

A Level Mechanics

Practice Test 2: Elasticity and Springs

Instructions:

Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Draw diagrams where appropriate to illustrate your solutions.
Time allowed: 2 hours

Section A: Hooke's Law Fundamentals [40 marks]

1. [12 marks] Define elasticity and explain Hooke's Law:

- (a) Define elastic deformation and give two examples.
- (b) State Hooke's Law in words and write its mathematical expression.
- (c) Define the spring constant and state its SI unit.
- (d) Explain what is meant by the limit of proportionality.
- (e) Sketch a force-extension graph for an elastic spring, labeling the key regions.
- (f) Explain what happens beyond the elastic limit.

2. [15 marks] A spring has a natural length of 20 cm. When a 5 N force is applied, it extends to 25 cm.

- (a) Calculate the extension of the spring.
- (b) Find the spring constant.
- (c) What force would be needed to extend the spring to 30 cm?
- (d) Calculate the extension when a 12 N force is applied.
- (e) If the spring obeys Hooke's Law up to a maximum extension of 15 cm, what is the maximum force that can be applied?

3. [13 marks] Two springs A and B have spring constants 200 N/m and 300 N/m respectively.

- (a) If both springs are stretched by 8 cm, calculate the force in each spring.
- (b) The springs are connected in series and a 60 N force is applied. Find the extension of each spring.
- (c) Calculate the total extension of the series combination.
- (d) Find the effective spring constant of the series combination.
- (e) The same springs are now connected in parallel. If a 60 N force is applied, find the extension and the effective spring constant.

Section B: Elastic Potential Energy [35 marks]

4. [12 marks] Explain elastic potential energy and its applications:

- (a) Define elastic potential energy.
- (b) Derive the formula for elastic potential energy stored in a spring.
- (c) State the relationship between work done and elastic potential energy.
- (d) Explain how elastic potential energy can be converted to kinetic energy.

5. [15 marks] A spring with spring constant 400 N/m is compressed by 15 cm from its natural length.

- (a) Calculate the force needed to compress the spring by this amount.
- (b) Find the elastic potential energy stored in the compressed spring.
- (c) The spring is released and pushes a 2 kg mass along a smooth horizontal surface. Calculate the velocity of the mass when the spring returns to its natural length.
- (d) If the surface has friction with coefficient 0.3, find the velocity of the mass when the spring reaches its natural length.
- (e) How far will the mass slide on the rough surface before coming to rest?

6. [8 marks] A bungee jumper of mass 70 kg jumps from a height of 50 m. The bungee cord has a spring constant of 100 N/m and natural length 20 m.

- (a) Calculate the extension of the cord at the lowest point of the jump.
- (b) Find the elastic potential energy stored in the cord at this point.
- (c) Calculate the velocity of the jumper when the cord just becomes taut.
- (d) Verify energy conservation throughout the jump.

Section C: Stress, Strain and Young's Modulus [45 marks]

7. [15 marks] Define stress, strain and Young's modulus:

- (a) Define stress and state its SI unit.
- (b) Define strain and explain why it has no units.
- (c) Write the equation linking stress and strain.
- (d) Define Young's modulus and state its SI unit.
- (e) Give the equation for Young's modulus in terms of force, area, length and extension.

8. [20 marks] A steel wire has length 2.0 m, diameter 1.2 mm and Young's modulus 2.0×10^{11} Pa.

- (a) Calculate the cross-sectional area of the wire.
- (b) When a 150 N force is applied, find the stress in the wire.
- (c) Calculate the strain in the wire.
- (d) Find the extension of the wire.

- (e) Calculate the elastic potential energy stored in the wire.
- (f) If the diameter of the wire is doubled while keeping all other factors the same, calculate the new extension.
- (g) If the length is doubled instead of the diameter, find the new extension.

9. [10 marks] The following data was obtained from a test on a rubber band:

Force (N)	Extension (mm)
0	0
2	5
4	10
6	15
8	20
10	25

- (a) Plot a graph of force against extension.
- (b) Calculate the spring constant of the rubber band.
- (c) Determine the elastic potential energy stored when the extension is 20 mm.
- (d) If the rubber band has cross-sectional area 4 mm^2 and original length 50 mm, calculate Young's modulus.

