

GCSE Higher Mathematics

Practice Test 5: Ratio, Proportion, and Rates of Change

Instructions:

Answer all questions. Show your working clearly.

Calculators may be used unless stated otherwise.

Time allowed: 90 minutes

Section A: Advanced Ratios

- Express these ratios in their simplest form:
 - $5.6 : 2.8 : 4.2$
 - $\frac{4}{9} : \frac{2}{15} : \frac{5}{18}$
 - $0.24 : 0.32 : 0.16$
 - $2\frac{2}{3} : 1\frac{1}{6} : 3\frac{1}{3}$
- A research grant of £84,000 is distributed among five universities in the ratio $7 : 5 : 8 : 3 : 4$.
 - How much does each university receive?
 - If additional funding of £27,000 is distributed in the same ratio, how much extra does each university get?
 - What percentage of the total grant does the smallest recipient receive?
- The angles around a point are in the ratio $5 : 7 : 4 : 8$. Find each angle.
- A cleaning solution mixes ammonia, water, and detergent in the ratio $3 : 14 : 8$ by volume.
 - How much of each component is needed for 500ml of solution?
 - If 84ml of water is used, find the volume of ammonia and detergent needed
 - What fraction of the solution is detergent?
- Split £1680 among three employees so that the manager gets £150 more than the supervisor, and the supervisor gets £90 more than the assistant.

Section B: Direct Proportion

- y is directly proportional to x . When $x = 21$, $y = 14$.
 - Find the equation connecting y and x
 - Calculate y when $x = 30$
 - Calculate x when $y = 22$
 - Sketch the graph of y against x

7. W varies directly as the square of Z . When $Z = 4$, $W = 96$.
- (a) Express W in terms of Z
 - (b) Find W when $Z = 6$
 - (c) Find Z when $W = 216$
 - (d) What happens to W when Z is multiplied by 1.5?
8. The cost of custom framing varies directly as the perimeter of the frame. A frame with perimeter 2.4m costs £72.
- (a) Find the cost per meter of perimeter
 - (b) How much does a frame with perimeter 3.8m cost?
 - (c) What perimeter can be framed for £105?
 - (d) Express the cost C in terms of perimeter P
9. The surface area of a cube varies directly as the square of its edge length. A cube with edge 6cm has surface area 216 cm^2 .
- (a) Find the surface area of a cube with edge 9cm
 - (b) What edge length gives a surface area of 384 cm^2 ?
 - (c) Write the relationship as an equation

Section C: Inverse Proportion

10. y is inversely proportional to x . When $x = 12$, $y = 15$.
- (a) Find the equation connecting y and x
 - (b) Calculate y when $x = 18$
 - (c) Calculate x when $y = 20$
 - (d) Sketch the graph of y against x
11. The time to download a file is inversely proportional to the internet speed. At 10 Mbps, a file takes 24 minutes to download.
- (a) How long at 15 Mbps?
 - (b) What speed is needed to download in 16 minutes?
 - (c) Express time T in terms of speed S
 - (d) If the speed drops from 20 Mbps to 8 Mbps, how much longer does the download take?
12. U is inversely proportional to the square of V . When $V = 6$, $U = 25$.
- (a) Express U in terms of V
 - (b) Find U when $V = 10$
 - (c) Find V when $U = 16$
 - (d) What happens to U when V is multiplied by 1.2?
13. The volume of gas is inversely proportional to the pressure applied. At 4 atmospheres, the gas occupies 150 liters.
- (a) Find the volume at 6 atmospheres
 - (b) At what pressure does the gas occupy 200 liters?
 - (c) Write the relationship as an equation

