GCSE Higher Mathematics
Practice Test 3: Probability

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Time allowed: 90 minutes

Section A: Conditional Probability Fundamentals

1. A survey of 200 students shows:
   - 125 study French
   - 95 study Spanish
   - 65 study both French and Spanish

   (a) Draw a Venn diagram
   (b) Find P(studies French — studies Spanish)
   (c) Find P(studies Spanish — studies French)
   (d) Find P(studies exactly one subject)
   (e) Are studying French and Spanish independent? Justify your answer

2. A box contains 9 orange balls and 11 pink balls. Two balls are drawn without replacement.

   (a) Find P(second ball is orange — first ball is orange)
   (b) Find P(second ball is pink — first ball is orange)
   (c) Find P(both balls are the same color)
   (d) Find P(balls are different colors)
   (e) Verify that your probabilities sum to 1

3. Events E and F are such that:
   - P(E) = 0.5
   - P(F) = 0.45
   - P(E F) = 0.2

   Calculate:

   (a) P(E F)
   (b) P(E')
   (c) P(E — F)
   (d) P(F — E)

(e) P(E' — F')
(f) P(E — F')

4. A card is drawn from a standard 52-card deck. Let E = "card is a diamond" and F = "card is a King".
   (a) Find P(E), P(F), and P(E F)
   (b) Calculate P(E — F)
   (c) Calculate P(F — E)
   (d) Are events E and F independent? Show your working
   (e) Find P(E' F')

Section B: Tree Diagrams and Sequential Events

5. A bag contains 8 blue counters and 5 orange counters. A counter is drawn, its color noted, and then replaced. This process is repeated twice more.
   (a) Draw a tree diagram for all three draws
   (b) Find P(all three counters are blue)
   (c) Find P(exactly two counters are orange)
   (d) Find P(at least one counter is blue)
   (e) Find P(first counter is orange — exactly two are orange)
6. Box M contains 5 red balls and 5 white balls. Box N contains 6 red balls and 4 white balls. A fair coin is flipped to choose a box, then a ball is drawn.
   (a) Draw a tree diagram
   (b) Find P(red ball)
   (c) Find P(white ball)
   (d) Given a red ball was drawn, find P(it came from Box M)
   (e) Given a white ball was drawn, find P(it came from Box N)
7. Three machines produce components with different defect rates:
   - Machine J: produces 45% of components, 2% defective
   - Machine K: produces 30% of components, 7% defective
   - Machine L: produces 25% of components, 10% defective

   (a) Find the overall probability of a defective component
   (b) If a component is defective, find the probability it came from Machine J
   (c) If a component is defective, find the probability it came from Machine L
   (d) If a component is not defective, which machine most likely produced it?
8. A student takes three tests. The probability of passing each test is 0.65, and the tests are independent.
   (a) Find P(passes all three tests)
   (b) Find P(fails all three tests)
   (c) Find P(passes exactly two tests)
   (d) Find P(passes at least one test)
   (e) Given the student passed at least two tests, find P(passed all three)

Section C: Bayes' Theorem Applications

9. A medical test for a disease has the following characteristics:
   - If a person has the disease, the test is positive 89% of the time
   - If a person doesn't have the disease, the test is negative 94% of the time
   - 1.2% of the population has the disease

   (a) Find P(positive test)
   (b) If someone tests positive, find P(they have the disease)
   (c) If someone tests negative, find P(they don't have the disease)
   (d) Comment on the reliability of a positive test result
   (e) How would the results change if 12% of the population had the disease?

10. A security system has three sensors. The probability each sensor detects an intruder is:
    - Sensor P: 0.91
    - Sensor Q: 0.86
    - Sensor R: 0.93

    The sensors operate independently.

    (a) Find P(all three sensors detect an intruder)
    (b) Find P(at least one sensor detects an intruder)
    (c) Find P(exactly two sensors detect an intruder)
    (d) If exactly two sensors detect an intruder, find P(Sensor Q failed)
    (e) Which single sensor is most reliable for detection?

