A Level Mechanics
Practice Test 5: Circular Motion

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Draw diagrams where appropriate to illustrate your solutions.
Time allowed: 3 hours

Section A: Circular Motion Fundamentals [29 marks]

Question 1 [11 marks]

(a) Define radian measure and explain why it is the natural unit for angular measurement in physics. **[3 marks]**

(b) Derive the relationship between arc length, radius, and angular displacement: $s = r\theta$. **[3 marks]**

(c) Explain the vector nature of angular velocity and how its direction relates to the plane of rotation. **[3 marks]**

(d) For variable circular motion, write expressions for both radial and tangential components of acceleration. **[2 marks]**

Question 2 [14 marks] A wind turbine blade of length 45 m accelerates from rest according to $\omega(t) = 0.8t - 0.02t^2$ rad/s for the first 20 seconds, then rotates at constant angular velocity.

(a) Find the angular acceleration during the variable phase. **[3 marks]**

(b) Calculate the maximum angular velocity reached and the time at which it occurs. **[4 marks]**

(c) Find the total angular displacement during the first 20 seconds. **[3 marks]**

(d) Calculate the linear speed of the blade tip at maximum angular velocity. **[2 marks]**

(e) Find the centripetal acceleration at the blade tip when at maximum speed. **[2 marks]**

Question 3 [4 marks] A particle moves in a circle of radius 2.8 m with angular position given by $\theta = 5t^2 - 2t^3$ radians.

(a) Find the angular velocity and acceleration at $t = 1$ second. **[3 marks]**

(b) Determine when the particle momentarily stops. **[1 mark]**

Section B: Centripetal Force [31 marks]

Question 4 [13 marks]

(a) Explain why centripetal acceleration is always directed toward the center, even when speed is constant. [**4 marks**]

(b) Derive the alternative form of centripetal force: $F_c = \frac{mv^2}{r}$ using calculus and vector methods. [**5 marks**]

(c) Explain what happens when insufficient centripetal force is available for the required circular motion. [**2 marks**]

(d) Give examples where the following forces provide centripetal force: electrostatic, normal force, combination of forces. [**2 marks**]

Question 5 [18 marks] A discus thrower spins a 2 kg discus in a circle of radius 1.5 m, reaching a release speed of 25 m/s.

(a) Calculate the centripetal force required just before release. [**3 marks**]

(b) Find the angular velocity at release. [**2 marks**]

(c) Calculate the centripetal acceleration in terms of g. [**3 marks**]

(d) If the thrower's grip can provide maximum force of 1200 N, find the maximum safe rotational speed. [**3 marks**]

(e) For vertical circular motion at 20 m/s, calculate the grip force at the top and bottom positions. [**4 marks**]

(f) Find the minimum speed at the top to maintain circular motion. [**3 marks**]

Section C: Horizontal Circular Motion [33 marks]

Question 6 [18 marks] A sports car of mass 1400 kg navigates a flat circular section of a test track with radius 140 m.

(a) If the tire-road friction coefficient is 0.9, find the maximum cornering speed. [**4 marks**]

(b) Calculate the maximum lateral acceleration achievable. [**2 marks**]

(c) Find the maximum friction force available for cornering. [**2 marks**]

(d) If the car corners at 26 m/s, calculate the required centripetal force. [**2 marks**]

(e) Determine the friction force utilization as a percentage of maximum available. [**3 marks**]

(f) Calculate the margin of safety at this speed. [**2 marks**]

(g) If racing slick tires increase the friction coefficient to 1.3, find the new maximum speed. [**3 marks**]

Question 7 [15 marks] A figure skater of mass 55 kg performs a spin with arms extended, moving in a circle of radius 0.8 m on ice with coefficient of friction 0.08.

(a) Calculate the maximum centripetal force available from friction. [**2 marks**]

(b) Find the maximum speed for horizontal circular motion. [**3 marks**]

(c) If the skater spins at 4 rad/s, calculate the required centripetal force. [**3 marks**]

(d) Find the linear speed and centripetal acceleration. [**3 marks**]

(e) The skater must lean to provide additional inward force. Calculate the required lean angle. [**4 marks**]

Section D: Banked Curves [28 marks]

Question 8 [12 marks]

(a) Explain how banking transfers some of the centripetal force requirement from friction to the normal force. [**4 marks**]

(b) For a banked curve, derive the speed range when friction acts both up and down the slope. [**8 marks**]

Question 9 [16 marks] A bobsled track section has radius 180 m and is banked at 25°. The coefficient of friction between sled and ice is 0.12.

(a) Calculate the design speed requiring no friction. [**3 marks**]

(b) A bobsled of mass 390 kg (including crew) travels at 38 m/s. Find the centripetal force needed. [**2 marks**]

(c) Calculate the normal force from the banked surface. [**4 marks**]

(d) Determine the friction force required and its direction. [**4 marks**]

(e) Find the maximum and minimum safe speeds on this banked section. [**3 marks**]

Section E: Vertical Circular Motion [31 marks]

Question 10 [13 marks]

(a) For an object moving in a vertical circle, explain how gravitational potential energy affects the motion. [**4 marks**]

(b) Derive the general expression for centripetal force at any point in vertical circular motion. [**5 marks**]

(c) Show that the speed variation in vertical circular motion follows $v^2 = v_0^2 + 2g(h_0 - h)$. [**4 marks**]

Question 11 [18 marks] A motorcycle stunt rider performs a vertical loop-the-loop with radius 12 m. The rider and bike have combined mass 280 kg.

