A Level Mechanics
Practice Test 6: Momentum and Impulse

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Draw diagrams where appropriate to illustrate your solutions.
Time allowed: 2 hours 45 minutes

Section A: Ultimate Momentum Theory [45 marks]

Question 1 [30 marks] A sophisticated space laboratory consists of seven modules with the following properties: Module A (mass 8 kg, position $(12, 8, -6)$ m, velocity $(-4, 6, 3)$ m/s), Module B (mass 12 kg, position $(-8, 15, 4)$ m, velocity $(7, -2, 5)$ m/s), Module C (mass 6 kg, position $(20, -12, 10)$ m, velocity $(-3, 8, -4)$ m/s), Module D (mass 15 kg, position $(-6, -8, 12)$ m, velocity $(5, 4, -6)$ m/s), Module E (mass 4 kg, position $(16, 6, -15)$ m, velocity $(-8, -3, 7)$ m/s), Module F (mass 10 kg, position $(-12, 18, 8)$ m, velocity $(6, -5, 2)$ m/s), and Module G (mass 5 kg, position $(8, -16, -4)$ m, velocity $(2, 9, -8)$ m/s).

(a) Calculate the total momentum vector of the space laboratory system. [**6 marks**]

(b) Find the position and velocity vectors of the center of mass. [**8 marks**]

(c) If modules A, C, and E dock together (perfectly inelastic collision), calculate their combined velocity and the new total momentum. [**8 marks**]

(d) Calculate the kinetic energy lost when these three modules dock. [**5 marks**]

(e) Determine the new center of mass velocity after docking. [**3 marks**]

Question 2 [15 marks] An advanced spacecraft of mass 5000 kg needs to perform a precise orbital maneuver. It simultaneously ejects three thruster packages: Package X (mass 400 kg) at velocity $(80, -120, 60)$ m/s relative to spacecraft, Package Y (mass 600 kg) at velocity $(-100, 80, -40)$ m/s relative to spacecraft, and Package Z (mass 500 kg) at velocity $(60, -60, 100)$ m/s relative to spacecraft. Before ejection, the system was moving at $(3000, -1500, 2000)$ m/s relative to Earth.

(a) Calculate the velocity of the spacecraft after all ejections. [**8 marks**]

(b) Find the velocities of all three packages relative to Earth. [**4 marks**]

(c) Calculate the total change in kinetic energy of the system. [**3 marks**]

Section B: Ultimate Collision Complexity [50 marks]

Question 3 [30 marks] A sophisticated collision experiment involves five balls arranged on a frictionless surface. Ball A (mass 2.5 kg, velocity 15 m/s) approaches four stationary balls in a line: Ball B (3.5 kg), Ball C (4.5 kg), Ball D (5.5 kg), and Ball E (6.5 kg). All collisions are elastic and occur with coefficient of restitution 1.0.

(a) Calculate the velocities after the collision between A and B. [**6 marks**]

(b) Find the velocities after the collision between B and C. [**6 marks**]

(c) Calculate the velocities after the collision between C and D. [**6 marks**]

(d) Find the velocities after the collision between D and E. [**6 marks**]

(e) Verify total momentum and kinetic energy conservation throughout the entire sequence. [**6 marks**]

Question 4 [**20 marks**] A complex ballistic pendulum system consists of a wooden block of mass 8 kg suspended by cables of length 3 m. A high-velocity projectile of mass 0.08 kg moving horizontally at 800 m/s embeds in the block. The combined system then swings and, at the bottom of its swing, collides elastically with a steel ball of mass 2 kg suspended by a separate cable.

(a) Calculate the velocity of the block immediately after the projectile embeds. [**4 marks**]

(b) Find the maximum angle the pendulum makes with the vertical. [**5 marks**]

(c) Calculate the velocity of the block at the bottom of its swing. [**4 marks**]

(d) Find the velocities of both the block and steel ball after their elastic collision. [**5 marks**]

(e) Calculate the total energy lost in the projectile embedding process. [**2 marks**]

Section C: Advanced Mathematical Force Analysis [40 marks]

Question 5 [**25 marks**] A particle of mass 1.5 kg experiences a complex three-dimensional force given by: $\vec{F}(t) = (24t - 6t^2 + t^3)\hat{\imath} + (15 - 10t + 2t^2 - 0.5t^3)\hat{\jmath} + (8t^2 - 4t^3 + t^4)\hat{k}$ N for $0 \leq t \leq 5$ seconds. The particle starts from rest.

