A Level Mechanics
Practice Test 4: Rotational Motion

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Draw diagrams where appropriate to illustrate your solutions.
Time allowed: 3 hours

Section A: Rotational Motion Fundamentals [27 marks]

Question 1 [8 marks]

(a) Define angular displacement, angular velocity, and angular acceleration, explaining how they relate to their linear motion counterparts. **[4 marks]**

(b) Derive the relationship $a_c = \omega^2 r$ for centripetal acceleration in terms of angular velocity. **[2 marks]**

(c) Write the three rotational kinematic equations for constant angular acceleration. **[2 marks]**

Question 2 [15 marks] A helicopter rotor blade rotates with angular velocity given by $\omega(t) = 2t^4 - 8t^3 + 6t^2 + 4$ rad/s.

(a) Find the expression for angular acceleration $\alpha(t)$. **[3 marks]**

(b) Calculate the angular velocity and acceleration at $t = 1.5$ seconds. **[3 marks]**

(c) Determine all times when the angular acceleration is zero. **[4 marks]**

(d) Find the angular displacement between $t = 0.5$ and $t = 2$ seconds. **[3 marks]**

(e) Calculate the average angular velocity over this time interval. **[2 marks]**

Question 3 [4 marks] A point on a rotating disc at distance 0.9 m from the center has tangential acceleration 6 m/s² and total acceleration of magnitude 14 m/s².

(a) Calculate the centripetal acceleration and hence find the tangential speed. **[2 marks]**

(b) Determine the angular velocity and angular acceleration of the disc. **[2 marks]**

Section B: Moment of Inertia - Point Masses [31 marks]

Question 4 [8 marks]

(a) Define moment of inertia mathematically and explain its role in rotational motion. **[3 marks]**

(b) State and prove the parallel axis theorem for a system of point masses. **[5 marks]**

Question 5 [23 marks] Calculate moments of inertia for the following systems:

(a) Eight point masses distributed as: 2.5 kg at (3,2), 1.8 kg at (-2,4), 3.1 kg at (4,-1), 2.2 kg at (-3,-3), 1.6 kg at (1,5), 2.8 kg at (-4,2), 3.4 kg at (2,-4), and 1.9 kg at (-1,-2) meters. Find the moment of inertia about the z-axis. **[6 marks]**

(b) Six masses of 3.5 kg each are positioned at the vertices of a regular hexagon with side length 1.5 m. Calculate the moment of inertia about the central axis. **[4 marks]**

(c) Four point masses are arranged linearly: 8 kg at x = -2 m, 12 kg at x = 0, 15 kg at x = 1.5 m, and 10 kg at x = 3 m. Find the moment of inertia about: (i) x = 0, (ii) the center of mass. **[7 marks]**

(d) A triangular arrangement has three masses: 6 kg at (0,0), 9 kg at (4,0), and 12 kg at (2,3) meters. Calculate the moment of inertia about an axis through (2,1) parallel to the z-axis. **[4 marks]**

(e) Use the parallel axis theorem to verify your calculation in part (d). **[2 marks]**

Section C: Moment of Inertia - Extended Bodies [33 marks]

Question 6 [9 marks]

(a) List the standard moment of inertia expressions for: uniform rod (center and end), solid disc, hollow disc, solid sphere, hollow sphere, and solid cylinder. **[6 marks]**

(b) Derive the moment of inertia of a solid sphere of mass M and radius R about a diameter using the result for a disc. **[3 marks]**

Question 7 [24 marks] Calculate moments of inertia for composite objects:

(a) A compound disc has a solid inner section (mass 10 kg, radius 0.25 m) and a hollow outer ring (mass 6 kg, inner radius 0.25 m, outer radius 0.4 m). Find the total moment of inertia about the central axis. **[5 marks]**

(b) A cross-shaped object consists of two identical uniform rods (each mass 7 kg, length 2.8 m) intersecting perpendicularly at their centers. Calculate the moment of inertia about the intersection point perpendicular to both rods. **[5 marks]**

