A Level Pure Mathematics
Practice Test 1: Algebra and Functions

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Time allowed: 2 hours

Section A: Algebraic Manipulation

1. Simplify these expressions:

   (a) $\frac{x^2-9}{x^2+6x+9}$

   (b) $\frac{2x^2-8}{x^2-5x+6}$

   (c) $\frac{x^3-8}{x^2-4}$

   (d) $\frac{x^4-16}{x^2+2x-8}$

2. Factorize completely:

   (a) $x^3 + 6x^2 + 12x + 8$

   (b) $8x^3 - 27$

   (c) $x^4 - 1$

   (d) $x^6 - 64$

   (e) $x^4 + 4x^2 + 4$

   (f) $x^3 - 3x^2 - 4x + 12$

3. Express as single fractions in simplest form:

   (a) $\frac{2}{x-1} + \frac{3}{x+2}$

   (b) $\frac{x}{x^2-4} - \frac{1}{x+2}$

   (c) $\frac{2x+1}{x^2+x-2} + \frac{x-3}{x^2-1}$

   (d) $\frac{1}{x} + \frac{1}{x-1} - \frac{2}{x^2-x}$

4. Use the binomial theorem to expand:

   (a) $(2x+3)^4$

   (b) $(x - \frac{1}{x})^5$

   (c) $(1 + 2x)^6$, and find the coefficient of x^4

   (d) Find the constant term in the expansion of $(x^2 + \frac{2}{x})^9$

5. Simplify using laws of indices:

   (a) $\frac{2^{3x} \cdot 4^{x-1}}{8^{x+2}}$

(b) $\frac{27^{2x-1} \cdot 9^{x+1}}{3^{5x-2}}$

(c) $\left(x^{\frac{2}{3}}\right)^{\frac{3}{4}} \cdot x^{-\frac{1}{2}}$

(d) $\frac{(2x)^3 \cdot (3x^2)^2}{6x^4}$

Section B: Linear and Quadratic Equations

6. Solve these equations:

   (a) $\frac{2x-1}{3} - \frac{x+2}{4} = \frac{1}{2}$

   (b) $\frac{x}{x-2} = \frac{3}{x+1}$

   (c) $\sqrt{2x+3} = x$

   (d) $\frac{1}{x-1} + \frac{1}{x+1} = \frac{1}{2}$

7. Solve these quadratic equations, leaving answers in exact form where appropriate:

   (a) $2x^2 - 7x + 3 = 0$

   (b) $x^2 + 4x - 1 = 0$

   (c) $3x^2 = 5x + 2$

   (d) $(2x-1)^2 = 3(x+2)$

8. For the quadratic equation $kx^2 + (k+3)x + 1 = 0$:

   (a) Find the discriminant in terms of k

   (b) Find the values of k for which the equation has equal roots

   (c) Find the values of k for which the equation has no real roots

   (d) When $k = 2$, find the sum and product of the roots

9. The quadratic $ax^2 + bx + c = 0$ has roots α and β.

   (a) Express $\alpha + \beta$ and $\alpha\beta$ in terms of a, b, and c

   (b) Find a quadratic equation with roots 2α and 2β

   (c) Find a quadratic equation with roots $\alpha + 1$ and $\beta + 1$

   (d) If $\alpha^2 + \beta^2 = 10$ and $\alpha + \beta = 4$, find $\alpha\beta$

Section C: Cubic and Higher Order Equations

10. Solve these cubic equations:

    (a) $x^3 - 6x^2 + 11x - 6 = 0$

    (b) $x^3 + 2x^2 - 5x - 6 = 0$

    (c) $2x^3 - x^2 - 13x - 6 = 0$

    (d) $x^3 - 7x + 6 = 0$

11. Given that $x = 2$ is a root of $x^3 + px^2 + qx - 8 = 0$:

    (a) Find a relationship between p and q

    (b) If the sum of all three roots is -1, find p and q

    (c) Hence find all three roots

    (d) Verify your answer by substitution

12. Solve these quartic equations:

(a) $x^4 - 5x^2 + 4 = 0$

(b) $x^4 - 13x^2 + 36 = 0$

(c) $(x^2 + x)^2 - 8(x^2 + x) + 12 = 0$

(d) $x^4 + x^3 - 7x^2 - x + 6 = 0$ (given that $x = 1$ is a root)

