area of the smaller pyramid?

Section H: Advanced Rate Problems

31. An investment grows by 8% each year. Starting with £15000:

    (a) Write an expression for value after t years
    (b) What is the value after 5 years?
    (c) When will the investment exceed £25000?
    (d) What is the rate of growth after 3 years (£per year)?

32. A reservoir is supplied by two streams and has one outlet. Stream A provides 45 liters/minute, stream B provides 35 liters/minute, and the outlet releases 28 liters/minute.

    (a) What is the net rate of filling when all are operating?
    (b) How long to fill a 3120-liter reservoir from empty?
    (c) If stream B dries up, what is the net rate?
    (d) What outlet rate would exactly balance both streams?

33. Currency exchange rates:

    - £1 = $1.28
    - £1 = €1.16
    - $1 = ¥108

    (a) Convert £280 to dollars
    (b) Convert €145 to pounds

(c) Convert $192 to yen

(d) Find the exchange rate from euros to dollars

34. A textile machine produces fabric at variable rates:

   - Warm-up: 0.5 hours at 40% speed (normal speed is 60 m/hour)
   - Production: 7 hours at full speed
   - Cool-down: 0.5 hours at 25% speed

   (a) Calculate total fabric produced

   (b) Find the average production rate over the whole cycle

   (c) How long would it take to produce the same amount at 80% speed?

Section I: Problem Solving and Applications

35. A cake recipe for 6 people uses:

   - 300g flour
   - 200g sugar
   - 4 eggs
   - 150g butter

   (a) Adapt the recipe for 15 people

   (b) How much of each ingredient for 4 people?

   (c) If you have 750g flour, what's the maximum number of people you can serve?

36. A theater has 2400 seats. The ratio of stalls to circle to balcony seats is 8:5:3. Due to renovation, 120 stalls seats are removed but no other seats change.

   (a) How many seats of each type were there originally?

   (b) What is the new ratio of stalls to circle to balcony seats?

   (c) How many balcony seats would need to be removed to restore the original ratio?

37. The stopping distance of a truck varies directly as the square of its speed for speeds above 20 mph. At 40 mph, the stopping distance is 48 meters.

   (a) Find the stopping distance at 60 mph

   (b) At what speed is the stopping distance 75 meters?

   (c) Compare the stopping distances at 30 mph and 50 mph

38. A software company's development cost C (in thousands) varies as the square of team size T up to 12 people, then becomes linear. When $T = 6$, $C = 180$. When $T = 15$, $C = 570$.

   (a) Find the cost function for $T \leq 12$

   (b) Find the cost function for $T > 12$

   (c) What team size gives the lowest cost per person for $T \leq 12$?

   (d) What is the marginal cost for teams larger than 12?

39. A gear train has four gears in sequence. Gear 1 has 24 teeth and rotates at 180 rpm. Gear 2 has 36 teeth, gear 3 has 18 teeth, and gear 4 has 48 teeth.

   (a) Find the rotation speed of each gear

   (b) What is the overall gear ratio from gear 1 to gear 4?

   (c) If gear 1's speed increases to 240 rpm, find the new speed of gear 4

   (d) How would the system change if gear 3 had 24 teeth instead?

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 100

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