

GCSE Higher Mathematics

Practice Test 4: Ratio, Proportion, and Rates of Change

Instructions:

Answer all questions. Show your working clearly.

Calculators may be used unless stated otherwise.

Time allowed: 90 minutes

Section A: Advanced Ratios

- Express these ratios in their simplest form:
 - $4.2 : 2.8 : 3.5$
 - $\frac{2}{7} : \frac{3}{14} : \frac{5}{21}$
 - $0.45 : 0.18 : 0.27$
 - $3\frac{1}{2} : 1\frac{3}{4} : 2\frac{5}{8}$
- A construction project allocates costs among materials, labor, equipment, and permits in the ratio $9 : 12 : 5 : 2$. If the total project cost is £168,000:
 - How much is allocated to each category?
 - If there's a cost overrun of £42,000 distributed in the same ratio, how much extra does each category receive?
 - What percentage of the total cost is allocated to labor?
- The exterior angles of a regular polygon sum to 360° . If three consecutive exterior angles are in the ratio $7 : 9 : 11$, find each angle and identify the polygon.
- An alloy mixture combines copper, tin, and zinc in the ratio $15 : 4 : 6$ by mass.
 - How much of each metal is needed for 200kg of alloy?
 - If 32kg of tin is used, find the mass of copper and zinc needed
 - What percentage of the alloy is copper?
- Distribute £1575 among three charities so that the first receives £125 more than the second, and the second receives £75 more than the third.

Section B: Direct Proportion

- y is directly proportional to x . When $x = 24$, $y = 16$.
 - Find the equation connecting y and x
 - Calculate y when $x = 36$
 - Calculate x when $y = 28$

- (d) Sketch the graph of y against x
7. S varies directly as the square of T . When $T = 5$, $S = 150$.
- (a) Express S in terms of T
 - (b) Find S when $T = 7$
 - (c) Find T when $S = 294$
 - (d) What happens to S when T is increased by 50%?
8. The cost of printing varies directly as the number of pages. 80 pages cost £12.
- (a) Find the cost per page
 - (b) How much does 135 pages cost?
 - (c) How many pages can be printed for £21?
 - (d) Express the cost C in terms of pages P
9. The gravitational potential energy of an object varies directly as its height above ground. At 15 meters, an object has 450 joules of potential energy.
- (a) Find the potential energy at 25 meters
 - (b) At what height does the object have 720 joules?
 - (c) Write the relationship as an equation

Section C: Inverse Proportion

10. y is inversely proportional to x . When $x = 9$, $y = 16$.
- (a) Find the equation connecting y and x
 - (b) Calculate y when $x = 12$
 - (c) Calculate x when $y = 18$
 - (d) Sketch the graph of y against x
11. The number of days to complete a harvest is inversely proportional to the number of machines used. With 8 machines, it takes 12 days.
- (a) How many days with 16 machines?
 - (b) How many machines are needed to complete it in 8 days?
 - (c) Express days D in terms of machines M
 - (d) If 3 machines break down from the original 8, how many extra days are needed?
12. V is inversely proportional to the square of W . When $W = 5$, $V = 20$.
- (a) Express V in terms of W
 - (b) Find V when $W = 8$
 - (c) Find W when $V = 5$
 - (d) What happens to V when W is reduced by 20%?
13. The magnetic field strength is inversely proportional to the square of the distance from a magnet. At 3 cm, the field strength is 80 tesla.
- (a) Find the field strength at 6 cm
 - (b) At what distance is the field strength 45 tesla?
 - (c) Write the relationship as an equation

