A Level Mechanics Practice Test 2: Statics and Dynamics of Particles

Instructions:

Answer all questions. Show your working clearly.

Calculators may be used unless stated otherwise.

Draw clear diagrams where appropriate to illustrate your solutions.

Time allowed: 2 hours 30 minutes

Section A: Equilibrium of Particles [30 marks]

Question 1 [12 marks] A particle is in equilibrium under the action of three forces: 15 N acting horizontally to the right, 20 N acting at 60° above the horizontal, and a third force F.

- (a) Resolve the 20 N force into horizontal and vertical components. [3 marks]
- (b) Calculate the horizontal component of force F required for equilibrium. [2 marks]
- (c) Calculate the vertical component of force F required for equilibrium. [2 marks]
- (d) Find the magnitude and direction of force F. [4 marks]
- (e) Verify your answer by showing that the sum of all forces is zero. [1 mark]

Question 2 [18 marks] A chandelier of mass 50 kg is suspended from the ceiling by three cables. Cable A is vertical, cable B makes an angle of 25° with the vertical, and cable C makes an angle of 40° with the vertical. Cables B and C are attached to the same point on the chandelier.

- (a) Draw a force diagram showing all forces acting at the attachment point. [3 marks]
- (b) If the tension in cable A is 300 N, calculate the weight supported by this cable. [2 marks]
- (c) Write equilibrium equations for the horizontal and vertical directions at the attachment point. [3 marks]
- (d) Calculate the tension in cable B. [5 marks]
- (e) Calculate the tension in cable C. [3 marks]
- (f) Find the total weight of the chandelier and verify this equals the given mass. [2 marks]

Section B: Particles on Inclined Planes [40 marks]

Question 3 [20 marks] A crate of mass 18 kg rests on a rough inclined plane that makes an angle of 28° with the horizontal. The coefficient of static friction is 0.65 and the coefficient of kinetic friction is 0.45.

(a) Calculate the component of weight acting down the inclined plane. [3 marks]

- (b) Find the normal reaction force between the crate and the plane. [3 marks]
- (c) Calculate the maximum static friction force available. [3 marks]
- (d) Determine whether the crate will remain stationary on the plane. Show your reasoning clearly. [4 marks]
- (e) If a horizontal force of 80 N is applied to push the crate up the plane, calculate the component of this force parallel to the plane. [3 marks]
- (f) For the situation in part (e), determine whether the crate will move up the plane and calculate its acceleration if it does. [4 marks]

Question 4 [20 marks] A skier of mass 70 kg slides down a slope inclined at 15° to the horizontal. The coefficient of kinetic friction between the skis and snow is 0.12.

- (a) Draw a force diagram showing all forces acting on the skier. [3 marks]
- (b) Calculate the component of the skier's weight parallel to the slope. [3 marks]
- (c) Find the normal reaction force from the slope. [3 marks]
- (d) Calculate the friction force acting on the skier. [3 marks]
- (e) Determine the net force acting down the slope. [2 marks]
- (f) Calculate the acceleration of the skier down the slope. [3 marks]
- (g) If the skier starts from rest, find the velocity after sliding 50 m down the slope. [3 marks]

Section C: Systems of Connected Particles [50 marks]

Question 5 [25 marks] Two blocks A and B have masses 8.0 kg and 12 kg respectively. They are connected by a light inextensible string passing over a smooth pulley. Block A rests on a rough horizontal table with coefficient of kinetic friction 0.25, while block B hangs freely.

- (a) Draw separate force diagrams for both blocks. [4 marks]
- (b) Calculate the weight of block B. [2 marks]
- (c) Find the maximum static friction force on block A if the coefficient of static friction is 0.30. [3 marks]
- (d) Determine whether the system will remain in equilibrium or start to move. [3 marks]
- (e) Write the equation of motion for block A in the horizontal direction. [3 marks]
- (f) Write the equation of motion for block B in the vertical direction. [2 marks]
- (g) Calculate the acceleration of the system. [5 marks]
- (h) Find the tension in the string. [3 marks]

Question 6 [25 marks] A particle P of mass 4.0 kg is connected by a light inextensible string to particle Q of mass 6.0 kg. Particle P rests on a smooth inclined plane at 35° to the horizontal, while particle Q rests on a rough inclined plane at 20° to the horizontal (coefficient of kinetic friction 0.4). The string passes over a smooth pulley at the junction of the two planes.

- (a) Draw force diagrams for both particles. [4 marks]
- (b) For particle P, calculate the component of weight down the inclined plane. [3 marks]

- (c) For particle Q, calculate:
 - (i) The component of weight down the inclined plane. [2 marks]
 - (ii) The normal reaction force. [2 marks]
 - (iii) The maximum friction force available. [2 marks]
- (d) Determine the direction of motion of the system by comparing the driving forces. [3 marks]
- (e) Write equations of motion for both particles and solve for the acceleration. [6 marks]
- (f) Calculate the tension in the string. [3 marks]

Section D: Motion with Resistance [25 marks]

Question 7 [15 marks] A particle of mass 3.0 kg moves in a straight line under the action of a driving force F = (20 - 4t) N, where t is time in seconds. The particle also experiences a constant resistance force of 5.0 N.

- (a) Write an expression for the net force acting on the particle at time t. [2 marks]
- (b) Calculate the acceleration of the particle when t = 2.0 s. [3 marks]
- (c) If the particle starts from rest, derive an expression for velocity as a function of time. [5 marks]
- (d) Find the velocity of the particle when t = 3.0 s. [2 marks]
- (e) Calculate the displacement of the particle during the first 3.0 seconds. [3 marks]

Question 8 [10 marks] A ball of mass 0.5 kg falls vertically through air. The air resistance force is proportional to the square of the velocity and is given by $R = 0.02v^2$ N, where v is the velocity in m/s.

