

A Level Mechanics

Practice Test 2: Work, Energy, and Power

Instructions:

Answer all questions. Show your working clearly.

Calculators may be used unless stated otherwise.

Draw diagrams where appropriate to illustrate your solutions.

Time allowed: 3 hours

Section A: Work and Energy Fundamentals [25 marks]

Question 1 [9 marks]

- (a) Define work done by a force and explain how it differs from the everyday use of the word "work." [3 marks]
- (b) State the conditions under which: (i) positive work is done, (ii) negative work is done, (iii) zero work is done. [3 marks]
- (c) Explain why kinetic energy is always positive while work can be positive, negative, or zero. [3 marks]

Question 2 [10 marks] A force $\vec{F} = (12\hat{i} + 8\hat{j})$ N acts on an object that undergoes a displacement $\vec{s} = (5\hat{i} - 3\hat{j})$ m.

- (a) Calculate the work done by the force using the dot product method. [3 marks]
- (b) Find the magnitude of the force and the displacement. [2 marks]
- (c) Calculate the angle between the force and displacement vectors. [3 marks]
- (d) Verify your work calculation using $W = Fd \cos \theta$. [2 marks]

Question 3 [6 marks] A 4 kg object moving at 6 m/s has a force of 20 N applied in the direction of motion for 3 seconds.

- (a) Calculate the initial kinetic energy. [1 mark]
- (b) Find the final velocity using Newton's second law. [2 marks]
- (c) Calculate the final kinetic energy and the change in kinetic energy. [2 marks]
- (d) Find the work done by the force and verify the work-energy theorem. [1 mark]

Section B: Work Done by Constant Forces [27 marks]

Question 4 [15 marks] A sled of mass 30 kg is pulled 22 m up a rough inclined plane at constant velocity. The plane makes an angle of 15° with the horizontal. The pulling force is 180 N and acts parallel to the incline. The coefficient of kinetic friction is 0.12.

- (a) Draw a force diagram showing all forces acting on the sled. [2 marks]
- (b) Calculate the work done by the pulling force. [2 marks]
- (c) Find the work done against gravity. [3 marks]
- (d) Calculate the normal force and the friction force. [3 marks]
- (e) Determine the work done against friction. [2 marks]
- (f) Verify that the net work done is zero (since velocity is constant). [3 marks]

Question 5 [12 marks] A motorcycle of mass 250 kg accelerates from 10 m/s to 35 m/s over a distance of 180 m. During this motion, it experiences air resistance and rolling friction totaling 320 N.

- (a) Calculate the change in kinetic energy. [3 marks]
- (b) Find the work done against resistance forces. [2 marks]
- (c) Determine the work done by the engine. [2 marks]
- (d) Calculate the engine force (driving force). [2 marks]
- (e) If the motorcycle then coasts to a stop from 35 m/s over 200 m with the same resistance, find the deceleration. [3 marks]

Section C: Variable Force and Work-Energy Theorem [30 marks]

Question 6 [16 marks] A particle of mass 2.5 kg moves along the x-axis under a variable force $F = 10x - 2x^3$ N, where x is in metres.

- (a) Calculate the work done by this force as the particle moves from $x = 0$ to $x = 2$ m. [4 marks]
- (b) Find the work done as the particle moves from $x = 1$ m to $x = 2.5$ m. [4 marks]
- (c) If the particle starts from rest at $x = 0$, calculate its speed at $x = 1$ m. [3 marks]
- (d) Find all positions where the force is zero. [3 marks]
- (e) Determine the position where the force is maximum in the interval $[0, 3]$. [2 marks]

Question 7 [14 marks]

- (a) State the work-energy theorem and explain its significance in mechanics. [3 marks]
- (b) A projectile of mass 0.8 kg is launched at 45° to the horizontal with initial speed 25 m/s. Use the work-energy theorem to find the speed when it is 15 m above the launch point. [4 marks]
- (c) A 15 kg box slides down a rough inclined plane (angle 25°) for 8 m. If the coefficient of kinetic friction is 0.18, calculate the final speed starting from rest. [4 marks]
- (d) A 5 kg object is pulled horizontally with a force that varies as $F(t) = 20 + 4t$ N for 6 seconds. If it starts from rest, find the final kinetic energy. [3 marks]

Section D: Gravitational Potential Energy [26 marks]**Question 8 [8 marks]**

- (a) Explain the concept of a conservative force and give three examples. [3 marks]
- (b) Prove that the work done by gravity is independent of the path taken. [3 marks]
- (c) Why is the choice of reference level for gravitational potential energy arbitrary? [2 marks]

Question 9 [18 marks] A projectile of mass 2.5 kg is fired from ground level at 60° to the horizontal with initial speed 40 m/s.