13. Use the substitution $y = x + \frac{1}{x}$ to solve:

(a) $x^2 + \frac{1}{x^2} = 7$

(b) $2x^2 + 3x + \frac{3}{x} + \frac{2}{x^2} = 0$

Section D: Functions - Definition and Notation

14. For the function $f(x) = \frac{2x+1}{x-3}$ where $x \neq 3$:

(a) Find $f(0)$, $f(1)$, and $f(-2)$

(b) Solve $f(x) = 1$

(c) Find the value of x for which $f(x)$ is undefined

(d) Find the range of $f(x)$

15. Given $g(x) = x^2 - 4x + 3$:

(a) Express $g(x)$ in the form $(x - a)^2 + b$

(b) State the minimum value of $g(x)$ and the value of x at which it occurs

(c) Solve $g(x) = 0$

(d) Find the range of $g(x)$

16. For $h(x) = \sqrt{9 - x^2}$:

(a) Find the domain of $h(x)$

(b) Find the range of $h(x)$

(c) Sketch the graph of $y = h(x)$

(d) Solve $h(x) = 2$

17. Define $k(x) = \begin{cases} x^2 & \text{if } x < 0 \\ 2x + 1 & \text{if } 0 \leq x < 2 \\ 5 & \text{if } x \geq 2 \end{cases}$

(a) Find $k(-2)$, $k(0)$, $k(1.5)$, and $k(3)$

(b) Is $k(x)$ continuous at $x = 0$? Justify your answer

(c) Is $k(x)$ continuous at $x = 2$? Justify your answer

(d) Sketch the graph of $y = k(x)$

Section E: Composite and Inverse Functions

18. Given $f(x) = 2x + 3$ and $g(x) = x^2 - 1$:

(a) Find $f(g(x))$ and $g(f(x))$

(b) Calculate $f(g(2))$ and $g(f(2))$

(c) Solve $f(g(x)) = 11$

(d) Find $(f \circ g)^{-1}(x)$

19. For $p(x) = \frac{x+1}{x-2}$ where $x \neq 2$:

    (a) Find $p^{-1}(x)$
    (b) Verify that $p(p^{-1}(x)) = x$
    (c) State the domain and range of $p^{-1}(x)$
    (d) Solve $p(x) = p^{-1}(x)$

20. Given $f(x) = 3x - 2$ and $g(x) = \frac{1}{x+1}$ where $x \neq -1$:

    (a) Find $(f \circ g)(x)$ and state its domain
    (b) Find $(g \circ f)(x)$ and state its domain
    (c) Find $(f \circ g)^{-1}(x)$
    (d) Verify your answer by showing $(f \circ g)((f \circ g)^{-1}(x)) = x$

21. The function $h(x) = x^2 + 4x + 1$ is defined for $x \geq -2$.

    (a) Explain why the domain restriction is necessary for h^{-1} to exist
    (b) Find $h^{-1}(x)$
    (c) State the domain and range of $h^{-1}(x)$
    (d) Sketch $h(x)$ and $h^{-1}(x)$ on the same axes

Section F: Graphing Functions

22. Sketch the graphs of these functions, clearly showing key features:

    (a) $y = x^3 - 3x^2 + 2$
    (b) $y = \frac{2x+1}{x-1}$
    (c) $y = |x^2 - 4x + 3|$
    (d) $y = \frac{x^2-1}{x^2+1}$

23. For the rational function $f(x) = \frac{x^2+x-2}{x^2-4}$:

    (a) Find the domain of $f(x)$
    (b) Find the x and y intercepts
    (c) Identify any vertical asymptotes
    (d) Find the horizontal asymptote
    (e) Sketch the graph of $y = f(x)$

24. Analyze the function $g(x) = \frac{2x^2-8}{x^2+x-6}$:

    (a) Factorize the numerator and denominator
    (b) Simplify $g(x)$ and state its domain
    (c) Find any asymptotes
    (d) Find the coordinates of any stationary points
    (e) Sketch the graph of $y = g(x)$

25. For the polynomial $p(x) = x^4 - 4x^3 + 4x^2$:

    (a) Factorize $p(x)$ completely
    (b) Find the roots and their multiplicities
    (c) Determine the behavior at each root
    (d) Find $p'(x)$ and locate stationary points
    (e) Sketch the graph of $y = p(x)$