(c) A linear system has four solid spheres (each mass 5 kg, radius 0.12 m) connected by three uniform rods (each mass 2 kg, length 1.5 m) in a straight line with spheres at the rod ends and junctions. Find the moment of inertia about the center perpendicular to the line. **[8 marks]**

(d) A fan consists of a solid disc hub (mass 8 kg, radius 0.2 m) with ten identical blade-spokes (each mass 1.5 kg, length 0.6 m) extending radially. Each blade can be modeled as a uniform rod. Calculate the total moment of inertia. **[6 marks]**

Section D: Torque and Rotational Dynamics [29 marks]

Question 8 [9 marks]

(a) Define torque as a vector quantity and explain how its magnitude and direction are determined. **[4 marks]**

(b) State the fundamental equation of rotational dynamics and explain each term. **[3 marks]**

(c) Explain the relationship between torque and the rate of change of angular momentum. **[2 marks]**

Question 10 [20 marks] A large wheel system consists of three coaxial discs: a large outer disc (mass 20 kg, radius 1.0 m), a medium disc (mass 12 kg, radius 0.6 m), and a small inner disc (mass 8 kg, radius 0.3 m).

(a) Calculate the total moment of inertia of the system. **[3 marks]**

(b) Four forces are applied: 40 N tangentially at the outer edge, 25 N at 60° to the radius at 0.8 m from center, 30 N tangentially at the medium disc edge in the opposite direction, and 15 N radially inward at the small disc edge. Calculate the net torque. **[7 marks]**

(c) Find the angular acceleration of the system. **[2 marks]**

(d) Starting from an initial angular velocity of 2 rad/s, find the angular velocity after 5 seconds. **[2 marks]**

(e) Calculate the angular displacement during these 5 seconds. **[2 marks]**

(f) Determine the work done by the net torque and verify using the change in kinetic energy. **[3 marks]**

(g) Find the instantaneous power at t = 5 seconds. **[1 mark]**

Section E: Rotational Energy [30 marks]

Question 11 [8 marks]

(a) Derive the formula for rotational kinetic energy by considering a rigid body as a collection of particles. **[3 marks]**

(b) Establish the work-energy theorem for rotational motion starting from the definition of work done by a torque. **[3 marks]**

(c) Show that the instantaneous power in rotational motion is given by $P = \tau\omega$. **[2 marks]**

Question 12 [22 marks] A hollow cylinder of mass 18 kg, inner radius 0.2 m, and outer radius 0.5 m rolls without slipping down a complex ramp that has an initial straight section (angle 40°, length 4 m) followed by a curved section that brings it to horizontal level 2 m below the starting point.

(a) Calculate the moment of inertia of the hollow cylinder. **[3 marks]**

(b) Write and explain the rolling constraint for this motion. **[2 marks]**

(c) Use energy conservation to find the linear speed when the cylinder reaches the bottom of the ramp. **[5 marks]**

(d) Calculate the corresponding angular velocity. **[2 marks]**

(e) If the cylinder then encounters a rough horizontal surface with coefficient of kinetic friction 0.3, find how far it rolls before stopping. **[5 marks]**

(f) Calculate the translational and rotational kinetic energies at the bottom of the ramp. **[3 marks]**

(g) Compare the fraction of rotational kinetic energy for this hollow cylinder with that of a solid cylinder. **[2 marks]**

Section F: Angular Momentum [26 marks]

Question 13 [6 marks]

(a) Define angular momentum for a rigid body and express it in terms of moment of inertia and angular velocity. **[2 marks]**

(b) State the conservation of angular momentum principle and derive it from the rotational equation of motion. **[4 marks]**

Question 14 [20 marks] An ice skater performs a spin, initially with arms and leg extended (moment of inertia 6.8 kg·m^2) rotating at 3.2 rad/s.

