A Level Mechanics
Practice Test 4: Gravitational Fields

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Draw diagrams where appropriate to illustrate your solutions.
Time allowed: 2 hours 55 minutes

Section A: Gravitational Force and Field Fundamentals [35 marks]

Question 1 [**15 marks**] A space probe is designed to study the gravitational field between Earth and the Moon. Earth has mass 5.97×10^{2} kg and the Moon has mass 7.35×10^{22} kg. They are separated by 3.84×10 m.

(a) Calculate the gravitational force between Earth and the Moon. [**4 marks**]

(b) Find the distance from Earth's center where the gravitational field strengths due to Earth and Moon are equal in magnitude but opposite in direction. [**6 marks**]

(c) Calculate the gravitational field strength at this neutral point. [**2 marks**]

(d) Determine the gravitational potential at this neutral point due to both Earth and Moon. [**3 marks**]

Question 2 [**20 marks**] Titan, Saturn's largest moon, has a mass of 1.35×10^{23} kg and a radius of 2.57×10 m.

(a) Calculate the gravitational field strength on Titan's surface. [**4 marks**]

(b) Find the gravitational field strength at a height of 1500 km above Titan's surface. [**4 marks**]

(c) A lander with mass 850 kg is placed on Titan's surface. Calculate its weight. [**3 marks**]

(d) At what height above Titan's surface would the gravitational field strength be 20% of its surface value? [**6 marks**]

(e) Calculate the ratio of gravitational field strengths on Titan's surface compared to Earth's surface. [**3 marks**]

Section B: Gravitational Potential and Energy Analysis [40 marks]

Question 3 [**18 marks**]

(a) Define gravitational potential energy and explain why it is considered negative. [**4 marks**]

(b) Starting from the definition of work done against gravity, derive the expression U = -GMm/r for gravitational potential energy. [**8 marks**]

(c) Show that the gravitational potential V = -GM/r follows from the potential energy expression. **[3 marks]**

(d) Explain the physical significance of the negative sign in the gravitational potential expression. **[3 marks]**

Question 4 [22 marks] A lunar mission requires sending a spacecraft from Earth's surface to the Moon's surface.

(a) Calculate the gravitational potential energy of a 5000 kg spacecraft on Earth's surface. **[3 marks]**

(b) Find the gravitational potential energy of the spacecraft on the Moon's surface. **[4 marks]**

(c) Calculate the change in potential energy for this journey, considering only Earth and Moon's gravitational fields. **[3 marks]**

(d) Determine the minimum launch velocity required for the spacecraft to reach the Moon. **[6 marks]**

(e) If the spacecraft is launched with velocity 12.0 km/s, calculate its velocity when it reaches the Moon's surface. **[4 marks]**

(f) Calculate the energy required per kilogram of spacecraft for this mission. **[2 marks]**

Section C: Orbital Mechanics and Satellite Motion [50 marks]

Question 5 [25 marks]

(a) For a satellite in circular orbit, explain what provides the centripetal force and derive the orbital velocity formula v = (GM/r). **[6 marks]**

(b) Show that the orbital period is T = $2(r^3/GM)$ and explain how this relates to Kepler's third law. **[6 marks]**

(c) For a satellite in circular orbit, prove that the kinetic energy is exactly half the magnitude of the potential energy. **[5 marks]**

(d) Derive the expression for the total energy of a satellite in circular orbit. **[4 marks]**

(e) Explain why more energy is required to move a satellite to a higher orbit, even though it moves more slowly there. **[4 marks]**

Question 6 [25 marks] A telecommunications satellite needs to be placed in geostationary orbit, and then later moved to a higher orbit at 50,000 km altitude above Earth's surface.

(a) Calculate the orbital radius and height above Earth's surface for geostationary orbit. **[5 marks]**

(b) Find the orbital velocity in geostationary orbit. **[3 marks]**

(c) Calculate the orbital velocity in the higher orbit at 50,000 km altitude. **[4 marks]**

(d) Determine the orbital period in the higher orbit. **[4 marks]**

(e) For a 3000 kg satellite, calculate the energy required to transfer it from geostationary orbit to the higher orbit. **[6 marks]**

(f) Explain why geostationary satellites are useful for telecommunications but have limitations for global coverage. **[3 marks]**

Section D: Planetary Systems and Kepler's Laws [30 marks]

Question 7 [18 marks] Exoplanet Kepler-452b orbits its star at a distance of 1.55×10^{11} m with a period of 385 days. The star has a similar mass to our Sun.

(a) Use Kepler's third law to calculate the mass of Kepler-452b's star. [**6 marks**]

(b) Calculate the orbital velocity of Kepler-452b. [**4 marks**]

(c) Compare this orbital velocity with Earth's orbital velocity around the Sun. [**3 marks**]

(d) If another planet in this system orbits at 0.85×10^{11} m from the star, predict its orbital period. [**5 marks**]

Question 8 [12 marks] The Galilean moons of Jupiter provide an excellent test of Kepler's laws. Io orbits at 4.22×10 m with period 1.77 days, while Callisto orbits at 1.88×10 m.

