GCSE Higher Mathematics
Practice Test 5: Number

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Time allowed: 90 minutes

Section A: Powers and Roots

1. Evaluate these expressions:
   (a) 7^3
   (b) 3^{-4}
   (c) 13^0
   (d) $(-7)^2$
   (e) $(-3)^5$
   (f) 9^{-2}

2. Write these as single powers:
   (a) $6^4 \times 6^9$
   (b) $9^6 \div 9^2$
   (c) $(4^6)^3$
   (d) $8^{-3} \times 8^{11}$
   (e) $\frac{11^8}{11^{-5}}$
   (f) $(6^4)^{-3}$

3. Evaluate these fractional indices:
   (a) $64^{\frac{1}{2}}$
   (b) $216^{\frac{1}{3}}$
   (c) $2401^{\frac{3}{4}}$
   (d) $100000^{\frac{2}{5}}$
   (e) $729^{\frac{2}{3}}$
   (f) $243^{-\frac{3}{5}}$

4. Simplify these expressions:
   (a) $\sqrt{169}$
   (b) $\sqrt[3]{512}$

(c) $\sqrt[4]{1296}$
(d) $\sqrt{0.81}$
(e) $\sqrt[3]{-343}$
(f) $\sqrt[5]{7776}$

5. Express in index form:

(a) $\sqrt{19}$
(b) $\sqrt[3]{13}$
(c) $\frac{1}{\sqrt{11}}$
(d) $\sqrt[4]{v^{11}}$
(e) $\frac{1}{\sqrt[3]{f^8}}$
(f) $\sqrt{v} \times \sqrt[3]{v}$

Section B: Laws of Indices

6. Simplify these expressions (no calculator):

(a) $6^5 \times 6^{-8} \times 6^9$
(b) $\frac{8^{10} \times 8^{-6}}{8^{-5}}$
(c) $(7^3)^{-5} \times 7^{17}$
(d) $\frac{9^{-6} \times 9^{13}}{9^5}$

7. Evaluate these expressions:

(a) $729^{\frac{2}{3}}$
(b) $625^{-\frac{3}{4}}$
(c) $3125^{\frac{3}{5}}$
(d) $2401^{-\frac{3}{4}}$
(e) $1728^{\frac{1}{3}}$
(f) $16807^{-\frac{4}{5}}$

8. Simplify these expressions:

(a) $x^{\frac{7}{8}} \times x^{\frac{1}{4}}$
(b) $\frac{u^{\frac{9}{4}}}{u^{\frac{1}{3}}}$
(c) $(f^{\frac{1}{6}})^{12}$
(d) $\sqrt{v} \times v^{\frac{1}{7}}$
(e) $\frac{\sqrt[3]{m^8}}{\sqrt{m}}$
(f) $(s^{-\frac{1}{6}})^{-12}$

9. Write these in the form a^n where a and n are rational:

(a) $\sqrt{13} \times 13^6$
(b) $\frac{17^4}{\sqrt[3]{17}}$
(c) $\sqrt[4]{10^{11}} \times 10^{-\frac{3}{4}}$
(d) $\frac{\sqrt{19}}{\sqrt[3]{19^8}}$

Section C: Surds

10. Simplify these surds:

    (a) $\sqrt{68}$
    (b) $\sqrt{99}$
    (c) $\sqrt{108}$
    (d) $\sqrt{175}$
    (e) $\sqrt{216}$
    (f) $\sqrt{700}$

11. Simplify these expressions:

    (a) $7\sqrt{13} + 11\sqrt{13}$
    (b) $15\sqrt{6} - 6\sqrt{6}$
    (c) $\sqrt{44} + \sqrt{99}$
    (d) $\sqrt{147} - \sqrt{75}$
    (e) $6\sqrt{20} + 8\sqrt{45}$
    (f) $\sqrt{175} - \sqrt{112} + \sqrt{28}$

12. Multiply and simplify:

    (a) $\sqrt{9} \times \sqrt{36}$
    (b) $\sqrt{21} \times \sqrt{84}$
    (c) $7\sqrt{3} \times 5\sqrt{12}$
    (d) $\sqrt{16} \times \sqrt{64}$
    (e) $\sqrt{7} \times \sqrt{28} \times \sqrt{112}$
    (f) $8\sqrt{6} \times 5\sqrt{24}$