(a) Calculate the initial angular momentum. **[2 marks]**

(b) The skater pulls arms and leg close to the body, reducing the moment of inertia to 1.9 kg·m^2. Find the new angular velocity. **[3 marks]**

(c) Calculate the initial and final rotational kinetic energies and explain the energy change. **[5 marks]**

(d) If the transition takes 1.2 seconds, calculate the average power supplied by the skater's muscles. **[3 marks]**

(e) While spinning in the extended position, the skater catches a 1.8 kg object thrown horizontally at 8 m/s that sticks to their hand at 1.5 m from the rotation axis. Find the final angular velocity. **[4 marks]**

(f) Compare the energy losses in parts (b) and (e) and explain the difference. **[3 marks]**

Section G: Conservation in Rotational Systems [25 marks]

Question 15 [16 marks] A large turntable (uniform disc, mass 400 kg, radius 3 m) rotates at 0.5 rad/s. Four people of masses 80 kg, 65 kg, 75 kg, and 70 kg jump onto it at distances 2.5 m, 2.0 m, 1.5 m, and 2.8 m from the center respectively.

(a) Calculate the initial angular momentum of the turntable. **[3 marks]**

(b) Find the total moment of inertia after all four people are on the turntable. **[4 marks]**

(c) Determine the final angular velocity using conservation of angular momentum. **[3 marks]**

(d) Calculate the kinetic energy change and explain what happens to the "lost" energy. **[4 marks]**

(e) If the 80 kg person runs tangentially onto the turntable at 7 m/s while others step on gently, find the final angular velocity. **[2 marks]**

Question 16 [9 marks] Two horizontal discs with different moments of inertia can rotate about the same vertical axis. Disc P (moment of inertia 3.2 kg·m^2) rotates at 15 rad/s, while disc Q (moment of inertia 2.1 kg·m^2) rotates at 8 rad/s in the opposite direction.

(a) Calculate the net angular momentum before the discs are coupled. **[3 marks]**

(b) Find the common angular velocity after coupling. **[3 marks]**

(c) Determine the kinetic energy dissipated and explain the physical mechanism of energy loss. **[3 marks]**

Section H: Combined Motion [23 marks]

Question 17 [**14 marks**] A uniform solid sphere (mass 12 kg, radius 0.18 m) is suspended by a string wound around its equator and released from rest.

(a) Calculate the moment of inertia about the central axis. [**2 marks**]

(b) Apply rotational and translational dynamics to derive the linear acceleration of the sphere. [**6 marks**]

(c) Calculate the tension in the string. [**2 marks**]

(d) Find the ratio of the sphere's acceleration to gravitational acceleration. [**2 marks**]

(e) If the sphere falls through a vertical distance of 1.8 m, calculate the final linear and angular velocities. [**2 marks**]

Question 18 [**9 marks**] A solid disc of mass 6 kg and radius 0.25 m rolls down an inclined plane (angle 28°) starting with an initial linear velocity of 4 m/s.

(a) Calculate the initial total kinetic energy. [**3 marks**]

(b) If the disc rolls 6 m down the incline, find the final linear velocity using energy conservation. [**4 marks**]

(c) Calculate the final angular velocity and total kinetic energy. [**2 marks**]

Section I: Complex Systems [24 marks]

Question 19 [**15 marks**] A sophisticated pulley system has two coaxial wheels: a large wheel (radius 0.8 m) and a small wheel (radius 0.45 m) with combined moment of inertia 8.5 kg·m^2. A 40 kg mass hangs from the large wheel, while a 25 kg mass hangs from the small wheel on the opposite side.

(a) Draw comprehensive free body diagrams for the system. [**4 marks**]

(b) Establish the complete system of equations of motion. [**5 marks**]

(c) Solve for the angular acceleration of the wheel system. [**3 marks**]

(d) Calculate the tensions in both cables. [**3 marks**]

Question 20 [**9 marks**] A uniform rod of mass 8 kg and length 2.4 m is pivoted at a point 0.9 m from one end. It is released from rest when the longer section is 45° above horizontal.