Section D: Combined Proportion

14. h varies directly as j and inversely as k . When $j = 18$ and $k = 12$, $h = 21$.
- (a) Express h in terms of j and k
 - (b) Find h when $j = 24$ and $k = 9$
 - (c) Find k when $j = 30$ and $h = 18$
 - (d) What happens to h if j is multiplied by 0.8 and k is divided by 1.5?
15. The period T of a pendulum varies directly as the square root of its length L and inversely as the square root of gravitational acceleration g . When $L = 1$ meter and $g = 9.8 \text{ m/s}^2$, $T = 2$ seconds.
- (a) Express T in terms of L and g
 - (b) Find T when $L = 2.25$ meters and $g = 9.8 \text{ m/s}^2$
 - (c) Find L when $T = 1.5$ seconds and $g = 9.8 \text{ m/s}^2$
 - (d) What happens to T if L is quadrupled?
16. The force F between two charged particles varies directly as the product of their charges q_1 and q_2 , and inversely as the square of the distance d between them. When $q_1 = 2$, $q_2 = 3$, and $d = 1.5$, $F = 8$ units.
- (a) Express F in terms of q_1 , q_2 , and d
 - (b) Find F when $q_1 = 4$, $q_2 = 2$, and $d = 2$
 - (c) Find d when $F = 12$, $q_1 = 3$, and $q_2 = 4$
17. p varies directly as q^2 and inversely as r^3 . When $q = 3$ and $r = 2$, $p = 27$.
- (a) Find the constant of proportionality
 - (b) Express p in terms of q and r
 - (c) Calculate p when $q = 4$ and $r = 3$

Section E: Speed, Distance, and Time

18. A helicopter travels 264 km in 1 hour 45 minutes. Calculate:
- (a) The average speed in km/h
 - (b) The average speed in m/s
 - (c) How far it travels in 2 hours 20 minutes at this speed
 - (d) How long it takes to travel 352 km at this speed
19. Convert these speeds:
- (a) 28 m/s to km/h
 - (b) 126 km/h to m/s
 - (c) 48 mph to km/h (use 1 mile = 1.6 km)
 - (d) 88 km/h to mph
20. A taxi drives from airport to hotel at 55 km/h and returns at 35 km/h. The total journey takes 2.4 hours.
- (a) Find the distance from airport to hotel
 - (b) Calculate the average speed for the whole journey

- (c) How much extra time was needed for the return journey?
21. Two ships start simultaneously from ports 480 km apart and sail towards each other. One sails at 25 km/h and the other at 35 km/h.
- (a) When do they meet?
- (b) How far from each starting port do they meet?
- (c) What is their relative speed of approach?
22. A swimmer completes a 3 km open water race. For the first 1.8 km, they swim at 2.4 km/h. For the remaining distance, they increase to 3.6 km/h.
- (a) Calculate the total time taken
- (b) Find the average speed for the whole race
- (c) How much faster would they need to swim the second part to achieve an overall average of 3 km/h?

Section F: Density and Flow Rates

23. A glass ornament has volume 240 cm^3 and mass 600 g.
- (a) Calculate its density in g/cm^3
- (b) Calculate its density in kg/m^3
- (c) What mass of this glass would have volume 320 cm^3 ?
- (d) What volume would 900 g of this glass occupy?
24. Different materials have these densities:
- Concrete: 2.4 g/cm^3
 - Rubber: 1.2 g/cm^3
 - Titanium: 4.5 g/cm^3
- (a) Compare the masses of 250 cm^3 of each material
- (b) What volume of titanium has the same mass as 180 cm^3 of concrete?
- (c) A component weighs 675 g and has volume 150 cm^3 . Which material is it most likely made from?
25. Steam flows through a turbine at a rate of 6.4 m^3 per minute.
- (a) How much steam flows in 18 minutes?
- (b) How long to process 768 m^3 of steam?
- (c) Express the flow rate in liters per second
- (d) If the turbine efficiency drops by 25%, what happens to the flow rate?
26. A bakery produces bread at different rates during the day:
- Early morning (4 hours): 450 loaves/hour
 - Morning (5 hours): 600 loaves/hour
 - Afternoon (3 hours): 360 loaves/hour
- (a) Calculate total daily production
- (b) Find the average production rate
- (c) How long to produce 2400 loaves at the average rate?

