

GCSE Higher Mathematics

Practice Test 4: Geometry and Measures

Instructions:

Answer all questions. Show your working clearly.

Calculators may be used unless stated otherwise.

Time allowed: 90 minutes

Section A: Pythagoras' Theorem

1. Use Pythagoras' theorem to find the missing lengths:
 - (a) A right-angled triangle with legs 9 cm and 12 cm. Find the hypotenuse.
 - (b) A right-angled triangle with hypotenuse 29 cm and one leg 21 cm. Find the other leg.
 - (c) A right-angled triangle with legs 13.5 cm and 18 cm. Find the hypotenuse.
 - (d) A right-angled triangle with hypotenuse 34 cm and one leg 30 cm. Find the other leg.
2. Find the distance between these pairs of coordinate points:
 - (a) A(4, 6) and B(12, 21)
 - (b) C(-1, 5) and D(11, -7)
 - (c) E(-8, -4) and F(4, 8)
 - (d) G(0, -3) and H(-5, 9)
3. A rectangle has length 21 cm and diagonal 29 cm. Find:
 - (a) The width of the rectangle
 - (b) The area of the rectangle
 - (c) The perimeter of the rectangle
4. A ladder 10.5 m long leans against a vertical wall. The foot of the ladder is 6.3 m from the wall.
 - (a) How high up the wall does the ladder reach?
 - (b) If the ladder slips so its foot is now 7.5 m from the wall, how far does the top slide down?
5. Determine whether these triangles are right-angled:
 - (a) Sides 7 cm, 24 cm, 25 cm
 - (b) Sides 11 cm, 60 cm, 61 cm
 - (c) Sides 16 cm, 18 cm, 24 cm
 - (d) Sides 21 cm, 28 cm, 35 cm

Section B: Basic Trigonometry

6. In a right-angled triangle with hypotenuse 25 cm and one leg 24 cm:
 - (a) Find the other leg using Pythagoras
 - (b) Calculate $\sin A$, $\cos A$, and $\tan A$ where A is the angle opposite the 24 cm side
 - (c) Find angle A to 1 decimal place
7. Use trigonometry to find the missing sides (give answers to 3 significant figures):
 - (a) Right-angled triangle: angle 53° , adjacent side 11 cm, find opposite side
 - (b) Right-angled triangle: angle 29° , hypotenuse 18 cm, find opposite side
 - (c) Right-angled triangle: angle 64° , opposite side 12 cm, find adjacent side
 - (d) Right-angled triangle: angle 37° , adjacent side 8 cm, find hypotenuse
8. Find the missing angles (to 1 decimal place):
 - (a) Right-angled triangle: opposite 8 cm, adjacent 15 cm
 - (b) Right-angled triangle: opposite 11.7 cm, hypotenuse 16.8 cm
 - (c) Right-angled triangle: adjacent 8.9 cm, hypotenuse 12.3 cm
 - (d) Right-angled triangle: opposite 21 cm, adjacent 13 cm
9. A 21 m ladder makes an angle of 76° with the ground.
 - (a) How high up the wall does it reach?
 - (b) How far is the foot of the ladder from the wall?
 - (c) What angle would the ladder make if its foot was 11 m from the wall?
10. From the top of a 42 m cliff, the angle of depression to a boat is 24° .
 - (a) Draw a diagram
 - (b) Find the horizontal distance from the cliff to the boat
 - (c) Find the direct distance from the top of the cliff to the boat

Section C: Advanced Trigonometry - Sine and Cosine Rules

11. Use the sine rule to find the missing parts:
 - (a) Triangle ABC: $a = 14$ cm, $A = 51^\circ$, $B = 73^\circ$. Find b and c .
 - (b) Triangle PQR: $p = 18$ cm, $q = 22$ cm, $P = 46^\circ$. Find angle Q .
 - (c) Triangle STU: $s = 10.8$ cm, $t = 14.2$ cm, $S = 39^\circ$. Find angle T .
12. Use the cosine rule to find the missing parts:
 - (a) Triangle ABC: $a = 15$ cm, $b = 12$ cm, $c = 8$ cm. Find angle C .
 - (b) Triangle VWX: $v = 17$ cm, $w = 13$ cm, $X = 62^\circ$. Find side x .
 - (c) Triangle YZA: $y = 9.7$ cm, $z = 12.4$ cm, $a = 7.8$ cm. Find the largest angle.
13. A triangle has sides 18 cm, 20 cm, and 22 cm.
 - (a) Use the cosine rule to find the largest angle
 - (b) Use the sine rule to find another angle
 - (c) Find the area using the formula $\text{Area} = \frac{1}{2}ab \sin C$

14. Two ships leave port at the same time. Ship A travels 42 km on a bearing of 075° . Ship B travels 35 km on a bearing of 145° .
- (a) Find the angle between their paths
 - (b) Calculate the distance between the ships
 - (c) Find the bearing of ship B from ship A
15. A parallelogram has sides 14 cm and 20 cm, with an included angle of 74° .
- (a) Find the length of the diagonals
 - (b) Find the area of the parallelogram
 - (c) Find the other angles of the parallelogram

