A Level Statistics
Practice Test 2: Hypothesis Testing

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Draw diagrams where appropriate to illustrate your solutions.
Time allowed: 3 hours

Section A: Advanced Hypothesis Testing Concepts [25 marks]

1. **[12 marks]** Define and explain advanced hypothesis testing concepts:

(a) Define statistical power and explain its relationship to Type II error.

(b) Explain what factors influence the power of a statistical test.

(c) Define effect size and explain why it's important in hypothesis testing.

(d) Distinguish between statistical significance and practical significance.

(e) Explain the concept of p-hacking and its implications for research.

(f) Describe what multiple testing correction means and why it's necessary.

2. **[8 marks]** Explain decision-making in hypothesis testing:

(a) Describe the relationship between confidence intervals and hypothesis tests.

(b) Explain how sample size affects the reliability of hypothesis test conclusions.

(c) Discuss when to use one-tailed versus two-tailed tests.

(d) Explain the interpretation of p-values in the context of scientific evidence.

3. **[5 marks]** Analyze hypothesis testing assumptions:

(a) List the key assumptions for parametric hypothesis tests.

(b) Explain what happens when normality assumptions are violated.

(c) Describe when non-parametric tests should be used instead.

Section B: Proportion and Binomial Tests [30 marks]

4. **[15 marks]** Explain proportion testing methods:

(a) State when to use exact binomial tests versus normal approximation.

(b) Write the test statistic formula for testing a single proportion.

(c) Explain the conditions required for normal approximation to binomial.

(d) Describe how to test for equality of two proportions.

(e) Explain continuity correction and when to apply it.

(f) Describe Fisher's exact test and its applications.

5. **[15 marks]** A pharmaceutical company claims their new medication is effective for 85

(a) State appropriate null and alternative hypotheses.

(b) Check whether the normal approximation conditions are satisfied.

(c) Calculate the test statistic using normal approximation.

(d) Find the p-value for a two-tailed test.

(e) Make a conclusion at = 0.05 and interpret in context.

(f) Calculate a 95

(g) Explain how the confidence interval relates to your hypothesis test conclusion.

(h) Calculate the exact p-value using the binomial distribution.

(i) Compare the exact and approximate p-values and comment on the difference.

Section C: ANOVA and Multiple Comparisons [35 marks]

6. **[18 marks]** Three different teaching methods are compared using test scores from independent groups:

Method A: 78, 82, 75, 80, 77, 79, 81, 76 (n = 8, $\bar{x_1} = 78.5$, $s_1 = 2.51$) **Method B:** 85, 88, 83, 86, 84, 87, 89, 82 (n = 8, $\bar{x_2} = 85.5$, $s_2 = 2.51$) **Method C:** 80, 83, 78, 81, 79, 82, 84, 77 (n = 8, $\bar{x_3} = 80.5$, $s_3 = 2.51$)

Given: Overall mean = 81.5, SST = 432, SSE = 120

(a) State the hypotheses for a one-way ANOVA test.

(b) Calculate the sum of squares between groups (SSB).

(c) Complete the ANOVA table with degrees of freedom, mean squares, and F-statistic.

(d) Find the critical value for = 0.05.

(e) Make a conclusion about whether the teaching methods differ significantly.

(f) Calculate the p-value for the F-test.

(g) Perform pairwise t-tests between all method pairs.

(h) Apply Bonferroni correction and determine which methods differ significantly.

(i) Calculate Cohen's d for the comparison between Methods A and B.

7. **[17 marks]** A study examines the effect of different exercise programs on weight loss:
Control Group: Mean loss = 2.1 kg, SD = 1.8 kg, n = 15 **Cardio Group:** Mean loss = 4.3 kg, SD = 2.2 kg, n = 18 **Strength Training:** Mean loss = 3.7 kg, SD = 2.0 kg, n = 16 **Combined Program:** Mean loss = 5.8 kg, SD = 2.4 kg, n = 17

(a) Test whether the control group achieves the recommended minimum weight loss of 2.5 kg.

(b) Compare the cardio group with the strength training group using a two-sample t-test.

(c) Calculate the pooled standard deviation for the cardio vs. strength comparison.

(d) Test whether the combined program is significantly better than cardio alone.

(e) Calculate effect sizes (Cohen's d) for all pairwise comparisons with the control group.

(f) Rank the exercise programs by effectiveness and statistical significance.

(g) Discuss the multiple testing problem and suggest appropriate corrections.

(h) Calculate the required sample size for 80

Answer Space
Use this space for your working and answers.

Formulae and Key Concepts

Proportion Tests:
One proportion z-test: $z = \frac{\hat{p}-p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$
Two proportions: $z = \frac{\hat{p_1}-\hat{p_2}}{\sqrt{\hat{p}(1-\hat{p})(\frac{1}{n_1}+\frac{1}{n_2})}}$
Pooled proportion: $\hat{p} = \frac{x_1+x_2}{n_1+n_2}$

Normal Approximation Conditions:
For binomial: $np \geq 5$ and $n(1-p) \geq 5$
Continuity correction: Use 0.5 adjustment for discrete to continuous

ANOVA:
F-statistic: $F = \frac{MSB}{MSE} = \frac{SSB/(k-1)}{SSE/(N-k)}$
Total: $SST = SSB + SSE$
$MSB = \frac{SSB}{k-1}$, $MSE = \frac{SSE}{N-k}$
where k = number of groups, N = total sample size

Multiple Comparisons:
Bonferroni correction: $\alpha_{adj} = \frac{\alpha}{m}$ for m comparisons

Familywise error rate: $\alpha_{FW} = 1 - (1 - \alpha)^m$

Effect Size:
Cohen's d: $d = \frac{|\bar{x_1} - \bar{x_2}|}{s_{pooled}}$
Interpretation: 0.2 (small), 0.5 (medium), 0.8 (large)
Eta squared (^{2}): $\eta^2 = \frac{SSB}{SST}$ (proportion of variance explained)

Power Analysis:
Power = 1 - (probability of detecting true effect)
Factors affecting power: effect size, sample size, level, variability

Confidence Intervals:
Proportion: $\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
Mean difference: $(\bar{x_1} - \bar{x_2}) \pm t_{\alpha/2} \cdot SE_{diff}$

Sample Size (Two means):
$n = \frac{2(z_{\alpha/2}+z_\beta)^2\sigma^2}{(\mu_1-\mu_2)^2}$ per group

Critical Values:
Standard normal: z. = 1.96, z. = 2.58
t-values depend on degrees of freedom
F-values depend on numerator and denominator df

END OF TEST

Total marks: 90

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