GCSE Higher Mathematics
Practice Test 7: Ratio, Proportion, and Rates of Change

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Time allowed: 90 minutes

Section A: Advanced Ratios

1. Express these ratios in their simplest form:
   (a) $6.3 : 4.2 : 2.1$
   (b) $\frac{5}{9} : \frac{7}{18} : \frac{2}{27}$
   (c) $0.36 : 0.48 : 0.24$
   (d) $3\frac{2}{5} : 1\frac{7}{10} : 2\frac{1}{2}$

2. A film production budget of £180,000 is allocated among script, cast, crew, equipment, and post-production in the ratio $2 : 12 : 8 : 15 : 8$.
   (a) How much is allocated to each category?
   (b) If additional funding of £45,000 is distributed in the same ratio, how much extra does each category receive?
   (c) What percentage of the total budget is allocated to equipment?

3. The interior angles of a hexagon are in the ratio $4 : 5 : 6 : 7 : 8 : 9$. Find each angle.

4. A plant food mixture combines nitrogen, phosphorus, and potassium in the ratio $8 : 5 : 12$ by mass.
   (a) How much of each nutrient is needed for 300g of plant food?
   (b) If 40g of phosphorus is used, find the mass of nitrogen and potassium needed
   (c) What fraction of the mixture is potassium?

5. Share £2430 among three investors so that the first receives £240 more than the second, and the second receives £180 more than the third.

Section B: Direct Proportion

6. y is directly proportional to x. When $x = 35$, $y = 28$.
   (a) Find the equation connecting y and x
   (b) Calculate y when $x = 45$
   (c) Calculate x when $y = 40$

(d) Sketch the graph of y against x

7. M varies directly as the square of N. When $N = 8$, $M = 192$.

   (a) Express M in terms of N

   (b) Find M when $N = 12$

   (c) Find N when $M = 432$

   (d) What happens to M when N is reduced by 25%?

8. The cost of advertising varies directly as the duration in days. 14 days of advertising costs £420.

   (a) Find the cost per day

   (b) How much does 25 days of advertising cost?

   (c) How many days can be afforded with £780?

   (d) Express the cost C in terms of days D

9. The volume of a gas varies directly as the cube of its temperature (in Kelvin). At 300K, the volume is 2.7 liters.

   (a) Find the volume at 400K

   (b) At what temperature is the volume 6.4 liters?

   (c) Write the relationship as an equation

Section C: Inverse Proportion

10. y is inversely proportional to x. When $x = 16$, $y = 21$.

    (a) Find the equation connecting y and x

    (b) Calculate y when $x = 24$

    (c) Calculate x when $y = 28$

    (d) Sketch the graph of y against x

11. The time to scan documents is inversely proportional to the scanner speed. At 45 pages per minute, it takes 16 minutes to scan a document.

    (a) How long at 60 pages per minute?

    (b) What speed is needed to scan in 10 minutes?

    (c) Express time T in terms of speed S

    (d) If the speed decreases from 50 to 30 pages per minute, how much longer does scanning take?

12. T is inversely proportional to the square of U. When $U = 9$, $T = 16$.

    (a) Express T in terms of U

    (b) Find T when $U = 12$

    (c) Find U when $T = 9$

    (d) What happens to T when U is reduced by 40%?

13. The sound intensity from a speaker is inversely proportional to the square of the distance from it. At 4 meters, the intensity is 90 decibels.

    (a) Find the intensity at 6 meters

    (b) At what distance is the intensity 60 decibels?

    (c) Write the relationship as an equation

Section D: Combined Proportion

14. r varies directly as s and inversely as t. When $s = 30$ and $t = 20$, $r = 36$.

    (a) Express r in terms of s and t

    (b) Find r when $s = 40$ and $t = 15$

    (c) Find t when $s = 45$ and $r = 27$

    (d) What happens to r if s is decreased by 25% and t is increased by 60%?

15. The escape velocity v from a planet varies directly as the square root of the planet's mass M and inversely as the square root of its radius R. When $M = 16$ units and $R = 4$ units, $v = 8$ km/s.