Section D: Circle Theorems

16. Apply circle theorems to find the missing angles:
- (a) Angle at center is 124° . Find the angle at circumference subtending the same arc.
 - (b) Angle at circumference is 34° . Find the angle at center subtending the same arc.
 - (c) Two angles at circumference subtend the same arc. One is 51° . Find the other.
 - (d) Angle in semicircle JKL where JK is diameter. Find angle JLK.
17. In a circle with center O:
- (a) Chord HI subtends angle 64° at circumference point J. Find angle HOI.
 - (b) Tangent UV touches circle at V. If angle $UVO = 41^\circ$, find angle VUO.
 - (c) Two chords PQ and RS intersect at T inside the circle. If angles $PTR = 89^\circ$, find angle PTS.
 - (d) Tangents from external point T touch circle at H and I. If angle $HTI = 58^\circ$, find angle HOI.
18. Prove these circle theorems (provide written proof):
- (a) Angle in semicircle is 90°
 - (b) Angles in same segment are equal
 - (c) Angle at center is twice angle at circumference
 - (d) Tangent is perpendicular to radius at point of contact
19. A circle has center O and radius 14 cm. Chord HI has length 20 cm.
- (a) Find the perpendicular distance from O to chord HI
 - (b) Find angle HOI
 - (c) Find the area of sector HOI
 - (d) Find the area of triangle HOI

Section E: Geometrical Proof and Reasoning

20. Prove that the sum of angles in any triangle is 180° .
21. Prove that the exterior angle of a triangle equals the sum of the two interior opposite angles.
22. In triangle RST, N is the midpoint of ST. Prove that RN is a median and that the three medians of a triangle meet at a single point.

23. UVWX is a parallelogram. Prove that:
- (a) Opposite sides are equal
 - (b) Opposite angles are equal
 - (c) Diagonals bisect each other
24. Prove that if a quadrilateral has both pairs of opposite sides parallel, then it is a parallelogram.
25. In triangle JKL, the perpendicular bisectors of sides JK and KL meet at point T. Prove that T is equidistant from J, K, and L.
26. UVWX is a cyclic quadrilateral (vertices lie on a circle). Prove that opposite angles sum to 180° .
27. Prove that the angle bisectors of a triangle meet at a single point (the incenter).

Section F: Surface Area and Volume

28. Calculate the surface area and volume of these shapes:
- (a) Cuboid: length 16 cm, width 11 cm, height 8 cm
 - (b) Cylinder: radius 7 cm, height 18 cm
 - (c) Sphere: radius 11 cm
 - (d) Cone: radius 8 cm, height 21 cm
29. A triangular prism has:
- Triangular cross-section with base 12 cm and height 7 cm
 - Length 24 cm

Calculate:

- (a) The volume
 - (b) The surface area
 - (c) The total edge length
30. A square-based pyramid has:
- Base edge 14 cm
 - Vertical height 11 cm
- Find:
- (a) The volume
 - (b) The slant height
 - (c) The surface area
31. A hemisphere sits on top of a cylinder. Both have radius 6 cm and the cylinder has height 16 cm.
- (a) Find the total volume
 - (b) Find the total surface area
 - (c) If this shape is a water tank, how much water does it hold?
32. A cone has base radius 8 cm and slant height 17 cm.
- (a) Find the vertical height
 - (b) Calculate the volume
 - (c) Calculate the curved surface area
 - (d) Calculate the total surface area

Section G: Complex 3D Problems

33. A rectangular swimming pool is 40 m long, 20 m wide, and 3.5 m deep.
- (a) Calculate the volume of water needed to fill it
 - (b) Find the surface area of the pool (5 faces - no top)
 - (c) How much does it cost to tile the pool at £55 per m^2 ?
 - (d) If water is added at 1200 liters per minute, how long to fill the pool?
34. A solid consists of a cylinder with radius 11 cm and height 16 cm, with a cone of radius 11 cm and height 14 cm on top.
- (a) Calculate the total volume
 - (b) Calculate the total surface area
 - (c) If the solid is made of metal with density 8.8 g/cm^3 , find its mass
35. A spherical ball has radius 24 cm. A cylindrical hole of radius 7 cm is drilled through the center.
- (a) Calculate the volume of material removed
 - (b) Calculate the remaining volume
 - (c) Calculate the new surface area (including the cylindrical hole)
36. A water tank is a horizontal cylinder with length 8 m and radius 2.1 m.
- (a) Calculate its capacity in liters
 - (b) If it's half full, find the depth of water
 - (c) Calculate the surface area of water

Section H: Problem Solving and Applications

37. A regular dodecagon is inscribed in a circle of radius 18 cm.
- (a) Find the side length of the dodecagon
 - (b) Calculate the area of the dodecagon
 - (c) Find the area between the circle and dodecagon
38. From point R, the angle of elevation to the top of a tower is 35° . From point S, 70 m closer to the tower, the angle of elevation is 58° .
- (a) Draw a diagram
 - (b) Find the height of the tower
 - (c) Find the distances from R and S to the base of the tower
39. A field is in the shape of a triangle with sides 180 m, 220 m, and 260 m.
- (a) Find the area of the field using Heron's formula
 - (b) Find the area using trigonometry
 - (c) A path 3.5 m wide runs around the perimeter. Find the area of the path
40. A church spire consists of a square-based pyramid on top of a cuboid. The cuboid is $14 \text{ m} \times 14 \text{ m} \times 24 \text{ m}$ high. The pyramid has height 18 m.
- (a) Calculate the total volume
 - (b) Calculate the total surface area

- (c) Find the angle the pyramid faces make with the horizontal
41. Two circles with radii 15 cm and 11 cm have centers 32 cm apart.
- (a) Do the circles intersect? Justify your answer.
 - (b) Find the length of their common external tangent
 - (c) Calculate the area of overlap if they moved closer together
42. A satellite orbits Earth at height 600 km above the surface. Earth's radius is 6400 km.
- (a) Calculate the satellite's orbital radius
 - (b) Find the maximum distance the satellite can see on Earth's surface
 - (c) Calculate the area of Earth visible from the satellite
43. Prove that in any triangle, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$ where R is the circumradius.
44. A frustum (truncated cone) has top radius 6 cm, bottom radius 15 cm, and height 12 cm.
- (a) Find the height of the complete cone
 - (b) Calculate the volume of the frustum
 - (c) Calculate the curved surface area of the frustum

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 100

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