(a) Calculate the minimum speed at the top to complete the loop. [**3 marks**]

(b) Find the minimum speed at the bottom using energy conservation. [**4 marks**]

(c) If the motorcycle enters at 32 m/s, calculate the speed at the top. [**3 marks**]

(d) Find the normal force from the track at the top for this speed. [**3 marks**]

(e) Calculate the normal force at the bottom. [**3 marks**]

(f) Determine the g-forces experienced at both positions. [**2 marks**]

Section F: Conical Pendulums [26 marks]

Question 12 [12 marks]

(a) For a conical pendulum, derive the relationship $r = l \sin\theta$ and show how this affects the motion equations. [**4 marks**]

(b) Prove that the period depends only on the vertical component of string length: $T = 2\pi\sqrt{\frac{l\cos\theta}{g}}$. [**8 marks**]

Question 13 [14 marks] A conical pendulum apparatus consists of a 0.6 kg mass suspended by a 2.2 m string, rotating such that the string makes 32° with the vertical.

(a) Calculate the radius of the horizontal circular motion. **[2 marks]**

(b) Find the vertical height below the suspension point. **[2 marks]**

(c) Calculate the tension in the string. **[3 marks]**

(d) Find the angular velocity, linear speed, and period of revolution. **[4 marks]**

(e) Calculate the centripetal acceleration. **[2 marks]**

(f) If the string length is reduced to 1.8 m while maintaining the same period, find the new angle. **[1 mark]**

Section G: Motion in a Vertical Circle - Loops [30 marks]

Question 14 [18 marks] A hollow sphere of mass 1.5 kg and radius 0.06 m rolls without slipping down a track and enters a vertical loop of radius 0.9 m. The moment of inertia is $I = \frac{2}{3}mr^2$.

(a) Find the minimum translational speed at the top to maintain contact. **[3 marks]**

(b) Calculate the minimum translational speed at the bottom considering rolling motion. **[6 marks]**

(c) If the sphere starts from rest at height 2.8 m above the bottom, find its speed entering the loop. **[4 marks]**

(d) Calculate the translational speed at the top of the loop. **[3 marks]**

(e) Find the normal force at the top and bottom positions. **[2 marks]**

Question 15 [12 marks] A pendulum bob swings through a complete vertical circle. The bob has mass 0.8 kg and the string length is 1.8 m.

(a) Find the minimum speed at the bottom to just complete the circle. **[4 marks]**

(b) Calculate the tension in the string at the bottom for this minimum speed. **[3 marks]**

(c) If the bob is given speed 8 m/s at the bottom, find the maximum tension in the string. **[3 marks]**

(d) Calculate the speed and tension when the string is horizontal. **[2 marks]**

Section H: Applications and Problem Solving [30 marks]

Question 16 [16 marks] A rotating space station provides artificial gravity for long-duration missions. The cylindrical habitat has radius 250 m and rotates to simulate 0.9g at the outer rim.

(a) Calculate the required angular velocity. **[3 marks]**

(b) Find the period of rotation. **[2 marks]**

(c) Calculate the linear speed at the outer rim. **[2 marks]**

(d) Find the artificial gravity at radius 200 m from the center. **[3 marks]**

(e) Calculate the artificial gravity at radius 150 m. **[2 marks]**

(f) For an 85 kg astronaut at the rim, find the normal force from the floor. **[2 marks]**

(g) Analyze the gravity gradient across a 3 m laboratory spanning radially. [**2 marks**]

Question 17 [**14 marks**] A carnival ride called "The Rotor" consists of a vertical cylinder of radius 4 m that rotates rapidly. Riders stand against the wall and the floor drops away.

(a) If the coefficient of friction between rider and wall is 0.6, find the minimum angular velocity to prevent sliding. [**5 marks**]

(b) Calculate the minimum linear speed. [**2 marks**]

(c) Find the centripetal acceleration at this minimum speed. [**2 marks**]

(d) If a rider has mass 70 kg, calculate the normal force from the wall. [**2 marks**]

(e) Calculate the friction force supporting the rider. [**2 marks**]

(f) Find the apparent weight of the rider. [**1 mark**]

Section I: Advanced Circular Motion [27 marks]

Question 18 [**14 marks**] A racing circuit has a complex curve consisting of a flat entry section (radius 100 m) leading to a banked exit section (radius 120 m, banking 18°).

(a) For the flat section with friction coefficient 0.8, find the maximum speed. [**3 marks**]

(b) Calculate the design speed for the banked section (no friction). [**3 marks**]

(c) A race car travels at 30 m/s through both sections. Compare the lateral force requirements. [**5 marks**]

(d) Determine which section is more demanding on the tires. [**3 marks**]

Question 19 [**13 marks**] A particle moves on the inside surface of a smooth spherical bowl of radius 2 m. It travels in a horizontal circle at a depth of 0.8 m below the rim.

