

GCSE Higher Mathematics

Practice Test 5: Geometry and Measures

Instructions:

Answer all questions. Show your working clearly.

Calculators may be used unless stated otherwise.

Time allowed: 90 minutes

Section A: Pythagoras' Theorem

- Use Pythagoras' theorem to find the missing lengths:
 - A right-angled triangle with legs 12 cm and 16 cm. Find the hypotenuse.
 - A right-angled triangle with hypotenuse 37 cm and one leg 12 cm. Find the other leg.
 - A right-angled triangle with legs 14.5 cm and 19.5 cm. Find the hypotenuse.
 - A right-angled triangle with hypotenuse 41 cm and one leg 40 cm. Find the other leg.
- Find the distance between these pairs of coordinate points:
 - A(5, 8) and B(17, 23)
 - C(-2, 7) and D(10, -5)
 - E(-7, -6) and F(5, 6)
 - G(0, 11) and H(-12, -1)
- A rectangle has length 28 cm and diagonal 35 cm. Find:
 - The width of the rectangle
 - The area of the rectangle
 - The perimeter of the rectangle
- A ladder 12.5 m long leans against a vertical wall. The foot of the ladder is 7.5 m from the wall.
 - How high up the wall does the ladder reach?
 - If the ladder slips so its foot is now 9 m from the wall, how far does the top slide down?
- Determine whether these triangles are right-angled:
 - Sides 12 cm, 35 cm, 37 cm
 - Sides 13 cm, 84 cm, 85 cm
 - Sides 18 cm, 22 cm, 28 cm
 - Sides 24 cm, 32 cm, 40 cm

Section B: Basic Trigonometry

6. In a right-angled triangle with hypotenuse 29 cm and one leg 21 cm:
 - (a) Find the other leg using Pythagoras
 - (b) Calculate $\sin A$, $\cos A$, and $\tan A$ where A is the angle opposite the 21 cm side
 - (c) Find angle A to 1 decimal place
7. Use trigonometry to find the missing sides (give answers to 3 significant figures):
 - (a) Right-angled triangle: angle 49° , adjacent side 13 cm, find opposite side
 - (b) Right-angled triangle: angle 66° , hypotenuse 20 cm, find opposite side
 - (c) Right-angled triangle: angle 27° , opposite side 11 cm, find adjacent side
 - (d) Right-angled triangle: angle 81° , adjacent side 9 cm, find hypotenuse
8. Find the missing angles (to 1 decimal place):
 - (a) Right-angled triangle: opposite 9 cm, adjacent 16 cm
 - (b) Right-angled triangle: opposite 13.4 cm, hypotenuse 18.7 cm
 - (c) Right-angled triangle: adjacent 9.8 cm, hypotenuse 13.6 cm
 - (d) Right-angled triangle: opposite 24 cm, adjacent 15 cm
9. A 24 m ladder makes an angle of 78° with the ground.
 - (a) How high up the wall does it reach?
 - (b) How far is the foot of the ladder from the wall?
 - (c) What angle would the ladder make if its foot was 13 m from the wall?
10. From the top of a 48 m cliff, the angle of depression to a boat is 31° .
 - (a) Draw a diagram
 - (b) Find the horizontal distance from the cliff to the boat
 - (c) Find the direct distance from the top of the cliff to the boat

Section C: Advanced Trigonometry - Sine and Cosine Rules

11. Use the sine rule to find the missing parts:
 - (a) Triangle ABC: $a = 16$ cm, $A = 54^\circ$, $B = 69^\circ$. Find b and c .
 - (b) Triangle PQR: $p = 21$ cm, $q = 25$ cm, $P = 49^\circ$. Find angle Q .
 - (c) Triangle DEF: $d = 12.6$ cm, $e = 16.4$ cm, $D = 43^\circ$. Find angle E .
12. Use the cosine rule to find the missing parts:
 - (a) Triangle ABC: $a = 17$ cm, $b = 14$ cm, $c = 9$ cm. Find angle C .
 - (b) Triangle GHI: $g = 19$ cm, $h = 15$ cm, $I = 58^\circ$. Find side i .
 - (c) Triangle JKL: $j = 11.3$ cm, $k = 14.6$ cm, $l = 8.7$ cm. Find the largest angle.
13. A triangle has sides 19 cm, 21 cm, and 23 cm.
 - (a) Use the cosine rule to find the largest angle
 - (b) Use the sine rule to find another angle
 - (c) Find the area using the formula $\text{Area} = \frac{1}{2}ab \sin C$

14. Two ships leave port at the same time. Ship A travels 48 km on a bearing of 085° . Ship B travels 39 km on a bearing of 155° .
- (a) Find the angle between their paths
 - (b) Calculate the distance between the ships
 - (c) Find the bearing of ship B from ship A
15. A parallelogram has sides 16 cm and 22 cm, with an included angle of 76° .
- (a) Find the length of the diagonals
 - (b) Find the area of the parallelogram
 - (c) Find the other angles of the parallelogram

Section D: Circle Theorems

16. Apply circle theorems to find the missing angles:
- (a) Angle at center is 136° . Find the angle at circumference subtending the same arc.
 - (b) Angle at circumference is 39° . Find the angle at center subtending the same arc.
 - (c) Two angles at circumference subtend the same arc. One is 47° . Find the other.
 - (d) Angle in semicircle MNO where MN is diameter. Find angle MON.
17. In a circle with center O:
- (a) Chord JK subtends angle 71° at circumference point L. Find angle JOK.
 - (b) Tangent WX touches circle at X. If angle $W XO = 43^\circ$, find angle XWO.
 - (c) Two chords ST and UV intersect at W inside the circle. If angles $SWU = 92^\circ$, find angle SWV.
 - (d) Tangents from external point W touch circle at J and K. If angle $JWK = 64^\circ$, find angle JOK.
18. Prove these circle theorems (provide written proof):
- (a) Angle in semicircle is 90°
 - (b) Angles in same segment are equal
 - (c) Angle at center is twice angle at circumference
 - (d) Tangent is perpendicular to radius at point of contact
19. A circle has center O and radius 16 cm. Chord JK has length 24 cm.
- (a) Find the perpendicular distance from O to chord JK
 - (b) Find angle JOK
 - (c) Find the area of sector JOK
 - (d) Find the area of triangle JOK

