GCSE Higher Mathematics
Practice Test 6: Ratio, Proportion, and Rates of Change

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Time allowed: 90 minutes

Section A: Advanced Ratios

1. Express these ratios in their simplest form:
   (a) $7.2 : 3.6 : 5.4$
   (b) $\frac{3}{8} : \frac{5}{12} : \frac{7}{24}$
   (c) $0.28 : 0.42 : 0.14$
   (d) $4\frac{1}{2} : 2\frac{1}{4} : 3\frac{3}{8}$

2. A charity distributes aid packages worth £126,000 among six regions in the ratio $5 : 7 : 4 : 9 : 3 : 8$.
   (a) How much aid does each region receive?
   (b) If emergency funding of £54,000 is distributed in the same ratio, how much extra does each region get?
   (c) What percentage of the total aid does the largest recipient receive?

3. The sides of a quadrilateral are in the ratio $6 : 8 : 5 : 9$. If the perimeter is 84 cm, find each side length.

4. A vitamin supplement contains calcium, magnesium, and zinc in the ratio $20 : 7 : 3$ by mass.
   (a) How much of each mineral is needed for 150g of supplement?
   (b) If 35mg of magnesium is used, find the mass of calcium and zinc needed
   (c) What percentage of the supplement is calcium?

5. Divide £2016 among three contractors so that the first gets £180 more than the second, and the second gets £120 more than the third.

Section B: Direct Proportion

6. y is directly proportional to x. When $x = 28$, $y = 21$.
   (a) Find the equation connecting y and x
   (b) Calculate y when $x = 40$
   (c) Calculate x when $y = 33$

(d) Sketch the graph of y against x

7. D varies directly as the square of E. When $E = 7$, $D = 147$.

   (a) Express D in terms of E

   (b) Find D when $E = 9$

   (c) Find E when $D = 243$

   (d) What happens to D when E is increased by 100%?

8. The cost of catering varies directly as the number of guests. Catering for 45 guests costs £315.

   (a) Find the cost per guest

   (b) How much does catering for 72 guests cost?

   (c) How many guests can be catered for £525?

   (d) Express the cost C in terms of guests G

9. The power output of a solar panel varies directly as the square of the sunlight intensity. At intensity level 8, the power output is 128 watts.

   (a) Find the power output at intensity level 12

   (b) What intensity level produces 200 watts?

   (c) Write the relationship as an equation

Section C: Inverse Proportion

10. y is inversely proportional to x. When $x = 14$, $y = 18$.

    (a) Find the equation connecting y and x

    (b) Calculate y when $x = 21$

    (c) Calculate x when $y = 28$

    (d) Sketch the graph of y against x

11. The time to mow a field is inversely proportional to the number of mowers used. With 5 mowers, it takes 9 hours.

    (a) How long with 15 mowers?

    (b) How many mowers are needed to complete it in 5 hours?

    (c) Express time T in terms of mowers M

    (d) If 2 mowers break down from the original 5, how many additional hours are needed?

12. R is inversely proportional to the square of S. When $S = 8$, $R = 18$.

    (a) Express R in terms of S

    (b) Find R when $S = 12$

    (c) Find S when $R = 8$

    (d) What happens to R when S is increased by 25%?

13. The electrical resistance of a wire is inversely proportional to the square of its cross-sectional area. With area 0.5 mm^2, the resistance is 72 ohms.

    (a) Find the resistance with area 0.8 mm^2

    (b) What area gives resistance of 50 ohms?

    (c) Write the relationship as an equation

Section D: Combined Proportion

14. f varies directly as g and inversely as h. When $g = 24$ and $h = 16$, $f = 27$.

    (a) Express f in terms of g and h

    (b) Find f when $g = 32$ and $h = 12$

    (c) Find h when $g = 40$ and $f = 30$

    (d) What happens to f if g is increased by 50% and h is decreased by 20%?

15. The orbital period T of a planet varies directly as the cube of its distance d from the sun and inversely as the square root of the sun's mass M. When $d = 2$ units and $M = 4$ units, $T = 4$ years.