Section D: Material Testing and Properties [30 marks]

10. [12 marks] Compare the properties of different materials:

- (a) Explain why steel is used for construction rather than rubber.
- (b) Compare the Young's modulus values for steel (200 GPa) and rubber (0.01 GPa). What does this tell us about their properties?
- (c) Explain why copper is used for electrical wires rather than steel.
- (d) Give three factors that could affect the Young's modulus of a material.

11. [18 marks] A copper wire and an aluminum wire have the same length (3.0 m) and diameter (2.0 mm). Young's modulus for copper is $1.2 \times 10^{11} \text{ Pa}$ and for aluminum is $7.0 \times 10^{10} \text{ Pa}$.

- (a) Calculate the cross-sectional area of each wire.
- (b) If both wires support the same 100 N load, calculate the stress in each wire.
- (c) Find the strain in each wire.
- (d) Calculate the extension of each wire.
- (e) Which wire is stiffer and by what factor?
- (f) Calculate the elastic potential energy stored in each wire.
- (g) If both wires must not extend by more than 2 mm, what is the maximum load each can support?

Section E: Simple Harmonic Motion with Springs [35 marks]

12. [15 marks] A 0.5 kg mass is attached to a spring of spring constant 200 N/m and undergoes simple harmonic motion.

- (a) Calculate the period of oscillation.
- (b) Find the frequency of oscillation.
- (c) If the amplitude is 6 cm, calculate the maximum velocity.
- (d) Find the maximum acceleration.
- (e) Calculate the total energy of the oscillating system.

13. [20 marks] A vertical spring has spring constant 150 N/m. A 2 kg mass is attached and allowed to reach equilibrium.

- (a) Calculate the extension of the spring at equilibrium.
- (b) The mass is then pulled down a further 8 cm and released. Find the period of oscillation.
- (c) Calculate the amplitude of oscillation.
- (d) Find the maximum velocity during oscillation.
- (e) At what position(s) is the velocity maximum?
- (f) Calculate the elastic potential energy when the mass is at its lowest point.
- (g) Find the kinetic energy when the mass passes through the equilibrium position.
- (h) Verify that total energy is conserved during the motion.

Section F: Combined Applications [25 marks]

14. [15 marks] A spring-powered toy car has a spring with constant 800 N/m. The spring is compressed by 10 cm and then released to propel a 0.3 kg car.

- (a) Calculate the elastic potential energy stored in the compressed spring.
- (b) Find the maximum velocity of the car if there is no friction.
- (c) If the car travels on a rough surface with coefficient of friction 0.2, calculate the maximum velocity.
- (d) How far will the car travel before coming to rest on the rough surface?
- (e) What compression would be needed for the car to travel 5 m on the rough surface?

15. [10 marks] A ball of mass 0.2 kg is dropped from a height of 3 m onto a spring of spring constant 2000 N/m.

- (a) Calculate the velocity of the ball just before it hits the spring.
- (b) Find the maximum compression of the spring.
- (c) Calculate the maximum force exerted by the spring.
- (d) At what compression does the ball momentarily stop falling and start to rise?
- (e) Find the height to which the ball rebounds.

Answer Space

Use this space for your working and answers.

Formulae and Constants**Hooke's Law and Springs:**

Force: $F = kx$ where k is spring constant and x is extension

Series springs: $\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2}$

Parallel springs: $k_{eff} = k_1 + k_2$

Elastic Energy:

Elastic potential energy: $E_p = \frac{1}{2}kx^2 = \frac{1}{2}Fx$

Stress and Strain:

Stress: $\sigma = \frac{F}{A}$ (force per unit area)

Strain: $\epsilon = \frac{x}{L}$ (extension per unit length)

Young's modulus: $E = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{x/L} = \frac{FL}{Ax}$

Simple Harmonic Motion:

Period: $T = 2\pi\sqrt{\frac{m}{k}}$

Frequency: $f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

Maximum velocity: $v_{max} = \omega A = A\sqrt{\frac{k}{m}}$

Maximum acceleration: $a_{max} = \omega^2 A = A\frac{k}{m}$

Energy and Motion:

Kinetic energy: $KE = \frac{1}{2}mv^2$

Gravitational potential energy: $PE = mgh$

Work done against friction: $W = \mu mgd$

Constants:

Acceleration due to gravity: $g = 9.8 \text{ m/s}^2$

$\pi = 3.14159\dots$

END OF TEST

Total marks: 210

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