13. Expand and simplify:

    (a) $(6 + \sqrt{11})(3 - \sqrt{11})$
    (b) $(5 + \sqrt{17})(2 + 6\sqrt{17})$
    (c) $(8 - \sqrt{13})^2$
    (d) $(\sqrt{19} + \sqrt{5})(\sqrt{19} - \sqrt{5})$
    (e) $(6\sqrt{11} + 1)(6\sqrt{11} - 1)$
    (f) $(\sqrt{17} + 6)^2$

14. Rationalize the denominators:

    (a) $\frac{1}{\sqrt{17}}$
    (b) $\frac{11}{\sqrt{19}}$
    (c) $\frac{\sqrt{11}}{\sqrt{44}}$
    (d) $\frac{12}{6\sqrt{3}}$
    (e) $\frac{1}{5+\sqrt{11}}$
    (f) $\frac{7}{1-\sqrt{17}}$

Section D: More Complex Surd Operations

15. Rationalize these denominators:
    (a) $\frac{9}{6+\sqrt{13}}$
    (b) $\frac{13}{5-\sqrt{23}}$
    (c) $\frac{\sqrt{11}}{1+\sqrt{11}}$
    (d) $\frac{6\sqrt{17}}{5+\sqrt{17}}$
    (e) $\frac{1}{\sqrt{18}-\sqrt{11}}$
    (f) $\frac{\sqrt{17}+5}{\sqrt{17}-6}$
16. Simplify these expressions completely:
    (a) $\frac{\sqrt{28}+\sqrt{63}}{\sqrt{7}}$
    (b) $\frac{\sqrt{75}-\sqrt{48}}{\sqrt{3}}$
    (c) $\sqrt{(7+\sqrt{17})(7-\sqrt{17})}$
    (d) $\sqrt{108}-6\sqrt{3}+\sqrt{75}$
    (e) $(\sqrt{11}+\sqrt{44})^2$
    (f) $\frac{\sqrt{99}}{\sqrt{11}}+\frac{\sqrt{77}}{\sqrt{11}}$
17. Prove that:
    (a) $(\sqrt{i}+\sqrt{j})(\sqrt{i}-\sqrt{j})=i-j$
    (b) $\frac{1}{\sqrt{k}+\sqrt{l}}=\frac{\sqrt{k}-\sqrt{l}}{k-l}$
    (c) $(k+l\sqrt{n})^2=k^2+2kl\sqrt{n}+l^2n$

Section E: Standard Form

18. Write these numbers in standard form:
    (a) 789000
    (b) 0.000106
    (c) 5670000000
    (d) 0.0000000398
    (e) 1047.8
    (f) 0.01058
19. Write these in ordinary form:
    (a) 8.9×10^{2}
    (b) 6.78×10^{-8}
    (c) 9.214×10^{-11}
    (d) 5.47×10^{14}
    (e) 1.06×10^{-6}
    (f) 7.89×10^{9}
20. Calculate, giving answers in standard form:

(a) $(8 \times 10^8) \times (9 \times 10^{10})$
(b) $(6 \times 10^{-6}) \times (11 \times 10^{11})$
(c) $(18 \times 10^9) \div (6 \times 10^{-4})$
(d) $(15 \times 10^{-8}) \div (9 \times 10^{-11})$
(e) $(8 \times 10^7)^2$
(f) $\sqrt{49 \times 10^{16}}$

21. Calculate these more complex expressions:

(a) $(6.4 \times 10^7) \times (2.25 \times 10^{-9})$
(b) $\frac{12.8 \times 10^{10}}{3.2 \times 10^{-7}}$
(c) $(7.2 \times 10^{-6}) + (8.9 \times 10^{-7})$
(d) $(9.5 \times 10^9) - (6.3 \times 10^8)$
(e) $\frac{(6.3 \times 10^5) \times (5.4 \times 10^{-6})}{9.45 \times 10^{-4}}$
(f) $(6.76 \times 10^{14})^{\frac{1}{2}}$

Section F: Rational Numbers and Operations

22. Calculate these fractions (give answers in simplest form):

(a) $\frac{6}{11} + \frac{7}{22}$
(b) $\frac{13}{16} - \frac{9}{24}$
(c) $\frac{14}{15} \times \frac{25}{21}$
(d) $\frac{12}{25} \div \frac{18}{35}$
(e) $\frac{9}{10} - \frac{5}{12} + \frac{11}{60}$
(f) $\left(\frac{6}{11}\right)^{-2}$

