A Level Statistics
Practice Test 2: Advanced Topics

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Draw diagrams where appropriate to illustrate your solutions.
Time allowed: 3 hours

Section A: Fundamental Concepts [25 marks]

1. [**12 marks**] Define and explain fundamental concepts:

(a) Define confidence intervals and explain their interpretation.

(b) Explain what is meant by "confidence level" and "margin of error."

(c) State the relationship between confidence level and interval width.

(d) Define Type I and Type II errors in hypothesis testing.

(e) Distinguish between significance level and p-value.

(f) Explain how confidence intervals relate to hypothesis testing.

2. [**8 marks**] Explain the importance of these concepts:

(a) Why are confidence intervals preferred over point estimates?

(b) Explain how Type I and Type II errors affect decision-making in practice.

(c) Describe the trade-off between Type I and Type II error rates.

(d) Explain the relationship between sample size and statistical power.

3. [**5 marks**] Practical and theoretical context:

(a) Explain why we use sampling distributions in statistical inference.

(b) Describe the role of assumptions in statistical procedures.

(c) Explain how robust statistical methods handle assumption violations.

Section B: Confidence Intervals - Theory [30 marks]

4. **[15 marks]** State and explain confidence interval construction:

(a) Write the general formula for a confidence interval for a population mean.

(b) Explain the role of the critical value in interval construction.

(c) Describe when to use t-distribution versus normal distribution.

(d) State the assumptions required for confidence intervals for means.

(e) Explain how sample size affects interval width.

(f) Describe the relationship between confidence level and critical values.

5. **[15 marks]** Properties and interpretation of confidence intervals:

(a) Explain the correct interpretation of a 95

(b) Describe common misinterpretations of confidence intervals.

(c) Explain what happens to interval width as confidence level increases.

(d) Describe the concept of coverage probability.

(e) Explain how to determine appropriate sample sizes for desired precision.

(f) Discuss the effect of population variability on interval width.

(g) Explain confidence intervals for proportions and their assumptions.

(h) Describe the Wilson score interval and when it should be used.

(i) Explain the concept of exact versus approximate confidence intervals.

Section C: Confidence Interval Applications [35 marks]

6. **[18 marks]** A random sample of 25 measurements has mean $\bar{x} = 12.4$ and standard deviation $s = 3.2$:

(a) Construct a 95

(b) Construct a 99

(c) Compare the widths of the two intervals and explain the difference.

(d) If the sample size were 100 instead of 25, calculate the 95

(e) Explain how quadrupling the sample size affects the interval width.

(f) Determine the sample size needed for a margin of error of 1.0 with 95

(g) If the population standard deviation were known to be $= 3.0$, recalculate the 95

(h) Explain when it's appropriate to assume the population standard deviation is known.

(i) Discuss the practical implications of the interval estimates.

7. **[17 marks]** A survey of 400 voters finds that 216 support a particular candidate:

(a) Calculate the sample proportion $\hat{p}$.

(b) Construct a 95

(c) Check whether the normal approximation conditions are satisfied.

(d) Construct a 90

(e) Calculate the margin of error for each confidence level.

(f) Determine the sample size needed for a margin of error of 0.02 with 95

(g) If the true population proportion were 0.50, what's the probability the sample proportion exceeds 0.55?

(h) Explain how this relates to the confidence interval coverage.

(i) Discuss the political implications of the interval estimate.

Section D: Hypothesis Testing - Theory [25 marks]

8. [**12 marks**] Explain hypothesis testing framework:

(a) Define null and alternative hypotheses with examples.

(b) Explain the logic of proof by contradiction in hypothesis testing.

(c) Define the p-value and explain its interpretation.

(d) Describe the relationship between p-values and significance levels.

(e) Explain why we "fail to reject" rather than "accept" the null hypothesis.

(f) Relate the burden of proof concept to statistical testing.

9. [**13 marks**] Error types and statistical power:

(a) Define Type I error and explain its consequences.

(b) Define Type II error and explain its consequences.

(c) Define statistical power and explain its importance.

(d) Explain the relationship between , , and power.

(e) Describe factors that affect statistical power.

(f) Explain how to calculate required sample sizes for desired power.

(g) Describe the effect of effect size on power calculations.

(h) Explain the concept of practical versus statistical significance.

(i) Discuss the multiple testing problem and correction methods.

