A Level Pure Mathematics
Practice Test 6: Coordinate Geometry in the (x, y) Plane

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Time allowed: 2 hours

Section A: Distance and Midpoint Formulas

1. Find the distance between these pairs of points:
   (a) $A(3, 8)$ and $B(9, 0)$
   (b) $C(-4, 5)$ and $D(8, -3)$
   (c) $E(-7, -2)$ and $F(1, 6)$
   (d) $G(2t, 5t)$ and $H(-3t, t)$

2. Find the midpoint of the line segment joining:
   (a) $P(7, 11)$ and $Q(3, 5)$
   (b) $R(-6, 3)$ and $S(4, -9)$
   (c) $T(4u, 7u)$ and $U(-2u, u)$
   (d) The point $(6v, -3v)$ and $(-2v, 9v)$

3. The point $N(3, 4)$ is the midpoint of the line segment CD where $C(-1, 8)$.
   (a) Find the coordinates of point D
   (b) Find the length of the perpendicular from N to the y-axis
   (c) Calculate the area of triangle OCD where O is the origin

4. Points $A(2, 7)$, $B(8, 3)$, and $C(4, 11)$ form a triangle.
   (a) Prove that triangle ABC is scalene
   (b) Find the coordinates of the centroid G
   (c) Calculate the distance from each vertex to the centroid
   (d) Verify that the centroid divides each median in the ratio $2 : 1$

5. The points $P(-2, 3)$, $Q(6, 7)$, $R(8, 15)$, and $S(0, 11)$ form a quadrilateral.
   (a) Show that $PQRS$ is a parallelogram
   (b) Find the coordinates of the point of intersection of the diagonals
   (c) Calculate the area using the shoelace formula
   (d) Determine if the parallelogram is a rectangle

Section B: Equations of Straight Lines

6. Find the equation of the straight line:
   (a) With gradient $\frac{5}{4}$ passing through $(8, -3)$
   (b) Passing through $(-2, 9)$ and $(5, -5)$
   (c) With x-intercept 4 and y-intercept -6
   (d) Perpendicular to $3x + 5y = 15$ and passing through $(2, 7)$

7. Express these equations in parametric form $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + t\begin{pmatrix} a \\ b \end{pmatrix}$:
   (a) $3x - 4y = 12$
   (b) $y = 5x - 7$
   (c) $2x + 3y = 18$
   (d) $4x - y + 8 = 0$

8. Find the equation of the line that:
   (a) Is parallel to $3x - 7y = 21$ and is at distance 2 from it
   (b) Is the perpendicular bisector of the segment joining $(4, 9)$ and $(10, 1)$
   (c) Passes through $(5, 2)$ and makes a $45°$ angle with the line $y = 3x$
   (d) Has equal perpendicular distances from $(2, 5)$ and $(-4, 1)$

9. Four lines form a quadrilateral: $L_1 : 2x + y = 8$, $L_2 : x - 3y = -6$, $L_3 : 2x - y = 2$, and $L_4 : 3x + y = 12$.
   (a) Find all four vertices of the quadrilateral
   (b) Calculate the area of the quadrilateral
   (c) Determine if any sides are parallel
   (d) Find the equations of the diagonals

10. A triangle has vertices at $A(1, 5)$, $B(7, 2)$, and $C(3, 8)$.
    (a) Find the equation of the perpendicular bisector of side AB
    (b) Calculate the coordinates of the circumcenter
    (c) Find the equation of the circumcircle
    (d) Verify that all three vertices lie on the circumcircle

Section C: Angle Between Lines

11. Calculate the acute angle between these pairs of lines:
    (a) $y = \frac{3}{4}x + 1$ and $y = -\frac{4}{3}x - 2$
    (b) $3x + 4y = 12$ and $4x - 3y = 8$
    (c) $5x - 2y + 3 = 0$ and $2x + 5y - 7 = 0$
    (d) $y = \tan 37.5° \cdot x + 2$ and $y = \tan 127.5° \cdot x - 1$

12. A line passes through $(4, 3)$ and makes an angle of $150°$ with the positive x-axis.
    (a) Find the equation of the line
    (b) Find where this line intersects $3x - 4y = 0$