23. Convert these recurring decimals to fractions:

(a) $0.\overline{6}$
(b) $0.\overline{72}$
(c) $0.5\overline{9}$
(d) $0.\overline{857142}$
(e) $4.1\overline{5}$
(f) $0.84\overline{7}$

24. Work out these percentage calculations:

(a) Increase 680 by 55%
(b) Decrease 840 by 28%
(c) Find 32.5% of 960
(d) What percentage is 126 out of 168?
(e) If 75% of a number is 195, find the number
(f) A price increases from £105 to £115.5. Find the percentage increase

25. Solve these percentage problems:

(a) After a 60% increase, a price is £256. Find the original price
(b) After a 55% decrease, a quantity is 108. Find the original quantity
(c) The value of a car decreases by 40% each year. If it's worth £10800 now, what was it worth 2 years ago?
(d) An investment grows by 7% per year. After 2 years it's worth £2287. Find the initial investment

Section G: Complex Calculations

26. Simplify these mixed expressions:
    (a) $6^{-2} + 9^0 - 10^{-1}$
    (b) $\sqrt{64} \times 343^{\frac{1}{3}} - 6^{-2}$
    (c) $\frac{1024^{\frac{2}{3}} - 2401^{\frac{3}{4}}}{64^{\frac{1}{2}}}$
    (d) $36^{-\frac{1}{2}} + 81^{\frac{1}{2}} \times 5^{-1}$

27. Calculate exactly (leave surds in your answer):
    (a) $\frac{9}{\sqrt{11}} + \frac{6}{\sqrt{44}}$
    (b) $\sqrt{48} \times \sqrt{75} - \sqrt{192}$
    (c) $\frac{\sqrt{147}+\sqrt{108}}{\sqrt{3}}$
    (d) $(6\sqrt{3} - 5)^2$

28. Work with standard form in context:
    (a) The mass of a carbon atom is 1.99×10^{-26} kg. Find the mass of 6.02×10^{23} carbon atoms
    (b) Electromagnetic waves travel at 3×10^8 m/s. How far do they travel in one week (use 1 week $= 6.048 \times 10^5$ seconds)?
    (c) The diameter of a red blood cell is approximately 7.5×10^{-6} m. How many red blood cells would fit across a distance of 4.5 mm?
    (d) A distributed computer system processes 9.6×10^{13} operations per second. How many operations in 20 minutes?

Section H: Problem Solving

29. Prove that $\sqrt{11}$ is irrational. (Use proof by contradiction: assume $\sqrt{11} = \frac{m}{n}$ where m and n are integers with no common factors)

30. The number γ satisfies $\gamma^2 = 4\gamma - 2$.
    (a) Show that $\gamma = 2 + \sqrt{2}$
    (b) Calculate γ to 4 decimal places
    (c) Find $\frac{1}{\gamma}$ in surd form

31. Rationalize the denominator of $\frac{1}{\sqrt{7}+\sqrt{11}+\sqrt{13}}$. (Hint: First rationalize using $(\sqrt{7} + \sqrt{11}) - \sqrt{13}$)

32. A rectangle has sides of length $(6 + \sqrt{11})$ cm and $(6 - \sqrt{11})$ cm.
    (a) Find the exact area
    (b) Find the exact perimeter
    (c) Show that the area is rational but the perimeter is irrational

33. The population of microbes sextuples every 8 hours. If there are initially 7×10^2 microbes:
    (a) How many microbes after 32 hours?
    (b) Express your answer in standard form
    (c) After how many hours will there be more than 6×10^9 microbes?

34. Show that $\frac{1}{\sqrt{i}+\sqrt{j}} + \frac{1}{\sqrt{i}-\sqrt{j}} = \frac{2\sqrt{i}}{i-j}$

35. A cone has volume $V = \frac{1}{3}\pi r^2 h$. If the volume is 576π cm^3 and $h = 18$ cm:

    (a) Find the radius in surd form

    (b) Find the slant height (use $l^2 = r^2 + h^2$)

    (c) Express both answers exactly

36. The equation $x^2 - 12x + 1 = 0$ has solutions $x = 6 \pm \sqrt{35}$.

    (a) Verify this by substitution

    (b) Find $\frac{1}{6+\sqrt{35}} + \frac{1}{6-\sqrt{35}}$ without using a calculator

    (c) Hence find the sum of the reciprocals of the roots

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 100

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