11. A factory produces items using two processes. Process M is used 55% of the time and produces 5% defective items. Process N is used 45% of the time and produces 9% defective items.

    (a) A random item is selected and found to be defective. Use Bayes' theorem to find P(produced by Process M)
    (b) If 1000 items are produced, how many would you expect to be defective?
    (c) How many of the defective items would come from each process?
    (d) To reduce overall defect rate to 4%, what should Process N's defect rate be?

12. Three weather forecasting models predict rain independently:
    - Model P: 84% accurate when it will rain, 82% accurate when it won't rain
    - Model Q: 76% accurate when it will rain, 89% accurate when it won't rain
    - Model R: 81% accurate when it will rain, 86% accurate when it won't rain

    Historically, it rains 35% of days.

    (a) If all three models predict rain, find P(it actually rains)
    (b) If Model P predicts rain but Models Q and R predict no rain, find P(it rains)
    (c) Which model would you trust most for a "rain" prediction?
    (d) Which model would you trust most for a "no rain" prediction?

Section D: Introduction to Binomial Distribution

13. A fair coin is flipped 12 times.
    (a) Find P(exactly 7 heads)
    (b) Find P(at most 4 heads)
    (c) Find P(at least 8 heads)
    (d) Find the expected number of heads
    (e) Find the most likely number of heads
    (f) Calculate the variance of the number of heads

14. A multiple choice test has 18 questions, each with 3 possible answers. A student guesses randomly on all questions.
    (a) State the distribution of the number of correct answers
    (b) Find P(exactly 5 correct answers)
    (c) Find P(more than 7 correct answers)
    (d) Find the expected number of correct answers
    (e) Find P(passes the test) if the pass mark is 55%
    (f) Calculate the standard deviation of correct answers

15. The probability that a seed germinates is 0.78. A packet contains 16 seeds.
    (a) Find P(all seeds germinate)
    (b) Find P(exactly 13 seeds germinate)
    (c) Find P(fewer than 10 seeds germinate)
    (d) How many seeds would you expect to germinate?
    (e) Find P(at least 70% of seeds germinate)
    (f) What's the most likely number of seeds to germinate?

16. A manufacturing process produces 8% defective items. Quality control samples 22 items.
    (a) Find P(no defective items in the sample)
    (b) Find P(exactly 3 defective items)
    (c) Find P(more than 4 defective items)
    (d) Calculate the expected number of defective items
    (e) Find P(defect rate in sample exceeds 12%)
    (f) Calculate the probability that the sample defect rate is between 5% and 10%

Section E: Advanced Binomial Applications

17. A basketball player has a 68% free throw success rate. In a game, they attempt 24 free throws.
    (a) Model this situation and state any assumptions
    (b) Find P(makes at least 18 free throws)
    (c) Find P(makes between 14 and 18 free throws inclusive)
    (d) Calculate the expected number of successful free throws
    (e) Find the probability their success rate in this game is above 75%
    (f) What's the minimum number of attempts needed for P(at least 1 success) 0.9999?

18. A quality control inspector checks 35 items per hour. The probability any item is defective is 0.09.
    (a) Find P(finds exactly 4 defective items in one hour)
    (b) Find P(finds no defective items in one hour)
    (c) Over a 12-hour shift, find the expected number of defective items found
    (d) In what percentage of hours would you expect to find more than 5 defective items?
    (e) If the inspector finds 8 defective items in one hour, comment on whether this is unusual

19. A pharmaceutical company claims their drug is effective for 78% of patients. A trial involves 35 patients.
    (a) If the claim is true, find P(drug works for exactly 28 patients)
    (b) Find P(drug works for at least 25 patients)
    (c) Calculate the expected number of patients for whom the drug works
    (d) If the drug works for only 22 patients, test whether this supports the company's claim
    (e) What's the minimum number of successes that would support the 78% claim at 5% significance?