Section G: Scale Factors and Similar Shapes

27. Two similar decagons have corresponding sides in the ratio 6:11.
- (a) If the smaller decagon has area 108 cm^2 , find the area of the larger decagon
 - (b) If the larger decagon has perimeter 132 cm, find the perimeter of the smaller decagon
 - (c) Find the ratio of their areas
28. A scale model of a stadium is built to scale 1:200. The real stadium is 300 m long and 200 m wide.
- (a) Find the dimensions of the model
 - (b) If the model requires 2.25 m^2 of covering material, how much does the real stadium need?
 - (c) The model weighs 15 kg. Estimate the mass of the real stadium if made from the same material
29. A hiking map has scale 1:25000. Two peaks are 14.4 cm apart on the map.
- (a) Calculate the actual distance in km
 - (b) What map distance represents 4.5 km?
 - (c) A lake covers 9.6 cm^2 on the map. Find its actual area in km^2
30. Two similar spheres have radius ratio 5:8.
- (a) Find the ratio of their surface areas
 - (b) Find the ratio of their volumes
 - (c) If the smaller sphere has volume 500 cm^3 , what is the volume of the larger sphere?
 - (d) If the larger sphere has surface area 256 cm^2 , what is the surface area of the smaller sphere?

Section H: Advanced Rate Problems

31. A population of bacteria increases by 25% every 2 hours. Starting with 1200 bacteria:
- (a) Write an expression for population after t hours
 - (b) How many bacteria after 8 hours?
 - (c) When will the population reach 5000?
 - (d) What is the rate of increase after 6 hours (bacteria per hour)?
32. A water treatment plant has four input pipes and two output pipes. Input pipes deliver 40, 35, 28, and 22 liters/minute respectively. Output pipes remove 45 and 30 liters/minute respectively.
- (a) What is the net rate of filling when all pipes operate?
 - (b) How long to fill a 1800-liter treatment tank from empty?
 - (c) If the largest input pipe fails, what is the new net rate?
 - (d) What capacity would a third output pipe need to balance all inputs?
33. Currency exchange rates:
- $\text{£}1 = \$1.26$
 - $\text{£}1 = \text{€}1.14$
 - $\text{\$}1 = \text{¥}109$
- (a) Convert $\text{£}450$ to dollars

- (b) Convert €228 to pounds
 - (c) Convert \$315 to yen
 - (d) Find the exchange rate from euros to dollars
34. A solar panel manufacturing line operates at variable capacity:
- Morning setup: 0.5 hours at 25% capacity (full capacity is 80 panels/hour)
 - Peak production: 7.5 hours at full capacity
 - Evening cleanup: 1 hour at 15% capacity
- (a) Calculate total panels produced
 - (b) Find the average production rate over the whole day
 - (c) How long would it take to produce the same quantity at 85% capacity?

Section I: Problem Solving and Applications

35. A pizza dough recipe for 3 pizzas uses:
- 450g flour
 - 270ml water
 - 3 tablespoons oil
 - 9g yeast
- (a) Adapt the recipe for 7 pizzas
 - (b) How much of each ingredient for 5 pizzas?
 - (c) If you have 1.2kg flour, what's the maximum number of pizzas you can make?
36. A sports club has 2700 members. The ratio of junior to adult to senior members is 8:15:4. Due to a membership drive, 216 adult members join but no other categories change.
- (a) How many members of each type were there originally?
 - (b) What is the new ratio of junior to adult to senior members?
 - (c) How many junior members would need to join to restore the original ratio?
37. The fuel consumption of a car engine varies inversely as the square of its efficiency rating. An engine with efficiency rating 6 consumes 8 liters per 100km.
- (a) Find the consumption for an engine with efficiency rating 8
 - (b) What efficiency rating gives consumption of 12.8 liters per 100km?
 - (c) Compare the fuel costs for a 250km journey using engines with ratings 5 and 10
38. A delivery company's operating cost C (in thousands) varies as the square of fleet size F up to 15 vehicles, then increases at a constant rate. When $F = 6$, $C = 108$. When $F = 20$, $C = 550$.
- (a) Find the cost function for $F \leq 15$
 - (b) Find the cost function for $F > 15$
 - (c) What fleet size minimizes cost per vehicle for $F \leq 15$?
 - (d) What is the marginal cost for each additional vehicle beyond 15?
39. A watch mechanism uses a gear train with six gears. Gear 1 has 20 teeth and rotates at 300 rpm. The subsequent gears have 30, 15, 45, 12, and 36 teeth respectively, connected in sequence.

- (a) Find the rotation speed of each gear
- (b) What is the overall gear ratio from gear 1 to gear 6?
- (c) If gear 1's speed increases to 450 rpm, find the new speed of gear 6
- (d) How would the final speed change if gear 4 had 30 teeth instead of 45?

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 100

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