Section G: Function Transformations

26. Given the function $f(x) = x^2$, describe the transformations and sketch:

    (a) $y = f(x-2) + 3$
    (b) $y = -2f(x+1)$
    (c) $y = f(2x) - 4$
    (d) $y = \frac{1}{2}f(-x) + 1$

27. The graph of $y = f(x)$ has vertex at $(3, -2)$ and passes through $(1, 2)$ and $(5, 2)$. Find the vertex and two other points for:

    (a) $y = f(x) + 4$
    (b) $y = f(x-2)$
    (c) $y = 3f(x)$
    (d) $y = f(2x)$
    (e) $y = -f(x)$
    (f) $y = f(-x)$

28. Given that $g(x) = |x-1| + 2$:

    (a) Describe the transformations applied to $y = |x|$
    (b) State the vertex of the graph
    (c) Find the range of $g(x)$
    (d) Solve $g(x) = 5$
    (e) Sketch the graph of $y = g(x)$

29. The function $h(x) = \sin x$ is transformed to $k(x) = 3\sin(2x + \frac{\pi}{3}) - 1$.

    (a) Identify each transformation in the correct order
    (b) State the amplitude of $k(x)$
    (c) State the period of $k(x)$
    (d) Find the phase shift
    (e) Find the vertical shift
    (f) Sketch one complete cycle of $y = k(x)$

Section H: Special Functions and Applications

30. For the exponential function $f(x) = 3^{x-1} + 2$:

    (a) State the domain and range
    (b) Find the y-intercept
    (c) Find the horizontal asymptote
    (d) Solve $f(x) = 11$
    (e) Find $f^{-1}(x)$ and state its domain and range

31. For the logarithmic function $g(x) = \log_2(x+3) - 1$:

    (a) State the domain and range
    (b) Find the x-intercept
    (c) Find the vertical asymptote

(d) Solve $g(x) = 2$

(e) Express $g(x)$ in terms of natural logarithms

32. A function is defined as $f(x) = \frac{ax+b}{cx+d}$ where $ad - bc \neq 0$.

    (a) Find the domain of $f(x)$

    (b) Find $f^{-1}(x)$

    (c) Show that $(f^{-1} \circ f)(x) = x$

    (d) Under what condition is $f(x) = f^{-1}(x)$?

33. The modulus function $|x|$ can be written as: $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

    (a) Sketch $y = |2x - 3|$

    (b) Solve $|2x - 3| = 5$

    (c) Solve $|2x - 3| < 4$

    (d) Find the range of values for which $|2x - 3| \geq 1$

Section I: Problem Solving and Applications

34. A rectangular enclosure is to be built against a wall using 60 meters of fencing. Let x be the width perpendicular to the wall.

    (a) Express the length parallel to the wall in terms of x

    (b) Show that the area $A = x(60 - 2x)$

    (c) Find the value of x that maximizes the area

    (d) Calculate the maximum area

    (e) State the domain of the function in this context

35. The profit P (in thousands of pounds) from selling x thousand items is given by: $P(x) = -2x^2 + 16x - 24$

    (a) Express $P(x)$ in completed square form

    (b) Find the break-even points (where $P(x) = 0$)

    (c) Determine the production level for maximum profit

    (d) Calculate the maximum profit

    (e) For what range of production levels is the profit positive?

36. A water tank is being filled at a rate that depends on time. The volume V (in liters) after t minutes is: $V(t) = 100t - 2t^2$ for $0 \leq t \leq 25$

    (a) Find when the tank is being filled fastest

    (b) Calculate the maximum volume in the tank

    (c) Determine when the tank starts emptying

    (d) Find the total time to fill and empty the tank

37. A function $f(x) = \frac{x^2 - 4}{x^2 + 1}$ models a physical process.

    (a) Find the domain and range of $f(x)$

    (b) Determine the horizontal asymptote and explain its physical meaning

    (c) Find the values of x where $f(x) = 0$

(d) Analyze the behavior as $x \to \pm\infty$

(e) Sketch the graph and discuss any symmetry

38. Two functions are related by $g(x) = f(2x - 1) + 3$ where $f(x) = x^2$.

    (a) Find an explicit expression for $g(x)$

    (b) Describe the transformations that map f to g

    (c) Find the vertex of the parabola $y = g(x)$

    (d) If f has domain $[0, 4]$, find the domain of g

    (e) Solve $g(x) = f(x)$ and interpret geometrically

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 150

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