A Level Mechanics
Practice Test 3: Circular Motion

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Draw diagrams where appropriate to illustrate your solutions.
Time allowed: 3 hours

Section A: Circular Motion Fundamentals [27 marks]

Question 1 [10 marks]

(a) Define period and frequency in circular motion, stating their relationship and SI units. **[3 marks]**

(b) Derive the formula $\omega = 2\pi f$ where ω is angular velocity and f is frequency. **[3 marks]**

(c) Explain the difference between angular speed and angular velocity. **[2 marks]**

(d) Show that centripetal acceleration can be written as $a_c = \omega^2 r$. **[2 marks]**

Question 2 [12 marks] A propeller blade of length 2.2 m rotates at 1800 revolutions per minute.

(a) Convert the rotational speed to rad/s. **[2 marks]**

(b) Calculate the linear speed of the blade tip. **[2 marks]**

(c) Find the centripetal acceleration at the blade tip. **[2 marks]**

(d) Calculate the centripetal acceleration at a point 1.5 m from the center. **[2 marks]**

(e) If a small object of mass 0.05 kg is attached at the blade tip, find the centripetal force. **[2 marks]**

(f) Calculate the period of one revolution. **[2 marks]**

Question 3 [5 marks] A particle moves in a circle such that its angular displacement is given by $\theta = 3t^2 + 2t$ radians, where t is time in seconds.

(a) Find the angular velocity as a function of time. **[2 marks]**

(b) Calculate the angular velocity at $t = 4$ seconds. **[1 mark]**

(c) Find the angular acceleration. **[2 marks]**

Section B: Centripetal Force [30 marks]

Question 4 [12 marks]

(a) Explain why an object moving in a circle at constant speed is accelerating. **[3 marks]**

(b) Derive Newton's second law for circular motion: $F = \frac{mv^2}{r}$. **[4 marks]**

(c) Explain what happens to an object if the centripetal force suddenly disappears. **[3 marks]**

(d) Give examples of centripetal force in: (i) planetary motion, (ii) a car on a curve, (iii) a ball on a string. **[2 marks]**

Question 5 [18 marks] A ball of mass 1.5 kg is attached to a rope of length 2.5 m and swung in a horizontal circle.

(a) If the ball moves with speed 8 m/s, calculate the tension in the rope. **[3 marks]**

(b) Find the angular velocity. **[2 marks]**

(c) If the rope breaks when tension exceeds 80 N, find the maximum safe speed. **[3 marks]**

(d) Calculate the maximum angular velocity. **[2 marks]**

(e) When swung in a vertical circle, find the minimum speed at the top to keep the rope taut. **[4 marks]**

(f) For this minimum speed, calculate the tension at the bottom of the vertical circle. **[4 marks]**

Section C: Horizontal Circular Motion [33 marks]

Question 6 [18 marks] A rally car of mass 1100 kg negotiates a flat circular bend of radius 90 m.

(a) If the coefficient of static friction between tires and road is 0.8, find the maximum speed without skidding. **[4 marks]**

(b) Calculate the maximum centripetal acceleration available. **[2 marks]**

(c) Find the maximum friction force. **[2 marks]**

(d) If the car travels at 16 m/s, calculate the centripetal force required. **[2 marks]**

(e) Will the car skid at this speed? Justify your answer. **[3 marks]**

(f) What is the minimum coefficient of friction needed to travel at 20 m/s? **[3 marks]**

(g) Calculate the centripetal acceleration at 20 m/s. **[2 marks]**

Question 7 [15 marks] A cyclist of total mass 75 kg travels around a horizontal circular track of radius 40 m. The cyclist leans inward to maintain balance.

(a) If the cyclist travels at 12 m/s, calculate the required centripetal force. **[2 marks]**

(b) Find the centripetal acceleration. **[2 marks]**

(c) Draw a force diagram showing the forces acting on the cyclist. **[3 marks]**

(d) If the coefficient of friction is 0.6, calculate the lean angle from vertical. **[5 marks]**

(e) Calculate the normal force from the track surface. **[3 marks]**

Section D: Banked Curves [25 marks]

Question 8 [**9 marks**]

(a) Explain the advantages of banking a curve compared to a flat curve. [**3 marks**]

(b) For a banked curve with no friction, derive $v = \sqrt{rg\tan\theta}$. [**6 marks**]

Question 9 [**16 marks**] A motorway exit ramp has radius 200 m and is banked at 18°.

