GCSE Higher Mathematics
Practice Test 2: Ratio, Proportion, and Rates of Change

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Time allowed: 90 minutes

Section A: Advanced Ratios

1. Express these ratios in their simplest form:
   (a) $1.8 : 2.7 : 0.9$
   (b) $\frac{3}{5} : \frac{2}{3} : \frac{1}{4}$
   (c) $0.15 : 0.225 : 0.075$
   (d) $2\frac{1}{3} : 1\frac{2}{3} : 3\frac{1}{6}$
2. Four friends start a business in the ratio $3 : 5 : 7 : 2$. If the total startup capital is £68,000:
   (a) How much does each friend contribute?
   (b) If the first year profit is £20,400, how much does each friend receive?
   (c) What percentage of the total capital does the smallest investor contribute?
3. The sides of a triangle are in the ratio $5 : 7 : 8$. If the perimeter is 60 cm, find each side length.
4. A paint mixture uses red, blue, and white paint in the ratio $4 : 3 : 8$ by volume.
   (a) How much of each color is needed for 750ml of paint?
   (b) If 120ml of blue paint is used, find the volume of red and white paint needed
   (c) What fraction of the mixture is white paint?
5. Divide £945 between three charities so that the first gets £35 more than the second, and the second gets £50 more than the third.

Section B: Direct Proportion

6. y is directly proportional to x. When $x = 15$, $y = 9$.
   (a) Find the equation connecting y and x
   (b) Calculate y when $x = 20$
   (c) Calculate x when $y = 24$
   (d) Sketch the graph of y against x
7. A varies directly as the square of B. When $B = 3$, $A = 27$.

(a) Express A in terms of B

(b) Find A when $B = 5$

(c) Find B when $A = 108$

(d) What happens to A when B is tripled?

8. The cost of fabric varies directly as its area. 12 m^2 costs £36.

(a) Find the cost per m^2

(b) How much does 18.5 m^2 cost?

(c) What area can be bought for £75?

(d) Express the cost C in terms of area A

9. The power generated by a wind turbine varies directly as the cube of the wind speed. At 8 m/s, it generates 512 watts.

(a) Find the power generated at 12 m/s

(b) What wind speed generates 1000 watts?

(c) Write the relationship as an equation

Section C: Inverse Proportion

10. y is inversely proportional to x. When $x = 8$, $y = 15$.

(a) Find the equation connecting y and x

(b) Calculate y when $x = 12$

(c) Calculate x when $y = 20$

(d) Sketch the graph of y against x

11. The time taken to paint a fence is inversely proportional to the number of workers. With 4 workers, it takes 6 hours.

(a) How long does it take with 8 workers?

(b) How many workers are needed to complete it in 2 hours?

(c) Express time T in terms of number of workers W

(d) If one worker becomes ill, how much longer will the job take?

12. M is inversely proportional to the square of N. When $N = 4$, $M = 12$.

(a) Express M in terms of N

(b) Find M when $N = 6$

(c) Find N when $M = 3$

(d) What happens to M when N is doubled?

13. The gravitational force between two objects is inversely proportional to the square of the distance between them. At 4 meters, the force is 225 newtons.

(a) Find the force at 6 meters

(b) At what distance is the force 100 newtons?

(c) Write the relationship as an equation

Section D: Combined Proportion

14. w varies directly as a and inversely as b. When $a = 8$ and $b = 5$, $w = 12$.

    (a) Express w in terms of a and b

    (b) Find w when $a = 12$ and $b = 4$

    (c) Find b when $a = 15$ and $w = 9$

    (d) What happens to w if both a and b are tripled?

15. The surface area S of a sphere varies directly as the square of its radius r. When $r = 5$, $S = 100\pi$.

    (a) Express S in terms of r

    (b) Find S when $r = 8$

    (c) Find r when $S = 225\pi$

    (d) What happens to S if r is halved?

16. The current I in a wire varies directly as the voltage V and inversely as the resistance R. When $V = 24$ and $R = 6$, $I = 4$.

    (a) Express I in terms of V and R

    (b) Find I when $V = 36$ and $R = 9$

    (c) Find R when $I = 5$ and $V = 30$

17. p varies directly as q^3 and inversely as r^2. When $q = 2$ and $r = 4$, $p = 6$.

    (a) Find the constant of proportionality

    (b) Express p in terms of q and r

    (c) Calculate p when $q = 3$ and $r = 2$

Section E: Speed, Distance, and Time

18. A train travels 315 km in 2 hours 30 minutes. Calculate:

    (a) The average speed in km/h

    (b) The average speed in m/s

    (c) How far it travels in 1 hour 45 minutes at this speed

    (d) How long it takes to travel 210 km at this speed

19. Convert these speeds:

    (a) 30 m/s to km/h

    (b) 144 km/h to m/s

    (c) 35 mph to km/h (use 1 mile = 1.6 km)

    (d) 96 km/h to mph

20. A motorist drives from P to Q at 40 km/h and returns at 60 km/h. The total journey takes 4 hours.

    (a) Find the distance from P to Q

    (b) Calculate the average speed for the whole journey

    (c) How much time was saved on the return journey?

21. Two cars start simultaneously from towns 420 km apart and drive towards each other. One travels at 90 km/h and the other at 60 km/h.

(a) When do they meet?

(b) How far from each starting point do they meet?

(c) What is their combined speed of approach?

22. A cyclist completes a 15 km race. For the first 9 km, they maintain a speed of 18 km/h. For the remaining 6 km, they increase to 24 km/h.

(a) Calculate the total time taken

(b) Find the average speed for the whole race

(c) How much slower would they need to cycle the second part to achieve an overall average of 15 km/h?

