

GCSE Higher Mathematics

Practice Test 5: Statistics

Instructions:

Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Time allowed: 90 minutes

Section A: Advanced Averages and Spread

1. The table shows the distribution of movie ratings:

Rating (out of 10)	Frequency
2.0-2.9	7
3.0-3.9	12
4.0-4.9	18
5.0-5.9	24
6.0-6.9	22
7.0-7.9	16
8.0-8.9	10
9.0-9.9	4

Calculate:

- The total number of movies rated
 - An estimate of the mean rating
 - The modal class
 - An estimate of the median rating
 - An estimate of the range
2. For the data set: 13, 16, 19, 22, 25, 28, 31, 34, 37, 43
- Calculate the mean
 - Find the median and quartiles (Q1 and Q3)
 - Calculate the interquartile range
 - Calculate the standard deviation
 - Identify any outliers using the $1.5 \times \text{IQR}$ rule
3. Two data sets have the following properties:
- Set R: mean = 72, standard deviation = 13, $n = 24$
 - Set S: mean = 68, standard deviation = 18, $n = 36$
- Calculate the combined mean

- (b) Calculate the combined standard deviation
 - (c) Which set is more consistent? Explain.
 - (d) Calculate the coefficient of variation for each set
4. The distances (in km) traveled by 42 delivery vehicles are summarized:

$$\sum x = 5040, \sum x^2 = 615600$$

- (a) Calculate the mean distance
- (b) Calculate the variance
- (c) Calculate the standard deviation
- (d) If each vehicle travels an additional 8 km, find the new mean and standard deviation

Section B: Histograms and Frequency Density

5. The histogram shows the distribution of website loading times:

[Imagine a histogram with: 0-1 sec (density 5.4), 1-2 sec (density 3.8), 2-3 sec (density 2.6), 3-5 sec (density 1.2), 5-8 sec (density 0.7)]

- (a) Complete the frequency table
 - (b) Calculate the total number of websites tested
 - (c) Estimate the mean loading time
 - (d) Find the modal class
 - (e) What percentage of websites load in under 2 seconds?
6. Draw a histogram for this data about student absences per term:

Absences	Frequency
0-2	36
3-5	42
6-7	18
8-10	21
11-15	20
16-25	15

- (a) Calculate the frequency density for each class
 - (b) Draw the histogram
 - (c) Estimate the median number of absences
 - (d) What fraction of students have more than 7 absences?
7. A histogram shows data with unequal class widths. The class 60-65 has frequency density 18 and the class 65-75 has frequency 120.
- (a) Find the frequency for the 60-65 class
 - (b) Find the frequency density for the 65-75 class
 - (c) If the total frequency is 500, suggest frequencies for other classes

Section C: Cumulative Frequency and Box Plots

8. The table shows the cumulative frequency of rainfall amounts:

Rainfall (mm)	Cumulative Frequency
≤ 5	9
≤ 10	24
≤ 15	42
≤ 20	61
≤ 25	76
≤ 30	87
≤ 35	94
≤ 40	98

- Draw the cumulative frequency curve
- Find the median
- Find the quartiles Q1 and Q3
- Calculate the interquartile range
- Draw a box plot
- Estimate the 95th percentile

9. Two box plots show the distribution of weekly earnings for two professions:

[Imagine box plots: Profession A (min 420, Q1 520, median 620, Q3 720, max 920), Profession B (min 380, Q1 480, median 580, Q3 680, max 880)]

Compare the distributions by commenting on:

- Central tendency (medians)
 - Spread (ranges and IQRs)
 - Shape and outliers
 - Which profession has more variable earnings?
10. The cumulative frequency curve for processing times (in milliseconds) passes through these points: (50, 0), (75, 16), (100, 34), (125, 52), (150, 67), (175, 78), (200, 84)
- Find the median processing time
 - Find the quartiles
 - What percentage have processing times between 90ms and 160ms?
 - Draw the corresponding box plot

Section D: Scatter Graphs and Correlation

11. The table shows data for 10 restaurants:

Tables	12	18	24	30	36	42	48	54	60	66
Weekly revenue (£000s)	8.5	12.2	15.8	19.5	23.1	26.8	30.4	34.1	37.7	41.4

- Plot a scatter graph
- Describe the correlation
- Calculate the equation of the line of best fit
- Use your line to predict revenue for a 45-table restaurant

- (e) Estimate the number of tables for £22,000 weekly revenue
 - (f) Calculate the correlation coefficient
12. The equation of a regression line is $y = -0.8x + 150$.
- (a) Interpret the gradient
 - (b) Interpret the y-intercept
 - (c) If $x = 75$, predict y
 - (d) If $y = 102$, estimate x
 - (e) State assumptions made when using this model
13. Classify these correlation coefficients and describe the relationships:
- (a) $r = 0.91$
 - (b) $r = -0.64$
 - (c) $r = 0.27$
 - (d) $r = -0.89$
 - (e) $r = 0.78$