(a) Calculate the design speed requiring no friction. [**3 marks**]

(b) A lorry of mass 8000 kg travels at 32 m/s on this ramp. Find the centripetal force needed. [**2 marks**]

(c) Calculate the component of weight parallel to the banked surface. [**3 marks**]

(d) Find the normal force from the road surface. [**4 marks**]

(e) Calculate the friction force required and state its direction. [**4 marks**]

Section E: Vertical Circular Motion [31 marks]

Question 10 [**13 marks**]

(a) Explain why the speed of an object varies in vertical circular motion under gravity alone. [**3 marks**]

(b) For an object on a string in vertical circular motion, derive the tension equations at the top and bottom. [**5 marks**]

(c) Show that the minimum speed at the top for circular motion is $v = \sqrt{gr}$. [**3 marks**]

(d) Derive the relationship between top and bottom speeds using energy conservation. [**2 marks**]

Question 11 [**18 marks**] A child's toy consists of a ball of mass 0.3 kg on a string of length 0.6 m, swung in a vertical circle.

(a) Calculate the minimum speed at the top to complete the circle. [**2 marks**]

(b) Find the corresponding minimum speed at the bottom. [**3 marks**]

(c) Calculate the tension at the top for minimum speed conditions. [**2 marks**]

(d) Find the tension at the bottom for minimum speed conditions. [**3 marks**]

(e) If the speed at the bottom is 6 m/s, calculate the speed at the top. [**3 marks**]

(f) For this case, find the tensions at both top and bottom positions. [**5 marks**]

Section F: Conical Pendulums [23 marks]

Question 12 [**9 marks**]

(a) Define a conical pendulum and sketch the forces acting on the bob. [**3 marks**]

(b) Derive the period formula for a conical pendulum: $T = 2\pi\sqrt{\frac{l\cos\theta}{g}}$. [**6 marks**]

Question 13 [**14 marks**] A conical pendulum consists of a mass of 0.8 kg on a string of length 1.8 m. The mass moves in a horizontal circle of radius 1.2 m.

(a) Calculate the angle the string makes with the vertical. **[3 marks]**

(b) Find the height of the mass below the point of suspension. **[2 marks]**

(c) Calculate the tension in the string. **[3 marks]**

(d) Find the angular velocity and linear speed. **[3 marks]**

(e) Calculate the period of revolution. **[2 marks]**

(f) Find the centripetal acceleration. **[1 mark]**

Section G: Motion in a Vertical Circle - Loops [28 marks]

Question 14 [16 marks] A small vehicle of mass 400 kg travels through a vertical circular loop of radius 8 m.

(a) Find the minimum speed at the top to maintain contact with the track. **[3 marks]**

(b) Calculate the minimum speed at the bottom using energy conservation. **[4 marks]**

(c) If the vehicle enters the loop at 20 m/s, find its speed at the top. **[3 marks]**

(d) Calculate the normal force at the top for this speed. **[3 marks]**

(e) Find the normal force at the bottom when entering at 20 m/s. **[3 marks]**

Question 15 [12 marks] A marble rolls down a track and enters a vertical loop of radius 0.4 m. It starts from rest at height h above the bottom of the loop.

(a) Find the minimum height h for the marble to just complete the loop. **[5 marks]**

(b) If $h = 1.2$ m, calculate the speed at the top of the loop. **[3 marks]**

(c) Find the normal force at the top if the marble has mass 0.02 kg. **[2 marks]**

(d) Calculate the speed at the bottom for this case. **[2 marks]**

Section H: Applications and Problem Solving [29 marks]

Question 16 [15 marks] A geostationary satellite orbits Earth at altitude 35,800 km above the surface. Earth's radius is 6400 km and surface gravity is 9.8 m/s^2.

(a) Calculate the orbital radius from Earth's center. **[1 mark]**

(b) Find the gravitational acceleration at this altitude. **[3 marks]**

(c) Calculate the orbital speed. **[3 marks]**

(d) Verify that the orbital period is 24 hours. **[3 marks]**

(e) If the satellite has mass 2000 kg, find the centripetal force. **[2 marks]**

(f) Calculate the gravitational force and verify it provides the centripetal force. **[3 marks]**

Question 17 [14 marks] A carnival ride consists of chairs hanging from 5 m cables attached to a rotating platform. During operation, the cables make 35° with the vertical.

(a) Calculate the radius of the circular path of the chairs. **[2 marks]**

(b) Find the height of the chairs below the attachment point. **[2 marks]**

(c) Calculate the angular velocity of the platform. **[4 marks]**

(d) Find the linear speed of the chairs. **[2 marks]**

(e) If a chair and passenger have combined mass 100 kg, calculate the cable tension. **[2 marks]**

(f) Find the centripetal acceleration. **[2 marks]**

Section I: Advanced Circular Motion [27 marks]

Question 18 **[14 marks]** A vehicle crosses a hump in the road that can be modeled as part of a vertical circle with radius 45 m.