Section E: Geometrical Proof and Reasoning

20. Prove that the sum of angles in any triangle is 180° .
21. Prove that the exterior angle of a triangle equals the sum of the two interior opposite angles.
22. In triangle LMN, P is the midpoint of MN. Prove that LP is a median and that the three medians of a triangle meet at a single point.

23. ABCD is a parallelogram. Prove that:
- (a) Opposite sides are equal
 - (b) Opposite angles are equal
 - (c) Diagonals bisect each other
24. Prove that if a quadrilateral has both pairs of opposite sides parallel, then it is a parallelogram.
25. In triangle MNO, the perpendicular bisectors of sides MN and NO meet at point W. Prove that W is equidistant from M, N, and O.
26. ABCD is a cyclic quadrilateral (vertices lie on a circle). Prove that opposite angles sum to 180° .
27. Prove that the angle bisectors of a triangle meet at a single point (the incenter).

Section F: Surface Area and Volume

28. Calculate the surface area and volume of these shapes:
- (a) Cuboid: length 18 cm, width 13 cm, height 9 cm
 - (b) Cylinder: radius 8 cm, height 20 cm
 - (c) Sphere: radius 12 cm
 - (d) Cone: radius 9 cm, height 24 cm
29. A triangular prism has:
- Triangular cross-section with base 14 cm and height 8 cm
 - Length 26 cm

Calculate:

- (a) The volume
 - (b) The surface area
 - (c) The total edge length
30. A square-based pyramid has:
- Base edge 16 cm
 - Vertical height 12 cm
- Find:
- (a) The volume
 - (b) The slant height
 - (c) The surface area
31. A hemisphere sits on top of a cylinder. Both have radius 7 cm and the cylinder has height 18 cm.
- (a) Find the total volume
 - (b) Find the total surface area
 - (c) If this shape is a water tank, how much water does it hold?
32. A cone has base radius 9 cm and slant height 19 cm.
- (a) Find the vertical height
 - (b) Calculate the volume
 - (c) Calculate the curved surface area
 - (d) Calculate the total surface area

Section G: Complex 3D Problems

33. A rectangular swimming pool is 45 m long, 22 m wide, and 4 m deep.
- (a) Calculate the volume of water needed to fill it
 - (b) Find the surface area of the pool (5 faces - no top)
 - (c) How much does it cost to tile the pool at £62 per m^2 ?
 - (d) If water is added at 1500 liters per minute, how long to fill the pool?
34. A solid consists of a cylinder with radius 12 cm and height 18 cm, with a cone of radius 12 cm and height 16 cm on top.
- (a) Calculate the total volume
 - (b) Calculate the total surface area
 - (c) If the solid is made of metal with density 9.1 g/cm^3 , find its mass
35. A spherical ball has radius 26 cm. A cylindrical hole of radius 8 cm is drilled through the center.
- (a) Calculate the volume of material removed
 - (b) Calculate the remaining volume
 - (c) Calculate the new surface area (including the cylindrical hole)
36. A water tank is a horizontal cylinder with length 9 m and radius 2.4 m.
- (a) Calculate its capacity in liters
 - (b) If it's half full, find the depth of water
 - (c) Calculate the surface area of water

Section H: Problem Solving and Applications

37. A regular pentagon is inscribed in a circle of radius 20 cm.
- (a) Find the side length of the pentagon
 - (b) Calculate the area of the pentagon
 - (c) Find the area between the circle and pentagon
38. From point T, the angle of elevation to the top of a tower is 29° . From point U, 110 m closer to the tower, the angle of elevation is 54° .
- (a) Draw a diagram
 - (b) Find the height of the tower
 - (c) Find the distances from T and U to the base of the tower
39. A field is in the shape of a triangle with sides 200 m, 240 m, and 280 m.
- (a) Find the area of the field using Heron's formula
 - (b) Find the area using trigonometry
 - (c) A path 4 m wide runs around the perimeter. Find the area of the path
40. A church spire consists of a square-based pyramid on top of a cuboid. The cuboid is $16 \text{ m} \times 16 \text{ m} \times 26 \text{ m}$ high. The pyramid has height 20 m.
- (a) Calculate the total volume
 - (b) Calculate the total surface area

- (c) Find the angle the pyramid faces make with the horizontal
41. Two circles with radii 18 cm and 13 cm have centers 38 cm apart.
- (a) Do the circles intersect? Justify your answer.
 - (b) Find the length of their common external tangent
 - (c) Calculate the area of overlap if they moved closer together
42. A satellite orbits Earth at height 700 km above the surface. Earth's radius is 6400 km.
- (a) Calculate the satellite's orbital radius
 - (b) Find the maximum distance the satellite can see on Earth's surface
 - (c) Calculate the area of Earth visible from the satellite
43. Prove that in any triangle, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$ where R is the circumradius.
44. A frustum (truncated cone) has top radius 7 cm, bottom radius 18 cm, and height 14 cm.
- (a) Find the height of the complete cone
 - (b) Calculate the volume of the frustum
 - (c) Calculate the curved surface area of the frustum

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 100

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