Section F: Density and Flow Rates

23. A wooden block has volume 180 cm^3 and mass 135 g.

(a) Calculate its density in g/cm^3

(b) Calculate its density in kg/m^3

(c) What mass of this wood would have volume 320 cm^3?

(d) What volume would 200 g of this wood occupy?

24. Different liquids have these densities:

- Water: 1.0 g/cm^3
- Oil: 0.8 g/cm^3
- Mercury: 13.6 g/cm^3

(a) Compare the masses of 100 cm^3 of each liquid

(b) What volume of mercury has the same mass as 200 cm^3 of water?

(c) A container holds 500 ml of liquid with mass 680 g. Which liquid is it most likely to be?

25. Oil flows through a pipeline at a rate of 2.5 m^3 per minute.

(a) How much oil flows in 8 minutes?

(b) How long to fill a 750 m^3 storage tank?

(c) Express the flow rate in liters per second

(d) If the pipeline diameter doubles, what happens to the flow speed?

26. A printing press operates at different rates during the day:

- Morning shift (6 hours): 200 pages/hour
- Afternoon shift (4 hours): 250 pages/hour
- Night shift (2 hours): 180 pages/hour

(a) Calculate total daily output

(b) Find the average production rate

(c) How long to print 1500 pages at the average rate?

Section G: Scale Factors and Similar Shapes

27. Two similar rectangles have corresponding sides in the ratio 4:7.

    (a) If the smaller rectangle has area 32 cm^2, find the area of the larger rectangle

    (b) If the larger rectangle has perimeter 56 cm, find the perimeter of the smaller rectangle

    (c) Find the ratio of their areas

28. A scale model of a building is built to scale 1:50. The real building is 35 m tall and 20 m wide.

    (a) Find the dimensions of the model

    (b) If the model requires 0.8 m^2 of material, how much material does the real building need?

    (c) The model weighs 3.2 kg. Estimate the mass of the real building if made from the same material

29. An Ordnance Survey map has scale 1:50000. Two cities are 12.8 cm apart on the map.

    (a) Calculate the actual distance in km

    (b) What map distance represents 8 km?

    (c) A forest covers 5.6 cm^2 on the map. Find its actual area in km^2

30. Two similar cones have height ratio 3:5.

    (a) Find the ratio of their base areas

    (b) Find the ratio of their volumes

    (c) If the smaller cone has volume 162 cm^3, what is the volume of the larger cone?

    (d) If the larger cone has surface area 400 cm^2, what is the surface area of the smaller cone?

Section H: Advanced Rate Problems

31. The value of a car depreciates by 15% each year. Starting with a value of £18000:

    (a) Write an expression for value after t years

    (b) What is the value after 4 years?

    (c) When will the value drop below £8000?

    (d) What is the rate of depreciation after 3 years (£per year)?

32. A swimming pool is being filled by two pipes and emptied by one drain. Pipe A fills at 15 liters/minute, pipe B fills at 25 liters/minute, and the drain empties at 18 liters/minute.

    (a) What is the net rate of filling when all are operating?

    (b) How long to fill a 1320-liter pool from empty?

    (c) If only pipe A and the drain operate, what is the net rate?

    (d) What size drain would balance both pipes together?

33. Currency exchange rates:

    - £1 = \$1.32
    - £1 = €1.18
    - \$1 = ¥115

    (a) Convert £350 to dollars

    (b) Convert €185 to pounds

(c) Convert \$220 to yen
(d) Find the exchange rate from euros to dollars

34. A 3D printer operates at variable speeds during production:

- Setup phase: 1 hour at 20% speed (produces 8 cm^3/hour)
- Main printing: 6 hours at full speed (produces 40 cm^3/hour)
- Finishing: 0.5 hours at 60% speed

(a) Calculate total volume printed
(b) Find the average printing rate over the whole process
(c) How long would it take to print the same volume at 75% speed?

Section I: Problem Solving and Applications

35. A recipe for 8 servings uses:

- 600g pasta
- 400ml cream
- 4 tomatoes
- 120g cheese

(a) Adapt the recipe for 12 servings
(b) How much of each ingredient for 5 servings?
(c) If you have 1.5kg pasta, what's the maximum number of servings you can make?

36. A college has 1800 students. The ratio of full-time to part-time students is 5:4. Due to new enrollments, 90 more full-time students join but no new part-time students.

(a) How many full-time and part-time students were there originally?
(b) What is the new ratio of full-time to part-time students?
(c) How many more part-time students would need to join to restore the original ratio?

37. The braking distance of a vehicle varies directly as the square of its speed for speeds under 70 mph. At 30 mph, the braking distance is 18 meters.

(a) Find the braking distance at 50 mph
(b) At what speed is the braking distance 32 meters?
(c) Compare the braking distances at 40 mph and 60 mph

38. A telecommunications company's revenue R (in millions) varies as the square of marketing spend M (in millions) up to £8m, then levels off. When $M = 3$, $R = 27$. When $M = 12$, $R = 192$.

(a) Find the revenue function for $M \leq 8$
(b) Find the constant revenue for $M > 8$
(c) What marketing spend gives maximum efficiency (revenue per £spent)?
(d) What is the maximum revenue?

39. A pulley system connects three wheels. Wheel A has radius 15 cm and rotates at 240 rpm. Wheel B has radius 25 cm and is connected to wheel A. Wheel C has radius 18 cm and is connected to wheel B.

(a) Find the rotation speed of wheel B
(b) Find the rotation speed of wheel C
(c) What is the overall speed ratio from wheel A to wheel C?
(d) If wheel A's radius increases to 20 cm, find the new speed of wheel C

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 100

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