    (a) Express T in terms of d and M

    (b) Find T when $d = 3$ units and $M = 9$ units

    (c) Find d when $T = 6$ years and $M = 4$ units

    (d) What happens to T if d is doubled and M is quadrupled?

16. The pressure P in a gas varies directly as the temperature T and inversely as the volume V. When $T = 300$ K and $V = 2$ liters, $P = 6$ atmospheres.

    (a) Express P in terms of T and V

    (b) Find P when $T = 450$ K and $V = 1.5$ liters

    (c) Find V when $P = 8$ atmospheres and $T = 400$ K

17. x varies directly as y^3 and inversely as z^2. When $y = 4$ and $z = 3$, $x = 32$.

    (a) Find the constant of proportionality

    (b) Express x in terms of y and z

    (c) Calculate x when $y = 3$ and $z = 2$

Section E: Speed, Distance, and Time

18. A yacht sails 189 km in 3 hours 30 minutes. Calculate:

    (a) The average speed in km/h

    (b) The average speed in m/s

    (c) How far it sails in 2 hours 45 minutes at this speed

    (d) How long it takes to sail 216 km at this speed

19. Convert these speeds:

    (a) 32 m/s to km/h

    (b) 162 km/h to m/s

    (c) 42 mph to km/h (use 1 mile = 1.6 km)

    (d) 112 km/h to mph

20. A delivery truck travels from warehouse to store at 48 km/h and returns at 72 km/h. The total journey takes 2.5 hours.

    (a) Find the distance from warehouse to store

    (b) Calculate the average speed for the whole journey

    (c) How much time was saved on the return journey?

21. Two express trains start simultaneously from stations 540 km apart and travel towards each other. One travels at 135 km/h and the other at 165 km/h.

    (a) When do they meet?

    (b) How far from each starting station do they meet?

    (c) What is their combined approach speed?

22. A runner completes a 12 km cross-country race. For the first 7.5 km, they run at 9 km/h. For the remaining distance, they slow to 6 km/h.

    (a) Calculate the total time taken

    (b) Find the average speed for the whole race

    (c) How much faster would they need to run the second part to achieve an overall average of 8 km/h?

Section F: Density and Flow Rates

1. A plastic component has volume 320 cm^3 and mass 384 g.

    (a) Calculate its density in g/cm^3

    (b) Calculate its density in kg/m^3

    (c) What mass of this plastic would have volume 480 cm^3?

    (d) What volume would 576 g of this plastic occupy?

2. Different alloys have these densities:

    - Bronze: 8.7 g/cm^3
    - Brass: 8.4 g/cm^3
    - Pewter: 7.3 g/cm^3

    (a) Compare the masses of 120 cm^3 of each alloy

    (b) What volume of bronze has the same mass as 160 cm^3 of pewter?

    (c) A decorative item weighs 504 g and has volume 60 cm^3. Which alloy is it most likely made from?

3. Compressed air flows through a system at a rate of 7.2 m^3 per minute.

    (a) How much air flows in 25 minutes?

    (b) How long to process 432 m^3 of air?

    (c) Express the flow rate in liters per second

    (d) If the system pressure increases by 20%, what happens to the flow rate?

4. A textile factory operates looms at different rates during shifts:

    - Day shift (7 hours): 840 meters/hour
    - Evening shift (5 hours): 720 meters/hour
    - Night shift (4 hours): 600 meters/hour

    (a) Calculate total daily fabric production

    (b) Find the average production rate

    (c) How long to produce 6000 meters at the average rate?

Section G: Scale Factors and Similar Shapes

5. Two similar dodecagons have corresponding sides in the ratio 4:9.

   (a) If the smaller dodecagon has area 64 cm^2, find the area of the larger dodecagon

   (b) If the larger dodecagon has perimeter 108 cm, find the perimeter of the smaller dodecagon

   (c) Find the ratio of their areas

6. A scale model of an aircraft is built to scale 1:80. The real aircraft is 48 m long and 44 m wingspan.

   (a) Find the dimensions of the model

   (b) If the model uses 0.45 m^2 of paint, how much paint does the real aircraft need?