(c) Calculate the angle between the line and $3x - 4y = 0$

13. Two lines intersect at $(2, 6)$ at an angle of $120°$. If one line has gradient $\frac{3}{4}$:

(a) Find the two possible gradients for the second line

(b) Write the equations of both possible second lines

(c) Determine the angle each makes with the positive x-axis

14. Find the equations of the lines through $(3, 2)$ that make an angle of $45°$ with the line $3x - y = 7$.

(a) Find the gradient of the given line

(b) Apply the tangent addition formula

(c) Solve for both possible gradients

(d) Write the equations and verify using dot product method

Section D: Equation of a Circle

15. Write the equation of the circle with:

(a) Center $(0, 0)$ and radius $4\sqrt{3}$

(b) Center $(5, -7)$ and radius $3\sqrt{2}$

(c) Center $(-3, 6)$ and passing through $(5, 0)$

(d) Diameter with endpoints $(7, 4)$ and $(-1, 10)$

16. Express these equations in standard form and find the center and radius:

(a) $x^2 + y^2 - 12x + 16y + 75 = 0$

(b) $x^2 + y^2 + 8x - 6y - 11 = 0$

(c) $x^2 + y^2 - 10x + 4y + 13 = 0$

(d) $4x^2 + 4y^2 - 16x + 24y - 36 = 0$

17. A circle has center $(3, -4)$ and is tangent to both coordinate axes.

(a) Find the two possible radii

(b) Write the equations of both possible circles

(c) Find the points of tangency for each circle

(d) Determine which quadrants each circle lies in

18. Three circles: $C_1 : x^2 + y^2 = 36$, $C_2 : (x - 8)^2 + y^2 = 16$, and $C_3 : (x - 4)^2 + (y - 3)^2 = 4$:

(a) Determine the relationship between each pair of circles

(b) Find external common tangents to C_1 and C_2

(c) Calculate the length of external common tangent

(d) Find the radical axis of C_1 and C_2

19. Find the equation of the circle passing through $(2, 6)$, $(6, 4)$, and $(8, 8)$.

(a) Use the determinant method

(b) Verify the solution using perpendicular bisectors

(c) Calculate the circumradius

(d) Find the power of point $(0, 0)$ with respect to this circle

Section E: Parabolas

20. For parabolas with different parameters:
    (a) Find the focus and directrix of $(y-2)^2 = 16(x+3)$
    (b) Find the focus and directrix of $(x+1)^2 = -12(y-4)$
    (c) Find the equation with vertex at $(3,-2)$ and focus at $(6,-2)$
    (d) Sketch $(y+2)^2 = -12(x-1)$ showing focus and directrix

21. A parabola has vertex at $(2,4)$ and passes through $(6,8)$.
    (a) Find the equation if the axis is horizontal
    (b) Find the equation if the axis is vertical
    (c) Determine which orientation fits the given points
    (d) Find the focus and directrix for the correct orientation

22. The parabola $y = ax^2 + bx + c$ has vertex at $(4,-3)$ and passes through $(2,5)$.
    (a) Express in vertex form and find a
    (b) Use the point $(2,5)$ to find the complete equation
    (c) Find the focus and directrix
    (d) Calculate the length of the latus rectum

23. A parabolic satellite dish has equation $x^2 = 24y$ where measurements are in centimeters.
    (a) Find the focus coordinates
    (b) If the dish opening is 80 cm in diameter, find its depth
    (c) Where should the receiver be positioned?
    (d) Find the equation of the tangent at point $(12,6)$

Section F: Ellipses

24. For the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$:
    (a) When $a = 12$ and $b = 9$, find the foci and eccentricity
    (b) If the eccentricity is $\frac{3}{5}$ and $a = 10$, find b
    (c) Find the equation if foci are at $(0, \pm 6)$ and vertices at $(0, \pm 8)$
    (d) Sketch $\frac{x^2}{64} + \frac{y^2}{36} = 1$

25. An ellipse has center at the origin, major axis of length 16, and minor axis of length 10.
    (a) Write the equation with major axis along x-axis
    (b) Write the equation with major axis along y-axis
    (c) Find the eccentricity for both cases
    (d) Calculate the area for both orientations

26. The ellipse $\frac{(x-3)^2}{81} + \frac{(y+2)^2}{49} = 1$ has center at $(3,-2)$.
    (a) Find all four vertices
    (b) Calculate the foci coordinates
    (c) Find the equations of the directrices
    (d) Calculate the length of focal radii for point $(6,3)$

27. An ellipse has vertices at $(2, 5)$ and $(2, 13)$ and one focus at $(2, 7)$.

    (a) Find the center and the value of a

    (b) Calculate the value of c and hence b

    (c) Write the equation of the ellipse

    (d) Find the second focus and both directrices

Section G: Hyperbolas

28. For the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:

    (a) When $a = 6$ and $b = 8$, find the foci and eccentricity

    (b) Find the asymptotes when $a = 5$ and $b = 12$

    (c) If the asymptotes are $y = \pm\frac{3}{4}x$ and $a = 8$, find b

    (d) Sketch $\frac{x^2}{25} - \frac{y^2}{16} = 1$

29. A hyperbola has equation $\frac{y^2}{36} - \frac{x^2}{64} = 1$.

    (a) Identify the orientation and transverse axis

    (b) Find the vertices and foci

    (c) Write the asymptote equations

    (d) Calculate the eccentricity and latus rectum

30. For the rectangular hyperbola $xy = k$:

    (a) When $k = 36$, find intersections with circle $x^2 + y^2 = 20$

    (b) Find both tangents to $xy = 32$ from external point $(8, 4)$

    (c) Prove that the area of triangle formed by tangent at $(t, \frac{k}{t})$ and coordinate axes is $2k$

