A Level Mechanics
Practice Test 5: Elasticity and Springs

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Draw clear, labelled diagrams where appropriate.
Time allowed: 2 hours 30 minutes

Section A: Springs and Hooke's Law [45 marks]

Question 1 [12 marks] A spring manufacturer tests the elastic properties of their products. A spring has a natural length of 18.0 cm and the following force-extension data is collected:

Force (N)	Extension (cm)
0.0	0.0
3.0	1.5
6.0	3.0
9.0	4.5
12.0	6.0
15.0	7.5
18.0	9.0

(a) State Hooke's Law and explain when it applies. **[3 marks]**

(b) Plot a graph of force against extension for this spring. **[4 marks]**

(c) Calculate the spring constant from your graph. **[3 marks]**

(d) Determine the total length of the spring when a 21.0 N force is applied. **[2 marks]**

Question 2 [18 marks] Three springs with spring constants $k_1 = 150$ N/m, $k_2 = 200$ N/m, and $k_3 = 300$ N/m are used in different configurations.

(a) All three springs are connected in series. A force of 30 N is applied to the combination.

(i) Calculate the extension of each spring. **[6 marks]**

(ii) Find the total extension. **[2 marks]**

(iii) Calculate the effective spring constant of the series combination. **[3 marks]**

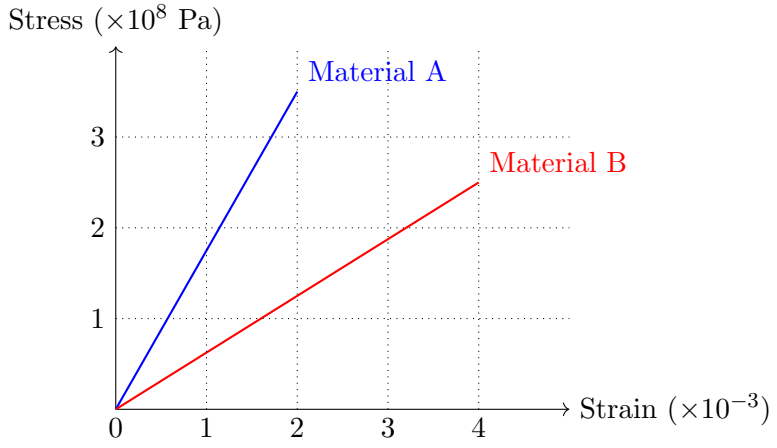
(b) The same three springs are now connected in parallel. The same 30 N force is applied.

(i) Calculate the force carried by each spring. **[3 marks]**

(ii) Find the extension of the parallel combination. **[2 marks]**

(iii) Calculate the effective spring constant of the parallel combination. **[2 marks]**

Question 3 [15 marks] The stress-strain graph below shows the behavior of two different elastic materials, A and B, when subjected to tensile forces:



(a) Calculate Young's modulus for material A. **[4 marks]**

(b) Calculate Young's modulus for material B. **[4 marks]**

(c) Which material is stiffer and by what factor? **[3 marks]**

(d) If both materials have the same cross-sectional area and are subjected to the same force, which will have the greater extension? Explain your answer. **[4 marks]**

Section B: Elastic Potential Energy [55 marks]

Question 4 [20 marks]

(a) Define elastic potential energy and explain how it arises. **[4 marks]**

(b) A spring with spring constant 250 N/m is compressed by 12.0 cm.

(i) Calculate the force required to compress the spring by this amount. **[2 marks]**

(ii) Find the elastic potential energy stored in the compressed spring. **[3 marks]**

(iii) Show that the work done in compressing the spring equals the elastic potential energy stored. **[4 marks]**

(c) The compressed spring is used to accelerate a 0.8 kg mass along a frictionless horizontal surface. Calculate the maximum velocity reached by the mass. **[4 marks]**

(d) If the surface has a coefficient of friction of 0.15, calculate how far the mass travels before coming to rest. **[3 marks]**

Question 5 [20 marks] A toy rocket is powered by a compressed spring mechanism. The spring has a spring constant of 800 N/m and is compressed by 20.0 cm. The rocket has a mass of 150 g.

