

GCSE Higher Mathematics

Practice Test 2: Algebra

Instructions:

Answer all questions. Show your working clearly.

Calculators may be used unless stated otherwise.

Time allowed: 90 minutes

Section A: Linear and Simultaneous Equations

1. Solve these linear equations:

(a) $4(3x - 2) = 2x + 14$

(b) $\frac{x+5}{3} - \frac{x-1}{4} = 2$

(c) $\frac{3x-2}{4} = \frac{2x+1}{3} - 1$

(d) $0.4x + 0.8 = 0.3x - 0.5$

2. Solve these simultaneous equations by elimination:

(a)
$$\begin{cases} 2x + 3y = 16 \\ 3x - y = 5 \end{cases}$$

(b)
$$\begin{cases} 4x + 5y = 23 \\ 3x - 2y = 4 \end{cases}$$

(c)
$$\begin{cases} 3x + 4y = 17 \\ 2x - 5y = 1 \end{cases}$$

(d)
$$\begin{cases} 5x + 2y = 24 \\ 3x - 4y = 2 \end{cases}$$

3. Solve these simultaneous equations by substitution:

(a)
$$\begin{cases} y = 3x - 2 \\ 2x + 5y = 26 \end{cases}$$

(b)
$$\begin{cases} x = 4y + 1 \\ 3x - y = 17 \end{cases}$$

(c)
$$\begin{cases} y = 7 - 3x \\ 2x + 4y = 18 \end{cases}$$

4. Find the graphical solution to these simultaneous equations by finding intersection points:

(a) $y = 3x - 1$ and $y = 7 - 2x$

(b) $y = x^2 - 3$ and $y = 3x + 1$

(c) $x^2 + y^2 = 16$ and $y = 2x - 1$

Section B: Quadratic Equations - Factoring

5. Factorize these quadratic expressions:

(a) $x^2 + 9x + 14$

(b) $x^2 - 7x - 18$

(c) $x^2 - 11x + 28$

(d) $x^2 + 3x - 18$

(e) $x^2 - 36$

(f) $x^2 - 12x + 36$

6. Solve these quadratic equations by factorizing:

(a) $x^2 + 9x + 20 = 0$

(b) $x^2 - 8x - 9 = 0$

(c) $x^2 - 6x = 0$

(d) $x^2 - 49 = 0$

(e) $x^2 + 14x + 49 = 0$

(f) $3x^2 - 12x = 0$

7. Factorize these harder quadratics:

(a) $3x^2 + 8x + 4$

(b) $2x^2 - 9x + 7$

(c) $9x^2 - 16$

(d) $4x^2 + 13x - 12$

(e) $16x^2 - 24x + 9$

(f) $6x^2 - 17x - 3$

8. Solve by factorizing:

(a) $3x^2 + 7x - 6 = 0$

(b) $2x^2 - 9x + 4 = 0$

(c) $9x^2 - 4 = 0$

(d) $4x^2 + 7x - 2 = 0$

Section C: Completing the Square and Quadratic Formula

9. Complete the square for these expressions:

(a) $x^2 + 8x + 7$

(b) $x^2 - 6x + 2$

(c) $x^2 + 2x - 5$

(d) $x^2 - 12x + 9$

(e) $3x^2 + 12x + 5$

(f) $2x^2 - 8x + 3$

10. Solve by completing the square:

(a) $x^2 + 8x + 5 = 0$

(b) $x^2 - 6x - 2 = 0$

(c) $x^2 + 4x + 1 = 0$

(d) $3x^2 + 6x - 2 = 0$

11. Use the quadratic formula to solve (leave in surd form where appropriate):

(a) $x^2 + 5x - 2 = 0$

(b) $3x^2 - 7x + 2 = 0$

(c) $x^2 - 8x + 3 = 0$

(d) $2x^2 + 3x - 4 = 0$

(e) $3x^2 + 8x + 2 = 0$

(f) $4x^2 - 5x - 1 = 0$

12. Find the discriminant and state the nature of the roots:

(a) $x^2 + 7x + 10 = 0$

(b) $x^2 - 6x + 9 = 0$

(c) $x^2 + 3x + 7 = 0$

(d) $3x^2 - 5x + 2 = 0$

Section D: Quadratic Graphs and Applications

13. For the quadratic $y = x^2 - 6x + 8$:

- (a) Find the y-intercept
- (b) Find the x-intercepts by factorizing
- (c) Complete the square to find the vertex
- (d) Sketch the graph
- (e) State the line of symmetry