- (a) Write the equation of motion for the ball when falling. [3 marks]
- (b) Calculate the terminal velocity of the ball. [4 marks]
- (c) If the ball has reached 80% of its terminal velocity, calculate the acceleration at this instant. [3 marks]

Section E: Projectile Motion [35 marks]

Question 9 [20 marks] A stone is thrown horizontally from the top of a building 80 m high with an initial velocity of 15 m/s.

- (a) Write expressions for the horizontal and vertical positions as functions of time. [4 marks]
- (b) Calculate the time taken for the stone to reach the ground. [3 marks]
- (c) Find the horizontal distance traveled when the stone hits the ground. [3 marks]
- (d) Calculate the horizontal and vertical components of velocity just before impact. [4 marks]
- (e) Find the magnitude and direction of the velocity just before impact. [4 marks]
- (f) At what time is the stone 30 m above the ground? [2 marks]

Question 10 [15 marks] A golf ball is struck from ground level at an angle of 42° above the horizontal with an initial speed of 28 m/s.

- (a) Calculate the initial horizontal and vertical components of velocity. [3 marks]
- (b) Find the maximum height reached by the ball. [4 marks]
- (c) Calculate the time of flight of the ball. [4 marks]
- (d) Determine the horizontal range of the ball. [2 marks]
- (e) Find the velocity components when the ball is at half its maximum height on the way up. [2 marks]

Section F: Complex Particle Problems [30 marks]

Question 11 [18 marks] A particle of mass 2.0 kg is in equilibrium under five forces: 25 N due north, 18 N due east, 30 N at 60° west of north, 20 N due south, and an unknown force F.

- (a) Set up a coordinate system with east as positive x-direction and north as positive y-direction. [1 mark]
- (b) Resolve each known force into x and y components. [6 marks]
- (c) Calculate the sum of x-components and y-components of all known forces. [3 marks]
- (d) Find the x and y components of force F required for equilibrium. [3 marks]
- (e) Calculate the magnitude and direction of force F. [4 marks]
- (f) Verify equilibrium by checking that the sum of all forces equals zero. [1 mark]

Question 12 [12 marks] A bead of mass 0.8 kg can slide on a smooth vertical circular wire of radius 1.2 m. The bead is in equilibrium under the action of its weight and a horizontal force P applied at the center of the circle.

- (a) If the bead is positioned at 30° from the vertical, draw a force diagram showing all forces. [3 marks]
- (b) Calculate the normal reaction force from the wire on the bead. [4 marks]
- (c) Find the magnitude of the horizontal force P required for equilibrium. [3 marks]
- (d) Determine the direction in which force P must act. [2 marks]

Physics Data and Formulae

Forces and Equilibrium:

For particle equilibrium: $\sum F_x = 0$ and $\sum F_y = 0$ Force resolution: $F_x = F \cos \theta$, $F_y = F \sin \theta$ Resultant magnitude: $R = \sqrt{F_x^2 + F_y^2}$ Resultant direction: $\tan \alpha = \frac{F_y}{F_z}$

Newton's Laws:

First Law: $\sum F = 0$ (equilibrium) Second Law: $\sum F = ma$

Third Law: Action and reaction forces are equal and opposite

Friction:

Static friction: $f_s \leq \mu_s N$ (maximum: $f_{s,max} = \mu_s N$) Kinetic friction: $f_k = \mu_k N$

Inclined Planes:

Component parallel to plane: $mg \sin \theta$ (down the plane) Component perpendicular to plane: $mg \cos \theta$ (into the plane) Normal reaction: $N = mg \cos \theta$ (for particle on plane)

Kinematics:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^{2}$$

$$v^{2} = u^{2} + 2as$$

$$s = \frac{u+v}{2}t$$

Projectile Motion:

Horizontal: $x = u_x t$ (constant velocity) Vertical: $y = u_y t - \frac{1}{2}gt^2$ Velocity components: $v_x = u_x$, $v_y = u_y - gt$ Maximum height: $h_{max} = \frac{u_y^2}{2g}$ Time of flight: $T = \frac{2u_y}{g}$ Range: $R = \frac{u_x \cdot 2u_y}{g}$

Energy and Work:

Kinetic energy: $KE = \frac{1}{2}mv^2$ Potential energy: PE = mghWork done: $W = Fs\cos\theta$

Constants:

Acceleration due to gravity: $g = 9.81 \text{ m/s}^2$

Trigonometric Values:

 $\sin 15 = 0.259, \cos 15 = 0.966, \tan 15 = 0.268$ $\sin 20 = 0.342, \cos 20 = 0.940, \tan 20 = 0.364$ $\sin 25 = 0.423, \cos 25 = 0.906, \tan 25 = 0.466$ $\sin 28 = 0.469, \cos 28 = 0.883, \tan 28 = 0.532$ $\sin 30 = 0.500, \cos 30 = 0.866, \tan 30 = 0.577$ $\sin 35 = 0.574, \cos 35 = 0.819, \tan 35 = 0.700$ $\sin 40 = 0.643, \cos 40 = 0.766, \tan 40 = 0.839$ $\sin 42 = 0.669, \cos 42 = 0.743, \tan 42 = 0.900$ $\sin 60 = 0.866, \cos 60 = 0.500, \tan 60 = 1.732$

END OF TEST

Total marks: 210

Grade boundaries: A* 189, A 168, B 147, C 126, D 105, E 84

END OF TEST

Total marks: 90

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