    (d) Find the locus of midpoints of chords of $xy = 25$ that pass through $(5, 5)$

31. A hyperbola has center at $(1, 4)$, one vertex at $(5, 4)$, and passes through $(7, 8)$.

    (a) Find the value of a

    (b) Use the point $(7, 8)$ to find b

    (c) Write the complete equation

    (d) Find the asymptote equations and foci

Section H: Mixed Conic Sections

32. Analyze and classify these conic sections:

    (a) $49x^2 + 36y^2 = 1764$

    (b) $25x^2 - 16y^2 = 400$

    (c) $(x - 2)^2 = 16(y + 1)$

    (d) $x^2 + y^2 - 6x + 8y + 9 = 0$

33. For conics with cross terms:

    (a) Classify: $x^2 + 4xy + 4y^2 - 8x - 16y + 16 = 0$

    (b) Classify: $9x^2 + 12xy + 4y^2 - 15x - 15y - 10 = 0$

    (c) Find rotation angle for: $3x^2 + 10xy + 3y^2 - 12x - 12y + 6 = 0$

(d) Apply rotation $\theta = 30°$ to: $3x^2 + 2\sqrt{3}xy + y^2 - 12 = 0$

34. Solve these intersection problems:

(a) Line $4x - 3y = 12$ and circle $x^2 + y^2 = 25$
(b) Line $y = 6$ and parabola $x^2 = 12y$
(c) Ellipse $\frac{x^2}{49} + \frac{y^2}{25} = 1$ and hyperbola $\frac{x^2}{16} - \frac{y^2}{25} = 1$
(d) Circle $(x-2)^2 + (y-3)^2 = 10$ and rectangular hyperbola $xy = 8$

35. Find tangent and normal equations:

(a) Tangent to circle $x^2 + y^2 + 4x - 6y - 12 = 0$ at $(2, 6)$
(b) Normal to parabola $(y-3)^2 = 16(x+2)$ at $(2, 11)$
(c) Tangent to ellipse $\frac{x^2}{81} + \frac{y^2}{49} = 1$ at $(9, 0)$
(d) Normal to hyperbola $\frac{x^2}{36} - \frac{y^2}{64} = 1$ at $(10, 8)$

Section I: Applications and Problem Solving

36. A tunnel entrance has a semi-elliptical arch with span 18 meters and maximum height 6 meters.

(a) Find the equation of the ellipse
(b) Calculate clearance heights at 2, 4, and 7 meters from center
(c) A truck convoy with vehicles 3 meters wide and 5.5 meters tall needs to pass through. Assess feasibility
(d) Find the area under the arch

37. A parabolic solar concentrator has diameter 2.4 meters and depth 0.3 meters.

(a) Find the equation of the parabolic reflector
(b) Calculate the focal length
(c) Where should the collector tube be positioned?
(d) If the collector must be 0.4 meters from the vertex, how should the parabola be redesigned?

38. A GPS navigation system uses hyperbolic positioning. Stations A and B are 240 km apart, and a receiver detects signals with time difference 0.0008 seconds.

(a) Calculate the distance difference between receiver and stations
(b) Set up coordinate system and find the hyperbola equation
(c) Find the eccentricity of this navigation hyperbola
(d) If stations C and D provide another hyperbola, describe how position is determined

39. A comet follows an elliptical orbit with perihelion 0.8 AU and aphelion 4.2 AU from the Sun.

(a) Calculate the semi-major and semi-minor axes of the orbit
(b) Find the eccentricity of the orbital ellipse
(c) Determine the focal distance and Sun's position
(d) Calculate the orbital period using Kepler's third law (period squared proportional to semi-major axis cubed)

40. A fountain water jet follows a parabolic path. The jet starts at origin, reaches maximum height 8 meters at horizontal distance 12 meters, and lands back at ground level.

(a) Find the equation of the jet's parabolic path
(b) Calculate the jet height at horizontal distances 3, 6, and 18 meters
(c) Determine the total horizontal range of the jet
(d) Find the angle of the jet path at the starting point

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 150

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