(a) Calculate the radius of the circular path. [**3 marks**]

(b) Find the angle the bowl surface makes with the horizontal at this level. [**2 marks**]

(c) Derive the equation of motion and find the speed of the particle. [**5 marks**]

(d) Calculate the normal force from the bowl if the particle has mass 0.4 kg. [**2 marks**]

(e) Find the period of circular motion. [**1 mark**]

Section J: Comprehensive Applications [31 marks]

Question 20 [**18 marks**] A large centrifuge used for materials testing has a specimen chamber at the end of a 10 m arm. The device can generate centripetal accelerations up to 15g.

(a) Calculate the maximum angular velocity. [**3 marks**]

(b) Find the maximum linear speed of the specimen chamber. [**2 marks**]

(c) Calculate the minimum period of rotation. [**2 marks**]

(d) If a 2 kg test specimen experiences 12g, find the centripetal force. [**2 marks**]

(e) Calculate the angular velocity for 12g acceleration. [**2 marks**]

(f) If the centrifuge accelerates uniformly from rest to 12g in 60 seconds, find the angular acceleration. [**3 marks**]

(g) Calculate the number of revolutions during this acceleration phase. [**2 marks**]

(h) Find the tangential acceleration of the specimen chamber during startup. [**2 marks**]

Question 21 [**13 marks**] Four particles with masses 1 kg, 2 kg, 3 kg, and 4 kg are arranged at the corners of a square with side length 2.4 m. The system rotates about an axis through the center perpendicular to the plane.

(a) Find the center of mass of the four-particle system. [**4 marks**]

(b) Calculate the distance of each particle from the center of the square. [**2 marks**]

(c) If the system rotates at 3 rad/s, find the centripetal force on each particle. [**4 marks**]

(d) Calculate the total moment of inertia about the center. [**2 marks**]

(e) Find the total kinetic energy of the rotating system. [**1 mark**]

Physics Data and Formulae

Circular Motion:

Angular velocity: $\omega = \frac{v}{r} = \frac{2\pi}{T} = 2\pi f$

Angular acceleration: $\alpha = \frac{d\omega}{dt}$

Arc length: $s = r\theta$ (where θ in radians)

Tangential acceleration: $a_t = r\alpha$

Centripetal acceleration: $a_c = \frac{v^2}{r} = \omega^2 r$

Total acceleration: $a = \sqrt{a_t^2 + a_c^2}$

Centripetal force: $F_c = ma_c = \frac{mv^2}{r} = m\omega^2 r$

Rotational Kinematics:

$\omega = \omega_0 + \alpha t$

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\omega^2 = \omega_0^2 + 2\alpha\theta$

Vertical Circular Motion:

At top: $T + mg = \frac{mv^2}{r}$ or $N + mg = \frac{mv^2}{r}$

At bottom: $T - mg = \frac{mv^2}{r}$ or $N - mg = \frac{mv^2}{r}$

At angle θ from vertical: $T\cos\theta - mg = \frac{mv^2}{r}$

Minimum speed at top: $v_{min} = \sqrt{gr}$

Speed variation: $v^2 = v_0^2 + 2g(h_0 - h)$

Banking:

No friction: $\tan\theta = \frac{v^2}{rg}$

Maximum speed: $v_{max} = \sqrt{rg\frac{\tan\theta+\mu}{1-\mu\tan\theta}}$

Minimum speed: $v_{min} = \sqrt{rg\frac{\tan\theta-\mu}{1+\mu\tan\theta}}$

Conical Pendulum:

$\cos\theta = \frac{g}{\omega^2 l}$, $\tan\theta = \frac{\omega^2 r}{g}$

$T\cos\theta = mg$, $T\sin\theta = m\omega^2 r$

Period: $T = 2\pi\sqrt{\frac{l\cos\theta}{g}}$

Radius: $r = l\sin\theta$

Energy Conservation:

$$\frac{1}{2}mv_1^2 + mgh_1 = \frac{1}{2}mv_2^2 + mgh_2$$

Rolling motion: $KE = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

Rolling constraint: $v = r\omega$

Artificial Gravity:

$$g_{artificial} = \omega^2 r$$

For desired gravity: $\omega = \sqrt{\frac{g_{desired}}{r}}$

Constants:

Acceleration due to gravity: $g = 9.8$ m/s^2

1 radian = 57.3°

Trigonometric Values:

$\sin 18 = 0.309$, $\cos 18 = 0.951$, $\tan 18 = 0.325$

$\sin 25 = 0.423$, $\cos 25 = 0.906$, $\tan 25 = 0.466$

$\sin 32 = 0.530$, $\cos 32 = 0.848$, $\tan 32 = 0.625$

$\sin 45 = 0.707$, $\cos 45 = 0.707$, $\tan 45 = 1.000$

END OF TEST

Total marks: 290

Grade boundaries: A* 261, A 232, B 203, C 174, D 145, E 116

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