(a) Use Io's orbital data to calculate Jupiter's mass. [**5 marks**]

(b) Predict Callisto's orbital period using Kepler's third law. [**4 marks**]

(c) Calculate and compare the orbital velocities of Io and Callisto. [**3 marks**]

Section E: Escape Velocity and Energy Transfer [25 marks]

Question 9 [15 marks]

(a) Define escape velocity and derive its expression starting from energy conservation principles. [**6 marks**]

(b) Calculate the escape velocity from Mercury's surface (mass 3.30×10^{23} kg, radius 2.44×10 m). [**3 marks**]

(c) Calculate the escape velocity from Neptune's surface (mass 1.02×10^{2} kg, radius 2.46×10 m). [**3 marks**]

(d) Explain why it would be easier to launch a spacecraft from Mercury than from Neptune, despite Mercury being closer to the Sun. [**3 marks**]

Question 10 [10 marks] A probe is launched from Earth with initial velocity 9.5 km/s to study the upper atmosphere.

(a) Calculate whether this probe will escape Earth's gravitational field. [**3 marks**]

(b) Find the maximum altitude reached by the probe above Earth's surface. [**5 marks**]

(c) Calculate the probe's velocity when it returns to an altitude of 2000 km during its descent. [**2 marks**]

Section F: Advanced Gravitational Field Applications [30 marks]

Question 11 [18 marks] A proposed asteroid mining mission involves placing a space station at the L2 Lagrange point beyond the Moon's orbit, where gravitational forces from Earth and Moon combine with centrifugal effects to create a stable point.

(a) Explain the concept of Lagrange points and why they are useful for space missions. [**4 marks**]

(b) The L2 point is approximately 1.5×10 m from Earth's center, beyond the Moon. Calculate the gravitational field strength at L2 due to Earth alone. [**3 marks**]

(c) Calculate the gravitational field strength at L2 due to the Moon alone. [**4 marks**]

(d) Find the gravitational potential at L2 due to both Earth and Moon. [**4 marks**]

(e) Calculate the energy required to move a 10,000 kg space station from Earth's surface to the L2 point. [**3 marks**]

Question 12 [12 marks] Tidal forces arise from gravitational field gradients. Consider a satellite in low Earth orbit experiencing tidal effects.

(a) Explain what causes tidal forces in terms of gravitational field variations. [**3 marks**]

(b) A 100 m long satellite orbits Earth at 400 km altitude. Calculate the difference in gravitational field strength between the Earth-facing end and the space-facing end of the satellite. [**6 marks**]

(c) Explain why tidal forces become more significant for larger objects or closer orbits. [**3 marks**]

Physics Data and Formulae

Gravitational Force and Field:

Newton's Law: $F = \frac{Gm_1m_2}{r^2}$

Field strength: $g = \frac{F}{m} = \frac{GM}{r^2}$

Field-potential relation: $g = -\frac{dV}{dr}$

Gravitational Potential and Energy:

Potential: $V = -\frac{GM}{r}$

Potential energy: $U = mV = -\frac{GMm}{r}$

Work done: $W = m\Delta V = \Delta U$

Orbital Motion:

Circular orbital velocity: $v = \sqrt{\frac{GM}{r}}$

Orbital period: $T = 2\pi\sqrt{\frac{r^3}{GM}}$

Kepler's Third Law: $\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$

Escape velocity: $v_e = \sqrt{\frac{2GM}{r}}$

Energy in Circular Orbits:

Kinetic energy: $E_k = \frac{1}{2}mv^2 = \frac{GMm}{2r}$

Potential energy: $E_p = -\frac{GMm}{r}$

Total energy: $E = E_k + E_p = -\frac{GMm}{2r}$

Circular Motion:

Centripetal force: $F_c = \frac{mv^2}{r}$

Centripetal acceleration: $a_c = \frac{v^2}{r}$

Angular velocity: $\omega = \frac{2\pi}{T}$

Physical Constants:

Gravitational constant: $G = 6.67 \times 10^{-11}$ Nm²/kg²

Earth's mass: $M_E = 5.97 \times 10^{24}$ kg

Earth's radius: $R_E = 6.37 \times 10^6$ m

Moon's mass: $M_M = 7.35 \times 10^{22}$ kg

Moon's radius: $R_M = 1.74 \times 10^6$ m

Earth-Moon distance: 3.84×10^8 m

Sun's mass: $M_S = 1.99 \times 10^{30}$ kg
Earth-Sun distance: 1.50×10^{11} m
Standard gravity: $g = 9.81$ m/s^2

END OF TEST

Total marks: 210

Grade boundaries: A* 189, A 168, B 147, C 126, D 105, E 84

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