(a) Calculate the moment of inertia about the pivot. [**3 marks**]

(b) Find the initial angular acceleration using torque analysis. [**3 marks**]

(c) Use energy methods to find the angular velocity when the rod is vertical. [**3 marks**]

Section J: Advanced Applications [24 marks]

Question 21 [**15 marks**] A Mars rover's wheel system consists of a central hub (solid cylinder, mass 40 kg, radius 0.3 m) with six spokes (each modeled as a uniform rod, mass 3 kg, length 0.5 m) and an outer rim (thin ring, mass 15 kg, radius 0.5 m).

(a) Calculate the moment of inertia of each component about the central axis. [**5 marks**]

(b) Find the total moment of inertia of the wheel. [**2 marks**]

(c) If the wheel rotates at 2.5 rad/s, calculate its angular momentum. [**2 marks**]

(d) During a maneuver, the motor applies a torque of 180 N·m for 15 seconds. Find the change in angular velocity. **[3 marks]**

(e) If the wheel hits an obstacle and one spoke breaks off completely, calculate the new angular velocity. **[3 marks]**

Question 22 [9 marks] A circular saw blade (solid disc, mass 2.5 kg, radius 0.15 m) operates at 3600 rpm. When cutting wood, the friction force at the rim is 45 N opposing the motion.

(a) Convert the operating speed to rad/s and calculate the angular momentum. **[3 marks]**

(b) Find the angular deceleration due to the friction force. **[3 marks]**

(c) Calculate the time taken to reduce the speed to 2400 rpm. **[3 marks]**

Physics Data and Formulae

Rotational Kinematics:

$\omega = \frac{d\theta}{dt}$, $\alpha = \frac{d\omega}{dt}$, $v = r\omega$, $a_t = r\alpha$, $a_c = \frac{v^2}{r} = \omega^2 r$

Moment of Inertia:

Point mass: $I = mr^2$

Parallel axis theorem: $I = I_{cm} + md^2$

Uniform rod (center): $I = \frac{1}{12}ML^2$, (end): $I = \frac{1}{3}ML^2$

Solid disc/cylinder: $I = \frac{1}{2}MR^2$, Thin ring: $I = MR^2$

Hollow cylinder: $I = \frac{1}{2}M(R_1^2 + R_2^2)$

Solid sphere: $I = \frac{2}{5}MR^2$, Hollow sphere: $I = \frac{2}{3}MR^2$

Rotational Dynamics:

$\tau = I\alpha$ (equation of rotational motion)

$\tau = \vec{r} \times \vec{F}$ (torque from force)

Work: $W = \tau\theta$, Power: $P = \tau\omega$

Rotational Energy:

$KE_{rot} = \frac{1}{2}I\omega^2$

Rolling motion: $KE_{total} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

Rolling constraint: $v = r\omega$ (no slipping)

Angular Momentum:

$L = I\omega$ (rigid body), $L = \vec{r} \times \vec{p}$ (point particle)

Conservation: $L_i = L_f$ (when $\sum \tau_{ext} = 0$)

Constants:

Acceleration due to gravity: $g = 9.8$ m/s²

Trigonometric Values:

$\sin 28 = 0.469$, $\cos 28 = 0.883$, $\tan 28 = 0.532$

$\sin 40 = 0.643$, $\cos 40 = 0.766$, $\tan 40 = 0.839$

$\sin 45 = 0.707$, $\cos 45 = 0.707$, $\tan 45 = 1.000$

$\sin 60 = 0.866$, $\cos 60 = 0.500$, $\tan 60 = 1.732$

END OF TEST

Total marks: 276

Grade boundaries: A* 248, A 221, B 193, C 166, D 138, E 110

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