- (a) Calculate the initial kinetic and potential energies (taking ground as reference). [3 marks]
- (b) Find the total mechanical energy of the projectile. [1 mark]
- (c) Use energy conservation to calculate the maximum height reached. [3 marks]
- (d) Find the speed when the projectile is at half the maximum height. [4 marks]
- (e) Calculate the speed when the projectile returns to ground level. [2 marks]
- (f) At what height is the kinetic energy three times the potential energy? [3 marks]
- (g) Find the speed and direction of velocity at the maximum height. [2 marks]

Section E: Elastic Potential Energy [24 marks]

Question 10 [14 marks] A spring with spring constant $k = 800 \text{ N/m}$ is compressed by 0.20 m from its natural length.

- (a) Calculate the elastic potential energy stored. [2 marks]
- (b) A mass of 1.6 kg is attached and released. Find the maximum speed achieved. [3 marks]
- (c) At what compression is the speed of the mass 3 m/s? [4 marks]
- (d) Calculate the acceleration when the spring is compressed by 0.15 m. [2 marks]
- (e) Find the position where the kinetic energy equals the potential energy. [3 marks]

Question 11 [10 marks] A 3 kg block is attached to a vertical spring with spring constant 600 N/m. The block is pulled down 0.3 m from equilibrium and released.

- (a) Find the equilibrium position of the block when hanging. [2 marks]
- (b) Calculate the total energy of the oscillating system. [2 marks]
- (c) Determine the maximum speed of the block. [2 marks]
- (d) Find the speed when the block is 0.1 m below the equilibrium position. [2 marks]
- (e) Calculate the maximum height reached above the release point. [2 marks]

Section F: Conservation of Mechanical Energy [28 marks]

Question 12 [16 marks] A pendulum bob of mass 1.2 kg hangs from a string of length 2 m. It is released from rest when the string makes 40° with the vertical.

- (a) Calculate the initial height above the lowest point. [2 marks]
- (b) Find the speed at the lowest point using energy conservation. [3 marks]
- (c) Calculate the tension in the string at the lowest point. [3 marks]
- (d) Find the speed when the string makes 20° with the vertical. [3 marks]
- (e) Determine the minimum release angle for the pendulum to just complete a vertical circle. [3 marks]
- (f) Calculate the tension at the top of the circle for the minimum case. [2 marks]

Question 13 [12 marks] A 800 kg roller coaster car starts from rest 40 m above ground level and travels along a frictionless track.

- (a) Find the speed at the bottom of the track (ground level). [2 marks]
- (b) The track then rises to 25 m above ground. Calculate the speed at this height. [3 marks]
- (c) If the track has a circular loop of radius 8 m at the bottom, find the normal force on the car at the bottom of the loop. [3 marks]
- (d) What is the minimum starting height needed for the car to just complete the loop? [4 marks]

Section G: Power Calculations [25 marks]

Question 14 [8 marks]

- (a) Distinguish between average power and instantaneous power with examples. [3 marks]
- (b) Derive the formula $P = F \cdot v$ for instantaneous power. [3 marks]
- (c) Explain why power is important in engineering applications. [2 marks]

Question 15 [17 marks] A lift of mass 1200 kg (including passengers) is raised by a motor.

- (a) Calculate the power required to lift the load at constant speed 2.5 m/s. [2 marks]
- (b) If the motor efficiency is 85%, find the electrical power input needed. [2 marks]
- (c) The lift accelerates upward at 1.5 m/s^2 from rest. Calculate the motor force required. [3 marks]
- (d) Find the power output after 3 seconds of acceleration. [3 marks]
- (e) Calculate the total work done by the motor in the first 4 seconds of acceleration. [4 marks]
- (f) If the lift then moves at constant speed for 10 seconds, find the total energy consumed. [3 marks]

Section H: Power in Vehicle Motion [20 marks]

Question 16 [12 marks] A truck engine provides constant power of 90 kW. The truck has mass 2500 kg and experiences air resistance $F_r = 500 + 2v^2$ N, where v is speed in m/s.

- (a) Find the maximum speed on level ground by setting driving force equal to resistance. [3 marks]
- (b) Calculate the acceleration when the truck speed is 20 m/s. [4 marks]
- (c) Determine the speed when acceleration is 1 m/s². [3 marks]
- (d) Estimate the time to accelerate from 10 m/s to 25 m/s. [2 marks]

Question 18 [8 marks] A racing cyclist (total mass 75 kg) maintains constant power output of 400 W.

- (a) On level ground with 30 N resistance, find the maximum speed. [2 marks]
- (b) When climbing a 12° incline at steady speed, calculate the new maximum speed if air resistance is negligible. [3 marks]
- (c) What total power would be needed to maintain 12 m/s up this incline? [3 marks]

Section I: Energy in Collisions and Springs [22 marks]

Question 19 [12 marks] A 4 kg block moving at 9 m/s undergoes an elastic collision with a 6 kg block initially at rest.