Section E: Advanced Probability

14. A bag contains 9 silver coins, 7 gold coins, and 6 bronze coins. Two coins are drawn without replacement.
- (a) Draw a tree diagram
 - (b) Find $P(\text{both silver})$
 - (c) Find $P(\text{both same metal})$
 - (d) Find $P(\text{at least one gold})$
 - (e) Find $P(\text{different metals})$
15. The probability that a server crashes on any day is 0.06, independently of other days.
- (a) Find the probability it crashes on exactly 2 out of 8 days
 - (b) Find the probability it crashes on at least 3 out of 8 days
 - (c) Find the expected number of crashes in 15 days
 - (d) In a 60-day period, find $P(\text{more than 5 crashes})$
16. A security quiz has 25 yes/no questions. An employee guesses randomly.
- (a) Find $P(\text{correct answer on one question})$
 - (b) Find $P(\text{exactly 15 correct answers})$
 - (c) Find $P(\text{at least 20 correct answers})$
 - (d) Find the expected number of correct answers
 - (e) Find the most likely number of correct answers
17. Events J and K are such that $P(J) = 0.42$, $P(K) = 0.58$, and $P(J \cap K) = 0.21$.
- (a) Find $P(J \cap K)$
 - (b) Find $P(J')$
 - (c) Find $P(J \cup K)$
 - (d) Find $P(K \cap J')$
 - (e) Are J and K independent? Justify your answer

Section F: Conditional Probability and Independence

18. A survey of 250 drivers about vehicle types gives:
- 140 drive petrol cars
 - 90 drive electric cars
 - 45 drive both types
- (a) Draw a Venn diagram
- (b) Find $P(\text{drives petrol} \mid \text{drives electric})$
- (c) Find $P(\text{drives electric} \mid \text{drives petrol})$
- (d) Find $P(\text{drives exactly one type})$
- (e) Are the vehicle types independent? Explain
19. In a textile mill, 75% of fabric is woven by Loom A and 25% by Loom B. Loom A produces 1.5% defective fabric, Loom B produces 4% defective fabric.
- (a) Draw a tree diagram
- (b) Find the probability fabric is defective
- (c) If fabric is defective, find the probability it came from Loom A
- (d) If fabric is not defective, find the probability it came from Loom B
20. A library has 16 hardcover books numbered 1-16 and 12 paperback books numbered 1-12. A book is selected at random.
- (a) Find $P(\text{hardcover and perfect square number})$
- (b) Find $P(\text{paperback} \mid \text{number greater than 10})$
- (c) Find $P(\text{number is prime})$
- (d) Are book format and number type independent?
21. A quality test is 91% accurate for approved items and 96% accurate for rejected items. 8% of items are rejected.
- (a) Find the probability of a failed test result
- (b) If an item fails the test, find the probability it should be rejected
- (c) If an item passes the test, find the probability it should be approved
- (d) Comment on the effectiveness of the quality test

Section G: Hypothesis Testing and Sampling

22. A vending machine is suspected of giving incorrect change. Out of 35 transactions tested, 8 give incorrect change.
- (a) State the null and alternative hypotheses (expected rate is 15%)
- (b) Calculate the probability of getting 8 or more incorrect changes if the rate is 15%
- (c) At the 5% significance level, is there evidence the machine has problems?
- (d) What would be a Type I error in this context?
23. A sample of 38 car tires has mean tread depth 6.8mm and standard deviation 1.4mm.
- (a) Calculate a 95% confidence interval for the population mean
- (b) Interpret your confidence interval

- (c) What assumptions are made?
- (d) How would the interval change with a larger standard deviation?
24. A hotel claims 92% guest satisfaction. In a sample of 150 guests, 134 report satisfaction.
- (a) Test at 5% level whether the claim is justified
- (b) Calculate the critical value
- (c) State your conclusion
- (d) What is the p-value for this test?

Section H: Problem Solving and Integration

25. A library records book checkout durations over 200 transactions. The data shows:
- Mean = 14 days
 - Standard deviation = 5 days
 - Distribution is approximately normal
- (a) Find $P(\text{checkout duration} < 22 \text{ days})$
- (b) Find the duration exceeded by only 12% of checkouts
- (c) What percentage of books are returned between 8 and 18 days?
- (d) If the library processes 500 checkouts weekly, estimate overdue books (assuming overdue after 21 days)
26. A packaging line samples 8 items every 45 minutes. Over 12 sampling periods, the number of damaged items found was: 0, 1, 2, 0, 1, 3, 1, 0, 2, 1, 0, 1.
- (a) Calculate the mean and standard deviation
- (b) Test whether the damage rate exceeds 10%
- (c) Create a control chart with warning limits
- (d) Comment on process stability
27. Compare these three savings account options over 5 years:
- Account A: Mean return 3.5%, standard deviation 1.2%
 - Account B: Mean return 5.8%, standard deviation 2.8%
 - Account C: Mean return 2.1%, standard deviation 0.8%
- (a) Calculate the coefficient of variation for each
- (b) Which offers the best risk-adjusted return?
- (c) Using normal distribution, find $P(\text{negative return})$ for each account
- (d) Recommend an option for a very conservative saver
28. A fitness center studies the relationship between workout frequency and weight loss. The correlation is 0.73.
- (a) What does this correlation suggest?
- (b) If workout frequency has mean 4.2 sessions/week and standard deviation 1.8, and weight loss has mean 2.8 kg/month and standard deviation 1.5, find the regression equation
- (c) Predict weight loss for someone working out 6 times per week
- (d) Calculate the coefficient of determination and interpret it

29. Design a statistical investigation to test whether a new medication reduces recovery time:
- (a) State hypotheses
 - (b) Describe the sampling method
 - (c) Identify variables and potential confounding factors
 - (d) Outline the analysis plan
 - (e) Discuss limitations and assumptions

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 100

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