    (a) Express v in terms of M and R

    (b) Find v when $M = 25$ units and $R = 9$ units

    (c) Find R when $v = 12$ km/s and $M = 36$ units

    (d) What happens to v if M is increased 9-fold and R is increased 4-fold?

16. The gravitational force F between two masses varies directly as the product of their masses m_1 and m_2, and inversely as the square of the distance d between them. When $m_1 = 6$ kg, $m_2 = 8$ kg, and $d = 2$ m, $F = 12$ newtons.

    (a) Express F in terms of m_1, m_2, and d

    (b) Find F when $m_1 = 9$ kg, $m_2 = 4$ kg, and $d = 3$ m

    (c) Find d when $F = 18$ newtons, $m_1 = 8$ kg, and $m_2 = 6$ kg

17. w varies directly as x^2 and inversely as y^3. When $x = 6$ and $y = 2$, $w = 54$.

    (a) Find the constant of proportionality

    (b) Express w in terms of x and y

    (c) Calculate w when $x = 4$ and $y = 3$

Section E: Speed, Distance, and Time

18. A cargo ship travels 252 km in 4 hours 12 minutes. Calculate:

    (a) The average speed in km/h

    (b) The average speed in m/s

    (c) How far it travels in 3 hours 15 minutes at this speed

    (d) How long it takes to travel 315 km at this speed

19. Convert these speeds:

    (a) 26 m/s to km/h

    (b) 135 km/h to m/s

    (c) 38 mph to km/h (use 1 mile = 1.6 km)

    (d) 84 km/h to mph

20. A commuter drives from home to work at 36 km/h and returns at 54 km/h. The total journey takes 3 hours.

    (a) Find the distance from home to work

    (b) Calculate the average speed for the whole journey

(c) How much time was saved on the return journey?

21. Two cyclists start simultaneously from towns 360 km apart and cycle towards each other. One cycles at 18 km/h and the other at 22 km/h.

    (a) When do they meet?

    (b) How far from each starting town do they meet?

    (c) What is their relative speed of approach?

22. An athlete completes a 16 km race. For the first 10 km, they maintain a pace of 12 km/h. For the remaining 6 km, they increase to 18 km/h.

    (a) Calculate the total time taken

    (b) Find the average speed for the whole race

    (c) How much slower would they need to run the second part to achieve an overall average of 13.5 km/h?

Section F: Density and Flow Rates

23. A metal alloy sample has volume 280 cm^3 and mass 2.24 kg.

    (a) Calculate its density in g/cm^3

    (b) Calculate its density in kg/m^3

    (c) What mass of this alloy would have volume 420 cm^3?

    (d) What volume would 3.36 kg of this alloy occupy?

24. Different ceramics have these densities:

    - Porcelain: 2.4 g/cm^3
    - Stoneware: 2.1 g/cm^3
    - Earthenware: 1.8 g/cm^3

    (a) Compare the masses of 150 cm^3 of each ceramic

    (b) What volume of porcelain has the same mass as 200 cm^3 of earthenware?

    (c) A vase weighs 630 g and has volume 300 cm^3. Which ceramic is it most likely made from?

25. Water flows through an irrigation system at a rate of 5.4 liters per minute.

    (a) How much water flows in 35 minutes?

    (b) How long to fill a 324-liter tank?

    (c) Express the flow rate in m^3 per hour

    (d) If the pipe diameter is reduced by 30%, what happens to the flow velocity?

26. A chocolate factory produces at different rates during production cycles:

    - Morning setup (2 hours): 360 bars/hour
    - Peak production (6 hours): 540 bars/hour
    - Afternoon cleanup (1 hour): 180 bars/hour

    (a) Calculate total daily production

    (b) Find the average production rate

    (c) How long to produce 2700 bars at the average rate?

Section G: Scale Factors and Similar Shapes

27. Two similar pentagons have corresponding sides in the ratio 8:15.

    (a) If the smaller pentagon has area 128 cm^2, find the area of the larger pentagon

    (b) If the larger pentagon has perimeter 150 cm, find the perimeter of the smaller pentagon

    (c) Find the ratio of their areas

28. A scale model of a cathedral is built to scale 1:160. The real cathedral is 96 m long and 64 m wide.

    (a) Find the dimensions of the model

    (b) If the model requires 0.24 m^2 of roofing material, how much does the real cathedral need?