14. For the quadratic $y = 3x^2 + 6x - 2$:

- (a) Complete the square
- (b) Find the coordinates of the vertex
- (c) Find the y-intercept
- (d) State the line of symmetry
- (e) Sketch the graph

15. A stone is thrown upward. Its height h (in meters) after t seconds is given by: $h = -4t^2 + 16t + 5$

- (a) What is the initial height?
- (b) At what times is the stone at ground level?
- (c) What is the maximum height reached?
- (d) At what time does it reach maximum height?

16. The revenue R (in thousands of pounds) from selling x thousand units is: $R = -3x^2 + 18x - 15$

- (a) How many units should be sold to maximize revenue?
- (b) What is the maximum revenue?
- (c) At what production levels is the revenue zero?

Section E: Linear Inequalities

17. Solve these linear inequalities:

(a) $4x + 7 > 23$

(b) $3x - 8 \leq 13$

(c) $7 - 3x < 4$

(d) $\frac{x+2}{3} \geq 4$

(e) $5 - 4x > 3x + 12$

(f) $\frac{3x-2}{4} < \frac{x+5}{3}$

18. Show these inequalities on number lines:

(a) $x > -3$

(b) $x \leq 5$

(c) $-2 < x \leq 6$

(d) $x < 2$ or $x > 7$

19. Solve these compound inequalities:

(a) $-4 < 3x + 2 < 8$

(b) $5 \leq 2x - 3 \leq 11$

(c) $-3 \leq \frac{2x+1}{3} < 5$

20. Find the integer solutions to:

(a) $3x + 4 > 10$ and $x < 6$

(b) $-2 \leq x + 3 < 5$

(c) $x^2 < 25$

Section F: Quadratic Inequalities

21. Solve these quadratic inequalities:

(a) $x^2 - 7x + 10 > 0$

(b) $x^2 - 8x + 15 \leq 0$

(c) $x^2 - 9 < 0$

(d) $x^2 + 3x - 10 \geq 0$

(e) $x^2 - 8x + 16 > 0$

(f) $3x^2 - 2x - 1 < 0$

22. Solve and show on number lines:

(a) $x^2 - 16 \leq 0$

(b) $x^2 + 2x - 8 > 0$

(c) $2x^2 - 7x + 3 \geq 0$

23. Find the values of x for which:

(a) $x^2 < 4x + 5$

(b) $3x^2 + 2x \geq 1$

(c) $x^2 + 6x + 9 \leq 0$

Section G: Algebraic Manipulation

24. Expand and simplify:

(a) $(x + 5)(3x - 2)$

(b) $(2x - 3)(x + 6)$

(c) $(3x + 4)^2$

(d) $(4x - 5)^2$

(e) $(x + 7)(x - 7)$

(f) $(3x + 4)(3x - 4)$

25. Expand these expressions:

(a) $(x + 3)(x^2 - 2x + 4)$

(b) $(3x - 2)(x^2 + x - 2)$

(c) $(x + 2)^3$

(d) $(3x - 1)^3$

26. Factorize completely:

(a) $8x^2 + 12x$

(b) $9x^2 - 25$

(c) $x^3 - 4x$

(d) $3x^3 + 12x^2 + 12x$

(e) $x^3 - 64$

(f) $27x^3 + 8$

27. Simplify these algebraic fractions:

(a) $\frac{x^2-9}{x+3}$

(b) $\frac{x^2+7x+12}{x+4}$

(c) $\frac{3x^2-12}{x^2-4}$

(d) $\frac{x^3-27}{x^2-9}$

Section H: Algebraic Fractions and Advanced Topics

28. Add and subtract these algebraic fractions:

(a) $\frac{3}{x} + \frac{4}{x}$

(b) $\frac{7}{x} - \frac{3}{x}$

(c) $\frac{2}{3x} + \frac{1}{6x}$

(d) $\frac{3}{x+2} + \frac{2}{x-3}$

(e) $\frac{x}{x+3} - \frac{2}{x-2}$

(f) $\frac{3x}{x^2-4} + \frac{2}{x+2}$

29. Multiply and divide these algebraic fractions:

(a) $\frac{x}{4} \times \frac{8}{x^2}$

(b) $\frac{x+3}{5} \times \frac{10}{x+3}$

(c) $\frac{x^2-9}{x+2} \div \frac{x-3}{x+2}$

(d) $\frac{3x+9}{x^2-16} \times \frac{x-4}{6}$

30. Solve these equations involving algebraic fractions:

(a) $\frac{x}{4} + \frac{x}{3} = 14$

(b) $\frac{3x+2}{5} = \frac{x-1}{2}$

(c) $\frac{4}{x} = \frac{3}{x-2}$

(d) $\frac{x+2}{x-3} = \frac{3x}{x+2}$

31. Make the subject of these formulae:

(a) $s = ut + \frac{1}{2}at^2$, make a the subject

(b) $V = \frac{4}{3}\pi r^3$, make r the subject

(c) $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$, make R_1 the subject

(d) $A = \frac{1}{2}(a+b)h$, make b the subject

Section I: Sequences

32. Find the n th term for these arithmetic sequences:

(a) 9, 14, 19, 24, ...

(b) 5, 11, 17, 23, ...

(c) 30, 26, 22, 18, ...

(d) $\frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, \dots$

33. For these geometric sequences, find the n th term:

(a) 3, 9, 27, 81, ...

(b) 5, 20, 80, 320, ...

(c) 96, 48, 24, 12, ...

(d) 2, -6, 18, -54, ...

34. Find the sum of these series:

(a) First 25 terms of $7 + 11 + 15 + 19 + \dots$

(b) First 8 terms of $4 + 12 + 36 + 108 + \dots$

(c) $2 + 6 + 10 + \dots + 98$ (arithmetic series)

(d) $5 + 10 + 20 + \dots + 320$ (geometric series)

35. These are quadratic sequences. Find the n th term:

(a) 3, 12, 27, 48, 75, ...

(b) 2, 9, 20, 35, 54, ...

(c) 1, 6, 15, 28, 45, ...

(d) 4, 13, 28, 49, 76, ...

36. A sequence is defined by $u_1 = 2$ and $u_{n+1} = 3u_n - 1$.

(a) Find the first 5 terms

(b) Find a formula for u_n

(c) Calculate u_8

Section J: Problem Solving

37. The sum of two numbers is 18 and their product is 77. Find the two numbers.
38. A rectangular field has perimeter 56m. If the length is 8m more than the width, find the dimensions.
39. The difference between a positive number and its reciprocal is $\frac{8}{3}$. Find the number.
40. A rocket's height h (in meters) after t seconds is given by: $h = 80t - 5t^2$
- (a) When does it hit the ground?
 - (b) What is its maximum height?
 - (c) When is it 300m high?
41. Prove that the sum of the first n even numbers is $n(n+1)$.
42. The quadratic $px^2 + qx + r = 0$ has roots α and β .
- (a) Show that $\alpha + \beta = -\frac{q}{p}$
 - (b) Show that $\alpha\beta = \frac{r}{p}$
 - (c) If the roots are 4 and -1, find p , q , and r when $p = 1$
43. A function is defined as $g(x) = x^2 + mx + n$. If $g(2) = 3$ and $g(4) = 15$, find m and n .
44. The sum of the first n terms of a sequence is $S_n = 3n^2 + 2n$. Find the n th term of the sequence.

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 100

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