   (c) The model weighs 1.8 kg. Estimate the mass of the real aircraft if made from the same material

7. A geological survey map has scale 1:10000. Two rock formations are 23.4 cm apart on the map.

   (a) Calculate the actual distance in km

   (b) What map distance represents 1.5 km?

   (c) A mineral deposit covers 12.8 cm^2 on the map. Find its actual area in hectares

8. Two similar cylinders have diameter ratio 7:12.

   (a) Find the ratio of their surface areas

   (b) Find the ratio of their volumes

   (c) If the smaller cylinder has volume 343 cm^3, what is the volume of the larger cylinder?

   (d) If the larger cylinder has surface area 576 cm^2, what is the surface area of the smaller cylinder?

Section H: Advanced Rate Problems

9. A chemical reaction doubles the concentration every 45 minutes. Starting with 80 mg/liter:

   (a) Write an expression for concentration after t minutes

   (b) What is the concentration after 3 hours?

   (c) When will the concentration reach 2560 mg/liter?

   (d) What is the rate of increase after 90 minutes (mg/liter per minute)?

10. A hydroelectric reservoir has three inflow rivers and two outflow channels. The rivers provide 65, 48, and 37 cubic meters/minute respectively. The channels remove 55 and 42 cubic meters/minute respectively.

    (a) What is the net rate of filling when all flows operate?

    (b) How long to fill a 3180 cubic meter reservoir from empty?

    (c) If the smallest inflow river dries up, what is the new net rate?

    (d) What capacity would a third outflow channel need to exactly balance all inflows?

11. Currency exchange rates:

    - £1 = \$1.29
    - £1 = €1.13
    - \$1 = ¥114

(a) Convert £380 to dollars

(b) Convert €195 to pounds

(c) Convert $234 to yen

(d) Find the exchange rate from euros to dollars

12. A semiconductor fabrication line operates at variable throughput:

- Initialization: 1.5 hours at 20% throughput (full throughput is 120 wafers/hour)
- Production run: 6 hours at full throughput
- Quality check: 0.5 hours at 40% throughput

(a) Calculate total wafers processed

(b) Find the average throughput rate over the whole cycle

(c) How long would it take to process the same quantity at 95% throughput?

Section I: Problem Solving and Applications

13. A soup recipe for 8 servings uses:

- 640ml broth
- 320g vegetables
- 160g meat
- 40ml cream

(a) Adapt the recipe for 12 servings

(b) How much of each ingredient for 5 servings?

(c) If you have 1.6 liters of broth, what's the maximum number of servings you can make?

14. A conference center has 4200 seats. The ratio of main hall to seminar rooms to workshop spaces is 14:8:3. Due to renovation, 280 main hall seats are converted to seminar room seats.

(a) How many seats of each type were there originally?

(b) What is the new ratio of main hall to seminar rooms to workshop spaces?

(c) How many workshop spaces would need to be added to restore the original ratio?

15. The heat loss from a building varies directly as the square of the temperature difference between inside and outside. When the difference is 15°C, heat loss is 450 watts.

(a) Find the heat loss when the difference is 25°C

(b) What temperature difference gives heat loss of 720 watts?

(c) Compare heat losses for differences of 10°C and 20°C

16. A manufacturing company's quality control cost Q (in hundreds) varies as the square of batch size B up to 25 units, then becomes constant. When $B = 10$, $Q = 200$. When $B = 30$, $Q = 1250$.

(a) Find the cost function for $B \leq 25$

(b) Find the constant cost for $B > 25$

(c) What batch size minimizes cost per unit for $B \leq 25$?

(d) At what batch size does the cost structure change?

17. A clockwork mechanism has a gear system with seven gears. Gear A has 24 teeth and rotates at 240 rpm. The subsequent gears have 18, 36, 12, 48, 16, and 32 teeth respectively, connected in sequence.

    (a) Find the rotation speed of each gear

    (b) What is the overall gear ratio from gear A to gear G?

    (c) If gear A's speed changes to 360 rpm, find the new speed of gear G

    (d) How would the final speed change if gear E had 40 teeth instead of 48?

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 100

For more resources and practice materials, visit:
stepupmaths.co.uk