(a) Calculate the impulse vector delivered during the entire 5-second interval. [**8 marks**]

(b) Find the final velocity vector of the particle. [**4 marks**]

(c) Determine when each component of the force reaches its extremum (maximum or minimum). [**8 marks**]

(d) Calculate the magnitude of the total impulse and final velocity. [**3 marks**]

(e) Find the time when the magnitude of the force is maximum. [**2 marks**]

Question 6 [**15 marks**] A cricket ball of mass 0.156 kg is struck by a bat in a complex collision. The force varies in three phases: Phase 1 ($0 \leq t \leq 0.001$ s): $F = 8000t^2$ N, Phase 2 ($0.001 \leq t \leq 0.003$ s): $F = 8 - 2000t$ N, Phase 3 ($0.003 \leq t \leq 0.004$ s): $F = 2000(0.004 - t)$ N. The ball approaches at 35 m/s.

(a) Calculate the impulse delivered in each phase. [**6 marks**]

(b) Find the total impulse and final velocity of the ball. [**4 marks**]

(c) Determine the maximum force during the collision and when it occurs. [**3 marks**]

(d) Calculate the average force over the entire contact period. [**2 marks**]

Section D: Ultimate Explosion Analysis [35 marks]

Question 7 [35 marks] A sophisticated explosive device of mass 25 kg traveling at velocity $(150, 80, -60)$ m/s explodes into six fragments. Fragment A (3 kg) flies at $(300, 200, 100)$ m/s, Fragment B (4 kg) at $(100, -250, 150)$ m/s, Fragment C (5 kg) at $(-150, 300, -80)$ m/s, Fragment D (6 kg) at $(250, -100, -200)$ m/s, Fragment E (3.5 kg) at $(-200, 150, 250)$ m/s.

(a) Calculate the velocity of Fragment F using momentum conservation. **[8 marks]**

(b) Find the total kinetic energy before and after explosion. **[8 marks]**

(c) Calculate the chemical energy released in the explosion. **[4 marks]**

(d) If the explosion lasted 0.0002 seconds, find the average force on Fragment C during explosion. **[5 marks]**

(e) Determine the impulse vectors experienced by Fragments A and E. **[6 marks]**

(f) Calculate the position of the center of mass 8 seconds after explosion. **[4 marks]**

Section E: Supreme Rocket Dynamics [45 marks]

Question 8 [25 marks] A revolutionary four-stage rocket system has the following specifications: - Stage 1: mass 20,000 kg (including fuel), burns 80 kg/s at 2000 m/s exhaust velocity for 100 seconds - Stage 2: mass 8,000 kg (including fuel), burns 40 kg/s at 2600 m/s exhaust velocity for 120 seconds - Stage 3: mass 3,000 kg (including fuel), burns 15 kg/s at 3200 m/s exhaust velocity for 150 seconds - Stage 4: mass 1,000 kg (including fuel), burns 5 kg/s at 4000 m/s exhaust velocity for 180 seconds

(a) Calculate the thrust force for each stage. **[8 marks]**

(b) Find the velocity increment for each stage (neglect gravity). **[12 marks]**

(c) Calculate the total final velocity of the payload. **[3 marks]**

(d) Determine the total impulse delivered by all four stages. **[2 marks]**

Question 9 [20 marks] An experimental plasma drive spacecraft of mass 3500 kg operates by ejecting plasma at 150,000 m/s relative to the spacecraft. The plasma ejection rate varies according to $\frac{dm}{dt} = 0.02(1 + 0.1\sin(0.01t))$ kg/s where t is in seconds.

(a) Calculate the instantaneous thrust as a function of time. **[4 marks]**

(b) Find the average thrust over the first 100 seconds. **[5 marks]**

(c) Calculate the total impulse delivered in the first 100 seconds. **[4 marks]**

(d) Find the velocity gained in the first 100 seconds (assume constant mass). **[4 marks]**

(e) Calculate the actual velocity accounting for mass change using the rocket equation. **[3 marks]**

Section F: Advanced Fluid Momentum Systems [35 marks]

Question 10 [20 marks] A sophisticated water jet system fires water at a curved blade. The water approaches at 60 m/s horizontally with flow rate 120 kg/s. The blade curves the water through an angle of 150° while reducing its speed to 45 m/s due to friction.