- (a) Use conservation of momentum and energy to find both final velocities. [5 marks]
- (b) Calculate the kinetic energies before and after collision and verify energy is conserved. [3 marks]
- (c) Find the velocity of the center of mass before and after collision. [2 marks]
- (d) Calculate the kinetic energy in the center of mass frame before collision. [2 marks]

Question 20 [10 marks] A 0.2 kg ball moving horizontally at 15 m/s strikes a vertical wall and rebounds horizontally at 12 m/s.

- (a) Calculate the change in momentum. [2 marks]
- (b) Find the impulse delivered by the wall. [1 mark]
- (c) Calculate the kinetic energy before and after collision. [3 marks]
- (d) Determine the energy lost and the coefficient of restitution. [3 marks]
- (e) If contact time is 0.05 seconds, find the average force. [1 mark]

Section J: Comprehensive Applications [28 marks]

Question 21 [16 marks] A 4 kg block slides down a frictionless curved track from height 5 m and then compresses a spring ($k = 1800$ N/m) on a horizontal surface with coefficient of friction 0.2.

- (a) Find the speed just before hitting the spring. [2 marks]
- (b) If the horizontal surface were frictionless, calculate the maximum compression. [3 marks]
- (c) With friction present, set up the energy equation for maximum compression. [3 marks]
- (d) Calculate the actual maximum compression with friction. [3 marks]

- (e) Find the speed when the block returns to the point where it first contacted the spring. **[3 marks]**
- (f) Determine how high the block will rise when it returns up the curved track. **[2 marks]**

Question 22 [12 marks] A hydroelectric power station has water flowing at $150 \text{ m}^3/\text{s}$ through turbines with a vertical drop of 120 m.

- (a) Calculate the gravitational potential energy converted per second. **[3 marks]**
- (b) If the turbine efficiency is 94%, find the electrical power output. **[2 marks]**
- (c) The power station supplies a city consuming 180 MW. What additional flow rate is needed? **[3 marks]**
- (d) If the reservoir holds 500 million cubic metres, calculate how long it can supply the city at full capacity. **[2 marks]**
- (e) Compare the environmental benefits of hydroelectric vs fossil fuel power generation. **[2 marks]**

Physics Data and Formulae

Work and Energy:

Work done: $W = \vec{F} \cdot \vec{s} = Fs \cos \theta$ (constant force)

Variable force: $W = \int_{s_1}^{s_2} F ds$

Kinetic energy: $KE = \frac{1}{2}mv^2$

Gravitational potential energy: $PE_g = mgh$

Elastic potential energy: $PE_e = \frac{1}{2}kx^2$

Work-Energy Theorem:

$$W_{net} = \Delta KE = KE_f - KE_i$$

Conservation of Energy:

$$E_{total} = KE + PE = \text{constant (conservative forces only)}$$

$$KE_i + PE_i = KE_f + PE_f + \text{energy dissipated}$$

Power:

$$\text{Average power: } P_{avg} = \frac{W}{t}$$

$$\text{Instantaneous power: } P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$$

$$\text{For constant force: } P = Fv \cos \theta$$

Springs and Oscillations:

$$\text{Hooke's Law: } F = -kx$$

$$\text{Spring potential energy: } U = \frac{1}{2}kx^2$$

$$\text{Equilibrium extension: } x_0 = \frac{mg}{k} \text{ (vertical spring)}$$

Circular Motion:

$$\text{Centripetal force: } F_c = \frac{mv^2}{r}$$

$$\text{At top of vertical circle: } mg + T = \frac{mv^2}{r}$$

$$\text{At bottom of vertical circle: } T - mg = \frac{mv^2}{r}$$

Constants:

$$\text{Acceleration due to gravity: } g = 9.8 \text{ m/s}^2$$

$$\text{Density of water: } \rho = 1000 \text{ kg/m}^3$$

Trigonometric Values:

$$\sin 12 = 0.208, \cos 12 = 0.978, \tan 12 = 0.213$$

$\sin 15 = 0.259$, $\cos 15 = 0.966$, $\tan 15 = 0.268$
 $\sin 20 = 0.342$, $\cos 20 = 0.940$, $\tan 20 = 0.364$
 $\sin 25 = 0.423$, $\cos 25 = 0.906$, $\tan 25 = 0.466$
 $\sin 40 = 0.643$, $\cos 40 = 0.766$, $\tan 40 = 0.839$
 $\sin 45 = 0.707$, $\cos 45 = 0.707$, $\tan 45 = 1.000$
 $\sin 60 = 0.866$, $\cos 60 = 0.500$, $\tan 60 = 1.732$

END OF TEST

Total marks: 275

Grade boundaries: A* 248, A 220, B 193, C 165, D 138, E 110

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