(a) Calculate the maximum speed to maintain contact with the road surface at the crest. **[4 marks]**

(b) If a car of mass 1400 kg travels at 12 m/s over the hump, find the normal force at the crest. **[4 marks]**

(c) Calculate the apparent weight of a 65 kg passenger at this speed. **[3 marks]**

(d) Find the centripetal acceleration at the crest. **[2 marks]**

(e) At what speed would the passenger feel weightless? **[1 mark]**

Question 19 **[13 marks]** A particle slides on the inside of a smooth vertical circular tube of radius 2.5 m. It is released from rest at a height of 2 m above the bottom.

(a) Calculate the speed at the bottom of the tube. **[3 marks]**

(b) Find the normal force from the tube wall at the bottom if the particle has mass 0.5 kg. **[3 marks]**

(c) Calculate the speed when the particle reaches the top of the circle. **[3 marks]**

(d) Determine if the particle will complete the full circular path. **[4 marks]**

Section J: Comprehensive Applications [29 marks]

Question 20 **[17 marks]** A coin sits on a horizontal turntable at distance 0.3 m from the center. The turntable starts from rest and accelerates at constant angular acceleration.

(a) If the coefficient of static friction is 0.4, find the maximum angular velocity before slipping. **[4 marks]**

(b) Calculate the maximum linear speed. **[2 marks]**

(c) If the turntable accelerates at 2 rad/s^2, how long before the coin starts to slip? **[3 marks]**

(d) At the moment just before slipping, calculate the friction force on a 5 g coin. **[3 marks]**

(e) If the coin is moved to 0.5 m from the center, find the new maximum angular velocity. **[3 marks]**

(f) Compare the maximum speeds at the two positions. **[2 marks]**

Question 21 **[12 marks]** Two masses of 3 kg and 5 kg are connected by a light rigid rod of length 2 m. The system rotates about an axis perpendicular to the rod passing through the center of mass.

(a) Find the position of the center of mass from the 3 kg mass. **[3 marks]**

(b) Calculate the distances of each mass from the rotation axis. **[2 marks]**

(c) If the system rotates at 6 rad/s, find the centripetal force on each mass. **[4 marks]**

(d) Calculate the total kinetic energy of the rotating system. **[3 marks]**

Physics Data and Formulae

Circular Motion:

Angular velocity: $\omega = \frac{v}{r} = \frac{2\pi}{T} = 2\pi f$

Angular displacement: $\theta = \omega t$ (constant ω)

Centripetal acceleration: $a_c = \frac{v^2}{r} = \omega^2 r$

Centripetal force: $F_c = ma_c = \frac{mv^2}{r} = m\omega^2 r$

Vertical Circular Motion:

At top: $T + mg = \frac{mv^2}{r}$ or $N + mg = \frac{mv^2}{r}$

At bottom: $T - mg = \frac{mv^2}{r}$ or $N - mg = \frac{mv^2}{r}$

Minimum speed at top: $v_{min} = \sqrt{gr}$

Energy conservation: $\frac{1}{2}mv^2_{bottom} = \frac{1}{2}mv^2_{top} + mg(2r)$

Banking:

No friction: $\tan\theta = \frac{v^2}{rg}$ or $v = \sqrt{rg\tan\theta}$

With friction: Resolve forces parallel and perpendicular to surface

Conical Pendulum:

$\cos\theta = \frac{g}{\omega^2 l}$

$T\cos\theta = mg$, $T\sin\theta = m\omega^2 r$

Period: $T = 2\pi\sqrt{\frac{l\cos\theta}{g}}$

Orbital Motion:

Gravitational acceleration: $g_h = g\left(\frac{R}{R+h}\right)^2$

Orbital speed: $v = \sqrt{g_h r}$ where $r = R + h$

Period: $T = \frac{2\pi r}{v}$

Energy Conservation:

$\frac{1}{2}mv_1^2 + mgh_1 = \frac{1}{2}mv_2^2 + mgh_2$

Constants:

Acceleration due to gravity: $g = 9.8$ m/s^2

Earth's radius: $R = 6.4 \times 10^6$ m

Trigonometric Values:

$\sin 18 = 0.309$, $\cos 18 = 0.951$, $\tan 18 = 0.325$

$\sin 35 = 0.574$, $\cos 35 = 0.819$, $\tan 35 = 0.700$

$\sin 45 = 0.707$, $\cos 45 = 0.707$, $\tan 45 = 1.000$

$\sin 60 = 0.866$, $\cos 60 = 0.500$, $\tan 60 = 1.732$

END OF TEST

Total marks: 282

Grade boundaries: A* 254, A 226, B 197, C 169, D 141, E 113

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