(a) Calculate the elastic potential energy stored in the compressed spring. **[3 marks]**

(b) Assuming 80% energy transfer efficiency, calculate:

(i) The kinetic energy of the rocket as it leaves the spring. **[2 marks]**

(ii) The initial velocity of the rocket. **[3 marks]**

(iii) The energy lost due to internal friction. **[2 marks]**

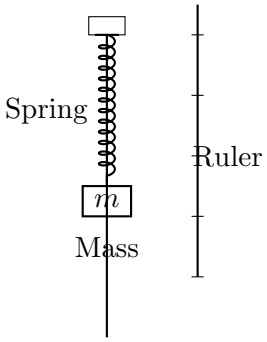
(c) The rocket is launched vertically upward. Calculate:

(i) The maximum height reached by the rocket. **[4 marks]**

(ii) The velocity of the rocket when it returns to its launch point. **[3 marks]**

(iii) The total time of flight. **[3 marks]**

Question 6 [15 marks] A student investigates elastic potential energy using the apparatus shown:



The following data is obtained:

Mass (kg)	Extension (m)	Elastic PE (J)
0.10	0.020	
0.20	0.040	
0.30	0.060	
0.40	0.080	
0.50	0.100	

(a) Calculate the spring constant from this data. **[4 marks]**

(b) Complete the table by calculating the elastic potential energy for each mass. **[5 marks]**

(c) If the 0.30 kg mass is lifted slightly and released, calculate its maximum speed during oscillation. **[4 marks]**

(d) Explain one source of systematic error in this experiment. **[2 marks]**

Section C: Stress, Strain and Material Testing [65 marks]

Question 7 [25 marks]

(a) Define the following terms clearly:

(i) Tensile stress, stating its SI unit. **[3 marks]**

(ii) Tensile strain, explaining why it is dimensionless. **[3 marks]**

(iii) Young's modulus, stating its SI unit. **[3 marks]**

(iv) Elastic limit. **[2 marks]**

(b) Write the mathematical relationships between:

(i) Stress, force and area. **[2 marks]**

(ii) Strain, extension and original length. **[2 marks]**

(iii) Young's modulus, stress and strain. **[2 marks]**

(c) A material has a high Young's modulus. Explain what this tells us about:

(i) The stiffness of the material. **[3 marks]**

(ii) Its suitability for construction applications. **[3 marks]**

(iii) How it compares to materials with low Young's modulus. **[2 marks]**

Question 8 [25 marks] An aluminum cable is used in a suspension bridge. The cable has the following specifications:

- Length: 120 m
- Diameter: 15.0 mm
- Young's modulus for aluminum: 7.0×10^1 Pa
- Density of aluminum: 2700 kg/m^3
- Maximum safe stress: 150×10 Pa

(a) Calculate:

(i) The cross-sectional area of the cable. **[3 marks]**

(ii) The mass of the cable. **[4 marks]**

(iii) The weight of the cable. **[2 marks]**

(b) The cable supports an additional load of 25,000 N. Calculate:

(i) The total force in the cable at the top support. **[2 marks]**

(ii) The tensile stress in the cable. **[3 marks]**

(iii) The tensile strain in the cable. **[3 marks]**

(iv) The extension of the cable. **[3 marks]**

(c) Determine whether the cable is operating within its safe stress limit. **[2 marks]**

(d) Calculate the elastic potential energy stored in the stretched cable. **[3 marks]**

Question 9 [15 marks] A materials engineer tests samples of three different wires, each 2.0 m long with diameter 1.0 mm. The results are:

Material	Young's Modulus ($\times 10^{11}$ Pa)	Extension under 100 N (mm)
Steel	2.0	
Copper	1.3	
Aluminum	0.7	

(a) Calculate the cross-sectional area of each wire. **[2 marks]**

(b) Complete the table by calculating the extension of each wire under a 100 N load. **[6 marks]**

(c) Rank the materials in order of stiffness, with the stiffest first. **[2 marks]**

(d) If all three wires were connected in parallel and a total force of 300 N applied, calculate the force carried by each wire. **[3 marks]**

(e) Which material would be most suitable for a guitar string? Justify your answer. **[2 marks]**

Section D: Simple Harmonic Motion with Springs [50 marks]

Question 10 [25 marks] A mass-spring system consists of a 0.6 kg mass attached to a horizontal spring with spring constant 360 N/m.