Section E: Hypothesis Testing Applications [30 marks]

10. [**15 marks**] A manufacturer claims the mean weight of cereal boxes is 500g. A sample of 36 boxes has mean 496.5g and standard deviation 8.2g:

(a) State appropriate null and alternative hypotheses.

(b) Check the assumptions for a one-sample t-test.

(c) Calculate the test statistic and degrees of freedom.

(d) Find the p-value for a two-tailed test.

(e) Make a decision using = 0.05 and state your conclusion.

(f) Calculate the 95

(g) If the true mean were 498g, calculate the probability of Type II error.

(h) Calculate the power of the test to detect a mean of 498g.

(i) Determine the sample size needed for 90

11. **[15 marks]** A new teaching method is tested. Before: mean = 72.3, n = 30, s = 8.5. After: mean = 76.8, n = 32, s = 9.2:

(a) State hypotheses for testing whether the new method improves scores.

(b) Check assumptions for a two-sample t-test.

(c) Calculate the pooled standard deviation assuming equal variances.

(d) Calculate the test statistic and degrees of freedom.

(e) Find the p-value and make a decision at = 0.01.

(f) Construct a 99

(g) Calculate the effect size (Cohen's d) and interpret its magnitude.

(h) If the true difference is 3 points, calculate the power of this test.

(i) Discuss the practical significance of the results.

Section F: Advanced Confidence Intervals [25 marks]

12. **[12 marks]** Non-standard confidence intervals:

(a) Explain confidence intervals for the variance of a normal population.

(b) Describe the chi-square distribution and its properties.

(c) Explain confidence intervals for the ratio of two variances.

(d) Describe bootstrap confidence intervals and their advantages.

(e) Explain non-parametric confidence intervals for medians.

(f) Discuss prediction intervals versus confidence intervals.

13. **[13 marks]** A sample of 20 observations from a normal population has variance $s^2 = 15.6$:

(a) Construct a 95

(b) Construct a 95

(c) Explain why the interval for is not symmetric around s.

(d) If the sample size were 50, recalculate the confidence interval for ^{2}.

(e) Compare the interval widths and explain the effect of sample size.

(f) Test H: ^{2} = 20 versus H: ^{2} 20 at = 0.05.

(g) Calculate the p-value for this test.

(h) Explain the relationship between the confidence interval and hypothesis test.

(i) Discuss applications where variance estimation is crucial.

Section G: Power Analysis and Sample Size [25 marks]

14. [**12 marks**] Power analysis concepts:

(a) Explain the four components of power analysis.

(b) Describe prospective versus retrospective power analysis.

(c) Explain the concept of minimum detectable difference.

(d) Discuss the ethics of underpowered studies.

(e) Explain interim analysis and stopping rules in clinical trials.

(f) Describe adaptive sample size designs.

15. [**13 marks**] A researcher wants to detect a difference of 5 units in mean response between two treatments, assuming = 12:

(a) Calculate the effect size for this study.

(b) Determine the sample size needed for 80

(c) Recalculate for 90

(d) Calculate the sample size needed if = 0.01 instead of 0.05.

(e) Explain how Type I error rate affects required sample sizes.

(f) If the true difference is actually 7 units, calculate the power with n = 30 per group.

(g) Create a power curve showing power versus effect size for n = 25 per group.

(h) Discuss the cost-benefit trade-offs in sample size determination.

(i) Explain how pilot studies inform power analysis.

Section H: Multiple Comparisons and ANOVA [20 marks]

16. [**10 marks**] Multiple comparison procedures:

(a) Explain why multiple testing increases Type I error rates.

(b) Describe the Bonferroni correction and its properties.

(c) Explain the False Discovery Rate and Benjamini-Hochberg procedure.

(d) Describe planned versus post-hoc comparisons.

(e) Explain the family-wise error rate concept.

17. [**10 marks**] An ANOVA study compares 4 treatment groups with the following results: F = 3.82, df = 3, df = 36:

(a) Calculate the p-value for the overall F-test.

(b) Make a decision at = 0.05 and state your conclusion.

(c) If we perform all pairwise comparisons (6 tests), what's the Bonferroni-corrected ?

(d) Calculate the critical F-value for = 0.01.

(e) Explain what the significant F-test tells us about the treatment means.

(f) Describe appropriate follow-up analyses after a significant ANOVA.

