A Level Mechanics
Practice Test 5: Gravitational Fields

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Draw diagrams where appropriate to illustrate your solutions.
Time allowed: 3 hours 10 minutes

Section A: Multi-Body Gravitational Systems [40 marks]

Question 1 [25 marks] Four masses are arranged at the corners of a square with side length 2.0 m: mass A (300 kg) at origin (0,0), mass B (400 kg) at (2,0), mass C (500 kg) at (2,2), and mass D (600 kg) at (0,2).

(a) Calculate the gravitational force on mass A due to each of the other three masses. **[9 marks]**

(b) Find the components of the net gravitational force on mass A. **[6 marks]**

(c) Calculate the magnitude and direction of the net force on mass A. **[4 marks]**

(d) Determine the gravitational field strength at the center of the square due to all four masses. **[6 marks]**

Question 2 [15 marks] A triple star system consists of three identical stars, each with mass 4.5 $\times$ 10^3 kg, arranged in an equilateral triangle with side length 8.0 $\times$ 10^{11} m.

(a) Calculate the gravitational force on one star due to the other two stars. **[6 marks]**

(b) Find the net gravitational force on this star. **[4 marks]**

(c) Calculate the gravitational field strength at the center of the triangle. **[5 marks]**

Section B: Gravitational Fields in Extended Objects [35 marks]

Question 3 [20 marks]

(a) State and prove the shell theorem for gravitational fields: that a uniform spherical shell of mass M acts gravitationally as if all its mass were concentrated at the center, for points outside the shell. **[8 marks]**

(b) Show that the gravitational field inside a uniform spherical shell is zero everywhere. **[6 marks]**

(c) For a uniform solid sphere of mass M and radius R, derive the expression for gravitational field strength at distance r from the center when r ¡ R. **[6 marks]**

Question 4 [15 marks] Earth can be modeled as a uniform sphere with mass 5.97 $\times$ 10^2 kg and radius 6.37 $\times$ 10 m.

(a) Calculate the gravitational field strength at Earth's surface. [**3 marks**]

(b) Find the gravitational field strength at a depth of 1000 km below Earth's surface. [**5 marks**]

(c) Calculate the gravitational field strength at Earth's center. [**2 marks**]

(d) Sketch a graph showing how gravitational field strength varies from Earth's center to a point 2R from the center. [**5 marks**]

Section C: Advanced Orbital Analysis [50 marks]

Question 5 [**25 marks**] A space agency plans to establish a lunar base and needs to analyze orbital mechanics around the Moon.

(a) Calculate the orbital velocity for a satellite in circular orbit 200 km above the Moon's surface. [**4 marks**]

(b) Find the orbital period for this satellite. [**4 marks**]

(c) A supply ship needs to transfer from this orbit to a higher orbit 800 km above the Moon's surface. Calculate:

- (i) The orbital velocity in the higher orbit. [**3 marks**]
- (ii) The orbital period in the higher orbit. [**3 marks**]

(d) For a 5000 kg supply ship, calculate the energy required for this orbital transfer. [**6 marks**]

(e) Determine the escape velocity from the Moon's surface and compare it with the orbital velocities calculated above. [**5 marks**]

Question 6 [**25 marks**]

(a) For an elliptical orbit, explain how gravitational potential energy and kinetic energy vary with position, and show that total energy remains constant. [**6 marks**]

(b) A comet has an elliptical orbit around the Sun with closest approach (perihelion) at 0.8 AU and farthest distance (aphelion) at 5.2 AU from the Sun. (1 AU = 1.50×10^{11} m)

- (i) Calculate the comet's velocity at perihelion. [**6 marks**]
- (ii) Find the comet's velocity at aphelion. [**4 marks**]
- (iii) Determine the orbital period of the comet. [**5 marks**]

(c) Explain how conservation of angular momentum affects the comet's motion during its orbit. [**4 marks**]

Section D: Gravitational Potential Theory [40 marks]

Question 7 [**22 marks**]

(a) Define gravitational potential and explain its relationship to gravitational potential energy. [**4 marks**]

(b) Show that the gravitational field strength is related to gravitational potential by g = -dV/dr. [**6 marks**]

(c) For two point masses M and M separated by distance d, derive an expression for the gravitational potential at any point along the line joining them. [**6 marks**]

(d) Calculate the gravitational potential at the midpoint between Earth and Moon. [**6 marks**]

Question 8 [**18 marks**] A spacecraft travels from Earth's surface to Mars' surface during a mission when the planets are 7.5×10^1 m apart.

(a) Calculate the gravitational potential energy of a 4000 kg spacecraft at Earth's surface. [**3 marks**]

(b) Find the gravitational potential energy of the spacecraft at Mars' surface (Mars mass = 6.39×10^{23} kg, radius = 3.39×10 m). [**4 marks**]

(c) Determine the change in potential energy for this interplanetary journey. [**3 marks**]

(d) Calculate the minimum launch velocity required from Earth's surface to reach Mars. [**6 marks**]

(e) If the spacecraft arrives at Mars with velocity 3.5 km/s, what was its actual launch velocity from Earth? [**2 marks**]

Section E: Satellite Applications and Orbital Transfers [35 marks]

Question 9 [**20 marks**] A weather monitoring system requires three types of satellites in different orbits around Earth.