(a) Explain why this system undergoes simple harmonic motion when displaced from equilibrium. **[4 marks]**

(b) The mass is displaced 5.0 cm from equilibrium and released. Calculate:

(i) The period of oscillation. **[3 marks]**

(ii) The frequency of oscillation. **[2 marks]**

(iii) The angular frequency. **[2 marks]**

(c) For this oscillation, calculate:

(i) The maximum velocity. **[3 marks]**

(ii) The maximum acceleration. **[3 marks]**

(iii) The velocity when the displacement is 3.0 cm from equilibrium. **[4 marks]**

(iv) The acceleration when the displacement is 2.0 cm from equilibrium. **[2 marks]**

(d) Calculate the total mechanical energy of the system and verify that energy is conserved at any point during the motion. **[2 marks]**

Question 11 [25 marks] A vertical spring with spring constant 200 N/m hangs from a fixed support. A mass of 1.5 kg is attached to the free end.

(a) Calculate the extension of the spring when the mass is in equilibrium. **[3 marks]**

(b) The mass is pulled down an additional 6.0 cm from its equilibrium position and released.

(i) State the amplitude of the resulting oscillation. **[1 mark]**

(ii) Calculate the period of oscillation. **[3 marks]**

(iii) Find the maximum speed during the oscillation. **[3 marks]**

(c) Calculate the elastic potential energy stored in the spring when the mass is:

(i) At its lowest point during oscillation. **[4 marks]**

(ii) At its highest point during oscillation. **[4 marks]**

(iii) Passing through the equilibrium position during oscillation. **[3 marks]**

(d) At what displacement from the equilibrium position is the kinetic energy equal to the elastic potential energy? **[4 marks]**

Formula Sheet and Physical Constants

Elasticity and Springs:

Hooke's Law: $F = kx$

Series springs: $\frac{1}{k_{total}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + ...$

Parallel springs: $k_{total} = k_1 + k_2 + k_3 + ...$

Elastic potential energy: $E_p = \frac{1}{2}kx^2$

Work done by variable force: $W = \frac{1}{2}Fx$

Stress and Strain:

Stress: $\sigma = \frac{F}{A}$

Strain: $\epsilon = \frac{\Delta L}{L_0}$

Young's modulus: $E = \frac{\sigma}{\epsilon} = \frac{F \cdot L_0}{A \cdot \Delta L}$

Energy Principles:

Kinetic energy: $E_k = \frac{1}{2}mv^2$

Gravitational potential energy: $E_p = mgh$

Work done against friction: $W_f = \mu mgs$

Conservation of energy: $E_{initial} = E_{final}$

Simple Harmonic Motion:

Restoring force: $F = -kx$

Period: $T = 2\pi\sqrt{\frac{m}{k}}$

Frequency: $f = \frac{1}{T}$

Angular frequency: $\omega = 2\pi f = \sqrt{\frac{k}{m}}$

Displacement: $x = A\cos(\omega t + \phi)$

Velocity: $v = -A\omega\sin(\omega t + \phi) = \pm\omega\sqrt{A^2 - x^2}$

Acceleration: $a = -A\omega^2\cos(\omega t + \phi) = -\omega^2 x$

Maximum velocity: $v_{max} = \omega A$

Maximum acceleration: $a_{max} = \omega^2 A$

Kinematics:

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

$s = \frac{u+v}{2}t$

Physical Constants:

Acceleration due to gravity: $g = 9.81$ m s^{-2}

$\pi = 3.14159...$

END OF TEST

Total marks: 215

Grade boundaries: A* 194, A 172, B 151, C 129, D 108, E 86

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stepupmaths.co.uk