Section D: Combined Proportion

14. k varies directly as m and inversely as n . When $m = 15$ and $n = 10$, $k = 18$.
- (a) Express k in terms of m and n
 - (b) Find k when $m = 20$ and $n = 8$
 - (c) Find n when $m = 25$ and $k = 15$
 - (d) What happens to k if m is increased by 20% and n is decreased by 25%?
15. The centripetal force F on an object varies directly as its mass m and as the square of its velocity v , and inversely as the radius r of its path. When $m = 2$ kg, $v = 8$ m/s, and $r = 4$ m, $F = 32$ newtons.
- (a) Express F in terms of m , v , and r
 - (b) Find F when $m = 3$ kg, $v = 12$ m/s, and $r = 6$ m
 - (c) Find r when $F = 50$ newtons, $m = 5$ kg, and $v = 10$ m/s
 - (d) What happens to F if m is doubled, v is halved, and r is tripled?
16. The heat generated H in a resistor varies directly as the square of the current I and as the time t . When $I = 2$ amps and $t = 5$ seconds, $H = 60$ joules.
- (a) Express H in terms of I and t
 - (b) Find H when $I = 3$ amps and $t = 8$ seconds
 - (c) Find t when $H = 180$ joules and $I = 3$ amps
17. a varies directly as b^3 and inversely as c^2 . When $b = 2$ and $c = 3$, $a = 16$.
- (a) Find the constant of proportionality
 - (b) Express a in terms of b and c
 - (c) Calculate a when $b = 3$ and $c = 2$

Section E: Speed, Distance, and Time

18. A ferry travels 156 km in 2 hours 36 minutes. Calculate:
- (a) The average speed in km/h
 - (b) The average speed in m/s
 - (c) How far it travels in 1 hour 55 minutes at this speed
 - (d) How long it takes to travel 195 km at this speed
19. Convert these speeds:
- (a) 18 m/s to km/h
 - (b) 72 km/h to m/s
 - (c) 65 mph to km/h (use 1 mile = 1.6 km)
 - (d) 100 km/h to mph
20. A courier drives from depot to client at 45 km/h and returns at 60 km/h. The total journey takes 3.5 hours.
- (a) Find the distance from depot to client
 - (b) Calculate the average speed for the whole journey
 - (c) How much time was saved on the return journey?

21. Two aircraft start simultaneously from airports 600 km apart and fly towards each other. One flies at 320 km/h and the other at 280 km/h.
- (a) When do they meet?
 - (b) How far from each starting airport do they meet?
 - (c) What is their combined approach speed?
22. A triathlete completes a 20 km cycling section. For the first 12 km, they cycle at 24 km/h. For the remaining 8 km, they increase speed to 32 km/h.
- (a) Calculate the total time taken
 - (b) Find the average speed for the whole cycling section
 - (c) How much slower would they need to cycle the second part to achieve an overall average of 25 km/h?

Section F: Density and Flow Rates

23. A ceramic tile has volume 360 cm^3 and mass 0.972 kg.
- (a) Calculate its density in g/cm^3
 - (b) Calculate its density in kg/m^3
 - (c) What mass of this ceramic would have volume 500 cm^3 ?
 - (d) What volume would 1.944 kg of this ceramic occupy?
24. Different substances have these densities:
- Wood (oak): 0.75 g/cm^3
 - Plastic (PVC): 1.4 g/cm^3
 - Steel: 7.8 g/cm^3
- (a) Compare the masses of 200 cm^3 of each material
 - (b) What volume of steel has the same mass as 150 cm^3 of plastic?
 - (c) A component weighs 546 g and has volume 70 cm^3 . Which material is it most likely made from?
25. Liquid flows through a valve at a rate of 4.8 liters per minute.
- (a) How much liquid flows in 15 minutes?
 - (b) How long to fill a 216-liter container?
 - (c) Express the flow rate in cm^3 per second
 - (d) If the valve opening is reduced by half, what happens to the flow velocity?
26. A bottling plant operates at different rates during shifts:
- Day shift (8 hours): 720 bottles/hour
 - Evening shift (6 hours): 600 bottles/hour
 - Night shift (4 hours): 480 bottles/hour
- (a) Calculate total daily production
 - (b) Find the average production rate
 - (c) How long to produce 3000 bottles at the average rate?

Section G: Scale Factors and Similar Shapes

27. Two similar octagons have corresponding sides in the ratio 3:7.
- (a) If the smaller octagon has area 54 cm^2 , find the area of the larger octagon
 - (b) If the larger octagon has perimeter 84 cm, find the perimeter of the smaller octagon
 - (c) Find the ratio of their areas
28. A scale model of a bridge is built to scale 1:120. The real bridge is 480 m long and 36 m wide.
- (a) Find the dimensions of the model
 - (b) If the model requires 0.6 m^2 of material, how much material does the real bridge need?
 - (c) The model weighs 2.5 kg. Estimate the mass of the real bridge if made from the same material
29. A city map has scale 1:15000. Two buildings are 18.9 cm apart on the map.
- (a) Calculate the actual distance in km
 - (b) What map distance represents 1.8 km?
 - (c) A shopping center covers 6.75 cm^2 on the map. Find its actual area in hectares
30. Two similar prisms have corresponding edge ratio 4:9.
- (a) Find the ratio of their surface areas
 - (b) Find the ratio of their volumes
 - (c) If the smaller prism has volume 512 cm^3 , what is the volume of the larger prism?
 - (d) If the larger prism has surface area 729 cm^2 , what is the surface area of the smaller prism?