    (c) The model weighs 4.5 kg. Estimate the mass of the real cathedral if made from the same material

29. A nature reserve map has scale 1:40000. Two bird watching hides are 17.2 cm apart on the map.

    (a) Calculate the actual distance in km

    (b) What map distance represents 3.2 km?

    (c) A woodland area covers 15.6 cm^2 on the map. Find its actual area in hectares

30. Two similar tetrahedra have edge ratio 5:9.

    (a) Find the ratio of their surface areas

    (b) Find the ratio of their volumes

    (c) If the smaller tetrahedron has volume 125 cm^3, what is the volume of the larger tetrahedron?

    (d) If the larger tetrahedron has surface area 729 cm^2, what is the surface area of the smaller tetrahedron?

Section H: Advanced Rate Problems

31. A viral infection spreads, tripling the number of cases every 6 hours. Starting with 40 cases:

    (a) Write an expression for cases after t hours

    (b) How many cases after 18 hours?

    (c) When will there be 8,740 cases?

    (d) What is the rate of increase after 12 hours (cases per hour)?

32. A water processing plant has five input streams and three output drains. The streams provide 28, 35, 42, 21, and 24 liters/minute respectively. The drains remove 38, 45, and 32 liters/minute respectively.

    (a) What is the net rate of filling when all systems operate?

    (b) How long to fill a 2100-liter processing tank from empty?

    (c) If the two largest input streams fail, what is the new net rate?

    (d) What capacity would a fourth drain need to exactly balance all inputs?

33. Currency exchange rates:

    - £1 = \$1.33
    - £1 = €1.19
    - \$1 = ¥106

(a) Convert £420 to dollars

(b) Convert €238 to pounds

(c) Convert $159 to yen

(d) Find the exchange rate from euros to dollars

34. A pharmaceutical production line operates at variable capacity:

- Preparation: 2 hours at 35% capacity (full capacity is 200 tablets/hour)
- Main production: 5 hours at full capacity
- Quality control: 1 hour at 60% capacity

(a) Calculate total tablets produced

(b) Find the average production rate over the whole cycle

(c) How long would it take to produce the same quantity at 75% capacity?

Section I: Problem Solving and Applications

35. A bread recipe for 6 loaves uses:

- 900g flour
- 540ml water
- 18g yeast
- 36g salt

(a) Adapt the recipe for 10 loaves

(b) How much of each ingredient for 4 loaves?

(c) If you have 2.7 kg flour, what's the maximum number of loaves you can make?

36. A library has 7200 books. The ratio of fiction to non-fiction to reference books is 9:7:2. Due to new acquisitions, 360 fiction books are added but no other categories change.

(a) How many books of each type were there originally?

(b) What is the new ratio of fiction to non-fiction to reference books?

(c) How many non-fiction books would need to be added to restore the original ratio?

37. The power consumption of a computer processor varies directly as the cube of its clock speed. At 2.5 GHz, it consumes 125 watts.

(a) Find the power consumption at 3.2 GHz

(b) What clock speed consumes 216 watts?

(c) Compare power consumptions at 2 GHz and 4 GHz

38. A shipping company's insurance cost I (in thousands) varies as the square of cargo value V (in millions) up to £30m, then increases linearly. When $V = 8$, $I = 64$. When $V = 35$, $I = 940$.

(a) Find the cost function for $V \leq 30$

(b) Find the cost function for $V > 30$

(c) What cargo value minimizes insurance cost per million for $V \leq 30$?

(d) What is the linear rate of increase for cargo values above £30m?

39. A grandfather clock has a gear mechanism with eight gears. Gear 1 has 30 teeth and rotates at 120 rpm. The subsequent gears have 20, 45, 15, 60, 18, 40, and 24 teeth respectively, connected in sequence.

    (a) Find the rotation speed of each gear

    (b) What is the overall gear ratio from gear 1 to gear 8?

    (c) If gear 1's speed decreases to 90 rpm, find the new speed of gear 8

    (d) How would the final speed change if gear 6 had 24 teeth instead of 18?

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 100

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