(a) Calculate the orbital parameters for each satellite type:

(i) Low Earth Orbit (LEO) at 500 km altitude: orbital velocity and period. [**4 marks**]

(ii) Medium Earth Orbit (MEO) at 20,000 km altitude: orbital velocity and period. [**4 marks**]

(iii) Geostationary Earth Orbit (GEO): orbital radius, height, and velocity. [**5 marks**]

(b) Compare the advantages and disadvantages of each orbit type for weather monitoring. [**4 marks**]

(c) Calculate the energy required to move a 2000 kg satellite from LEO to GEO. [**3 marks**]

Question 10 [**15 marks**] A Hohmann transfer orbit is an elliptical orbit used to transfer spacecraft between two circular orbits.

(a) Explain the principle of a Hohmann transfer and why it is energy-efficient. [**4 marks**]

(b) Calculate the transfer velocity changes (v) required to move a satellite from a 300 km circular orbit to a 600 km circular orbit using a Hohmann transfer. [**8 marks**]

(c) Determine the transfer time for this Hohmann transfer. [**3 marks**]

Section F: Advanced Gravitational Phenomena [25 marks]

Question 11 [**15 marks**] Gravitational time dilation is a consequence of general relativity, but can be approximated using gravitational potential for weak fields.

(a) The fractional change in time rate due to gravitational potential difference is approximately t/t V/c^2, where c is the speed of light. Calculate the time dilation effect for a clock at:

(i) Earth's surface compared to infinity. [**4 marks**]

(ii) Geostationary orbit compared to Earth's surface. [**5 marks**]

(b) Explain why GPS satellites must account for gravitational time dilation for accurate positioning. [**3 marks**]

(c) Calculate the daily time error that would accumulate if time dilation were not corrected for GPS satellites. [**3 marks**]

Question 12 [10 marks] Gravitational lensing occurs when massive objects bend the path of light due to spacetime curvature.

(a) In Newtonian approximation, the deflection angle for light passing a massive object is 4GM/(rc²), where r is the closest approach distance. Calculate the deflection angle for light grazing the Sun's surface. **[5 marks]**

(b) Explain how gravitational lensing is used in astronomy to study distant galaxies and dark matter. **[3 marks]**

(c) Describe the Einstein ring phenomenon and the conditions required for its observation. **[2 marks]**

Physics Data and Formulae

Gravitational Force and Field:
Newton's Law: $F = \frac{Gm_1m_2}{r^2}$
Field strength: $g = \frac{F}{m} = \frac{GM}{r^2}$
Field inside uniform sphere: $g = \frac{GMr}{R^3}$ (for $r < R$)
Field-potential relation: $g = -\frac{dV}{dr}$

Gravitational Potential and Energy:
Potential: $V = -\frac{GM}{r}$
Potential energy: $U = mV = -\frac{GMm}{r}$
Multiple masses: $V = \sum_i V_i = -G\sum_i \frac{M_i}{r_i}$

Orbital Motion:
Circular orbital velocity: $v = \sqrt{\frac{GM}{r}}$
Orbital period: $T = 2\pi\sqrt{\frac{r^3}{GM}}$
Kepler's Third Law: $\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$
Escape velocity: $v_e = \sqrt{\frac{2GM}{r}}$

Energy in Orbits:
Circular orbit kinetic energy: $E_k = \frac{GMm}{2r}$
Potential energy: $E_p = -\frac{GMm}{r}$
Total energy: $E = -\frac{GMm}{2r}$
Elliptical orbit: $E = -\frac{GMm}{2a}$ (where a is semi-major axis)

Conservation Laws:
Energy conservation: $E = \frac{1}{2}mv^2 - \frac{GMm}{r} = \text{constant}$
Angular momentum: $L = mvr = \text{constant}$ (for central forces)

Physical Constants:
Gravitational constant: $G = 6.67 \times 10^{-11}$ Nm²/kg²
Earth's mass: $M_E = 5.97 \times 10^{24}$ kg
Earth's radius: $R_E = 6.37 \times 10^{6}$ m
Moon's mass: $M_M = 7.35 \times 10^{22}$ kg
Moon's radius: $R_M = 1.74 \times 10^{6}$ m
Sun's mass: $M_S = 1.99 \times 10^{30}$ kg
Earth-Sun distance: 1.50×10^{11} m (1 AU)
Speed of light: $c = 3.00 \times 10^{8}$ m/s

END OF TEST

Total marks: 225

Grade boundaries: A* 203, A 180, B 158, C 135, D 113, E 90

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