(g) Calculate the minimum sample size per group for 80

Section I: Non-parametric Methods [20 marks]

18. **[10 marks]** Non-parametric testing concepts:

(a) Explain when non-parametric tests are preferred over parametric tests.

(b) Describe the advantages and disadvantages of non-parametric methods.

(c) Explain the concept of distribution-free tests.

(d) Describe rank-based test statistics and their properties.

(e) Explain the efficiency of non-parametric tests relative to parametric alternatives.

19. **[10 marks]** A Wilcoxon signed-rank test is performed on paired data with n = 15 pairs, yielding W = 89:

(a) State the null and alternative hypotheses for this test.

(b) Explain how the Wilcoxon test statistic is calculated.

(c) Determine the critical value for = 0.05 (two-tailed).

(d) Make a decision and state your conclusion.

(e) Calculate the normal approximation to the test statistic.

(f) Compare the exact and approximate p-values.

(g) Explain when the normal approximation is adequate for the Wilcoxon test.

Section J: Advanced Applications [25 marks]

20. **[12 marks]** Real-world statistical inference:

(a) Explain the role of statistical inference in clinical trials.

(b) Describe quality control applications of hypothesis testing.

(c) Explain how confidence intervals are used in engineering tolerances.

(d) Describe A/B testing in business analytics.

(e) Explain the statistical basis for opinion polling margins of error.

(f) Discuss the replication crisis and its implications for statistical practice.

21. **[13 marks]** A pharmaceutical company tests a new drug's effectiveness. In the treatment group (n = 120), 84 patients improved. In the control group (n = 110), 63 patients improved:

(a) Calculate the improvement rates for both groups.

(b) Construct 95

(c) Test whether the improvement rates differ significantly.

(d) Calculate the test statistic for comparing two proportions.

(e) Find the p-value and make a decision at = 0.05.

(f) Construct a 95

(g) Calculate the number needed to treat (NNT) and its confidence interval.

(h) Discuss the clinical significance of the results.

(i) Explain how regulatory agencies might interpret these findings.

Answer Space

Use this space for your working and answers.

Formulae and Key Concepts

Confidence Intervals:

Mean (known): $\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$

Mean (unknown): $\bar{x} \pm t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}$

Proportion: $\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

Hypothesis Testing:

One-sample t-test: $t = \frac{\bar{x}-\mu_0}{s/\sqrt{n}}$

Two-sample t-test: $t = \frac{\bar{x_1}-\bar{x_2}}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$

Pooled SD: $s_p = \sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}}$

Power and Sample Size:

Effect size: $d = \frac{\mu_1-\mu_0}{\sigma}$

Power: $1-\beta = P(\text{reject } H_0|H_1 \text{ true})$

Sample size: $n = \frac{(z_{\alpha/2}+z_\beta)^2\sigma^2}{(\mu_1-\mu_0)^2}$

Chi-square Tests:

Variance: $\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$ with df = n-1

CI for ²: $\left[\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}}, \frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}}\right]$

Two Proportions:

Test statistic: $z = \frac{\hat{p_1}-\hat{p_2}}{\sqrt{\hat{p}(1-\hat{p})(\frac{1}{n_1}+\frac{1}{n_2})}}$

Pooled proportion: $\hat{p} = \frac{x_1+x_2}{n_1+n_2}$

Sample Size for Proportions:

$n = \frac{z^2_{\alpha/2}p(1-p)}{E^2}$ where E is margin of error

Conservative: use p = 0.5

Multiple Comparisons:

Bonferroni: $\alpha_{adj} = \frac{\alpha}{k}$ for k comparisons

Family-wise error rate: $\alpha_{FW} = 1 - (1 - \alpha)^k$

Non-parametric Tests:

Wilcoxon signed-rank: Sum of positive ranks

Normal approximation: $z = \frac{W - n(n+1)/4}{\sqrt{n(n+1)(2n+1)/24}}$

Effect Sizes:

Cohen's d: $d = \frac{\bar{x_1} - \bar{x_2}}{s_{pooled}}$

Small: d = 0.2, Medium: d = 0.5, Large: d = 0.8

Critical Values:

Standard normal: z. = 1.96, z. = 2.58

Common t-values depend on df and level

Chi-square values depend on df and level

END OF TEST

Total marks: 300

For more resources and practice materials, visit:
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