20. A survey shows 38% of people support a proposal. A random sample of 28 people is surveyed.
    (a) Find P(exactly 11 people support the proposal)
    (b) Find P(fewer than 9 people support the proposal)
    (c) Calculate the expected number of supporters
    (d) Find P(between 25% and 45% of the sample support the proposal)
    (e) If 15 people in the sample support the proposal, is this significantly different from expected?

Section F: Combined Probability Scenarios

21. An online retailer has two suppliers. Supplier M provides 75% of goods with 4% defect rate. Supplier N provides 25% of goods with 9% defect rate.
    (a) A customer receives 15 items. Find P(exactly 2 are defective)
    (b) If a customer complains about a defective item, find P(it came from Supplier N)
    (c) A batch of 120 items arrives. Find the expected number from each supplier
    (d) Calculate the overall defect rate
    (e) If the company wants to reduce defects to 3%, what should Supplier N's rate be?

22. A casino game involves drawing 5 cards from a standard deck without replacement. The player wins if all 5 cards are clubs.
    (a) Calculate P(all 5 cards are clubs)
    (b) Calculate P(all 5 cards are the same suit)
    (c) If 800 people play this game, how many would you expect to win?
    (d) What should be the payout ratio for this to be a fair game?
    (e) How does the probability change if cards are replaced after each draw?

23. A communication system sends signals through 5 independent channels. Each channel has probability 0.82 of successful transmission.
    (a) Find P(message received successfully through all channels)

(b) Find P(message fails on exactly one channel)

(c) The system works if at least 4 channels succeed. Find P(system works)

(d) If the system sends 40 messages, find P(fewer than 35 are received successfully)

(e) What should be the individual channel reliability for 99.8% system reliability?

24. A hospital emergency department sees an average of 15% critical cases. On a particular shift, 24 patients arrive.

    (a) Model the number of critical cases and state assumptions

    (b) Find P(exactly 4 critical cases)

    (c) Find P(no critical cases)

    (d) Find P(more than 6 critical cases)

    (e) Calculate the expected number of critical cases

    (f) If there are 8 critical cases in one shift, is this unusually high?

Section G: Advanced Problem Solving

25. A genetic disorder affects 1 in 600 births. A screening test is 94% accurate for positive cases and 98.8% accurate for negative cases.

    (a) Calculate the probability of testing positive

    (b) If a baby tests positive, what's the probability they have the disorder?

    (c) How many false positives occur per 60,000 births?

    (d) Design a two-stage testing procedure to reduce false positives

    (e) Comment on the ethical implications of these probabilities

26. A software company releases updates with bugs 22% of the time. They use a testing protocol that catches 78% of buggy updates but also flags 8% of good updates as potentially buggy.

    (a) If an update is flagged, find P(it actually has bugs)

    (b) If an update passes testing, find P(it's actually bug-free)

    (c) In 150 updates, how many false alarms would you expect?

    (d) Suggest improvements to the testing protocol

    (e) Calculate the overall accuracy of the testing system

27. A lottery has the following structure: pick 4 numbers from 1-40. You win the jackpot if all 4 match.

    (a) Calculate P(winning the jackpot)

    (b) Find P(matching exactly 3 numbers)

    (c) Find P(matching exactly 2 numbers)

    (d) If 6 million tickets are sold, find P(no one wins the jackpot)

    (e) Model the number of jackpot winners as a binomial distribution

28. A cybersecurity system monitors network traffic. It correctly identifies 91% of malicious attacks and incorrectly flags 4% of normal traffic. On average, 0.3% of traffic is malicious.

    (a) Find the probability of an alert

    (b) If there's an alert, find P(it's a real attack)

    (c) In monitoring 750,000 data packets, how many false alarms occur?

(d) Design a cost-benefit analysis for this system

(e) How would increasing the detection rate to 95% affect false alarms?

29. Design and analyze a probability model for a real-world scenario of your choice:

(a) Define the scenario and identify random variables

(b) State all assumptions clearly

(c) Choose appropriate probability distributions

(d) Calculate relevant probabilities

(e) Discuss limitations and potential improvements

(f) Consider practical applications of your analysis

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 100

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