Section H: Advanced Rate Problems

31. A radioactive substance decays at 12% per hour. Starting with 800 grams:
- (a) Write an expression for mass after t hours
 - (b) What mass remains after 6 hours?
 - (c) When will the mass drop below 200 grams?
 - (d) What is the rate of decay after 4 hours (grams per hour)?
32. A storage tank is filled by three pumps and emptied by two drains. Pump A delivers 30 liters/minute, pump B delivers 25 liters/minute, pump C delivers 20 liters/minute. Drain X removes 15 liters/minute and drain Y removes 18 liters/minute.
- (a) What is the net rate of filling when all systems operate?
 - (b) How long to fill a 2520-liter tank from empty?
 - (c) If pump C fails, what is the new net rate?
 - (d) What rate would a third drain need to exactly balance all three pumps?
33. Currency exchange rates:
- $\text{£}1 = \$1.31$
 - $\text{£}1 = \text{€}1.17$
 - $\text{\$}1 = \text{¥}112$
- (a) Convert $\text{£}320$ to dollars

- (b) Convert €156 to pounds
- (c) Convert \$168 to yen
- (d) Find the exchange rate from euros to dollars

34. A robotic assembly line produces components at variable efficiency:

- Startup: 1 hour at 30% efficiency (full efficiency is 150 components/hour)
 - Peak production: 6 hours at full efficiency
 - Maintenance: 1 hour at 20% efficiency
- (a) Calculate total components produced
 - (b) Find the average production rate over the whole cycle
 - (c) How long would it take to produce the same quantity at 90% efficiency?

Section I: Problem Solving and Applications

35. A smoothie recipe for 4 servings uses:

- 240ml orange juice
 - 160ml yogurt
 - 2 bananas
 - 80ml honey
- (a) Adapt the recipe for 9 servings
 - (b) How much of each ingredient for 6 servings?
 - (c) If you have 600ml orange juice, what's the maximum number of servings you can make?

36. A university has 3600 students. The ratio of undergraduate to postgraduate to research students is 12:4:1. Due to new admissions, 180 undergraduate students join but no other categories change.

- (a) How many students of each type were there originally?
- (b) What is the new ratio of undergraduate to postgraduate to research students?
- (c) How many postgraduate students would need to join to restore the original ratio?

37. The air resistance on a cyclist varies directly as the square of their speed for speeds above 15 km/h. At 30 km/h, the air resistance is 36 newtons.

- (a) Find the air resistance at 45 km/h
- (b) At what speed is the air resistance 64 newtons?
- (c) Compare the air resistance at 25 km/h and 40 km/h

38. A logistics company's fuel cost F (in hundreds of pounds) varies as the square of the number of vehicles V up to 20 vehicles, then increases linearly. When $V = 8$, $F = 32$. When $V = 25$, $F = 120$.

- (a) Find the cost function for $V \leq 20$
- (b) Find the cost function for $V > 20$
- (c) What number of vehicles gives the lowest cost per vehicle for $V \leq 20$?
- (d) What is the additional cost for each vehicle beyond 20?

39. A clock mechanism has a gear system with five gears. Gear A has 16 teeth and rotates at 360 rpm. Gear B has 32 teeth, gear C has 12 teeth, gear D has 40 teeth, and gear E has 15 teeth. The gears are connected in sequence $A \rightarrow B \rightarrow C \rightarrow D \rightarrow E$.

- (a) Find the rotation speed of each gear
- (b) What is the overall gear ratio from gear A to gear E?
- (c) If gear A's speed decreases to 300 rpm, find the new speed of gear E
- (d) How would the final speed change if gear C had 16 teeth instead?

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 100

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