(a) Calculate the initial and final momentum vectors of the water per unit time. **[6 marks]**

(b) Find the rate of change of momentum of the water. **[4 marks]**

(c) Calculate the force components exerted by the water on the blade. [**5 marks**]

(d) Determine the power lost to friction in the water flow. [**3 marks**]

(e) Calculate the impulse delivered to the blade in 45 seconds. [**2 marks**]

Question 11 [**15 marks**] A wind turbine blade system experiences wind flowing at 25 m/s. The air density is 1.2 kg/m^3, and the effective blade area is 150 m^2. The blade system deflects the air downward at 15° while reducing its horizontal speed to 18 m/s.

(a) Calculate the mass flow rate of air through the blade system. [**3 marks**]

(b) Find the change in momentum of the air per unit time. [**6 marks**]

(c) Calculate the force exerted by the air on the blade system. [**3 marks**]

(d) Determine the power extracted by the turbine. [**3 marks**]

Section G: Ultimate System Integration [40 marks]

Question 12 [**40 marks**] An extremely complex collision scenario involves four objects in three-dimensional space. Object A (mass 6 kg) moves at $(10, 8, -6)$ m/s, Object B (mass 8 kg) moves at $(-6, 12, 4)$ m/s, Object C (mass 10 kg) moves at $(4, -8, 10)$ m/s, and Object D (mass 12 kg) moves at $(-8, 6, -12)$ m/s. After a complex collision, Object A moves at $(2, 6, -2)$ m/s, Object B moves at $(-4, 8, 2)$ m/s, and Object C moves at $(6, -4, 8)$ m/s.

(a) Calculate the total momentum vector before collision. [**5 marks**]

(b) Find the velocity vector of Object D after collision using momentum conservation. [**8 marks**]

(c) Calculate the total kinetic energy before and after collision. [**8 marks**]

(d) Determine the total energy lost in the collision. [**4 marks**]

(e) Calculate the impulse vector experienced by each object. [**8 marks**]

(f) If the collision lasted 0.025 seconds, estimate the average force magnitude between the objects. [**4 marks**]

(g) Verify that the center of mass motion is unaffected by internal forces. [**3 marks**]

Physics Data and Formulae

Momentum and Impulse Fundamentals:

Linear momentum: $\vec{p} = m\vec{v}$

Impulse-momentum theorem: $\vec{J} = \int \vec{F}\,dt = \Delta\vec{p}$

Conservation of momentum: $\sum \vec{p}_i = \sum \vec{p}_f$ (isolated system)

Newton's second law: $\vec{F} = \frac{d\vec{p}}{dt}$

Center of Mass Dynamics:

Position: $\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$

Velocity: $\vec{v}_{cm} = \frac{\sum m_i \vec{v}_i}{\sum m_i}$

External force: $\vec{F}_{ext} = M_{total}\vec{a}_{cm}$

Internal forces: $\sum \vec{F}_{internal} = 0$

Collision Mechanics:

Elastic collision (1D): $v_1' = \frac{(m_1-m_2)v_1+2m_2v_2}{m_1+m_2}$

$v_2' = \frac{(m_2-m_1)v_2+2m_1v_1}{m_1+m_2}$

Coefficient of restitution: $e = \frac{v_2'-v_1'}{v_1-v_2}$

Perfectly inelastic: $v_{final} = \frac{m_1v_1+m_2v_2}{m_1+m_2}$

Rocket Propulsion Theory:

Thrust equation: $F = \frac{dm}{dt}v_{rel}$

Tsiolkovsky rocket equation: $\Delta v = v_{rel} \ln\left(\frac{m_i}{m_f}\right)$

For time-varying mass loss: $v(t) = v_0 + v_{rel} \ln\left(\frac{m_0}{m(t)}\right)$

Energy Relations:

Kinetic energy: $E_k = \frac{1}{2}mv^2$

Rotational kinetic energy: $E_r = \frac{1}{2}I\omega^2$

Gravitational potential energy: $E_p = mgh$

Energy in collisions: $\Delta E = E_{initial} - E_{final}$

Vector Mathematics:

Magnitude: $|\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$

Unit vector: $\hat{v} = \frac{\vec{v}}{|\vec{v}|}$

Dot product: $\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z$

Component resolution: $v_x = v\cos\theta_x$

Advanced Calculus:

$\int t^n\,dt = \frac{t^{n+1}}{n+1} + C$

$\int \sin(at)\,dt = -\frac{1}{a}\cos(at) + C$

$\frac{d}{dt}[f(t)g(t)] = f'(t)g(t) + f(t)g'(t)$

Physical Constants:

Acceleration due to gravity: $g = 9.8$ m/s²

Standard air density: $\rho = 1.2$ kg/m³

END OF TEST

Total marks: 290

Grade boundaries: A* 261, A 232, B 203, C 174, D 145, E 116

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