A Level Mechanics
Practice Test 3: Statics and Dynamics of Particles

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Draw clear diagrams where appropriate to illustrate your solutions.
Time allowed: 2 hours 45 minutes

Section A: Vector Forces and Equilibrium [35 marks]

Question 1 [**15 marks**] A particle is acted upon by four forces: 30 N due east, 24 N at 30° north of east, 18 N due north, and a fourth force F that maintains equilibrium.

(a) Resolve the 24 N force into its horizontal (east) and vertical (north) components. [**3 marks**]

(b) Calculate the total horizontal component of the three known forces. [**2 marks**]

(c) Calculate the total vertical component of the three known forces. [**2 marks**]

(d) Find the horizontal and vertical components of force F. [**3 marks**]

(e) Calculate the magnitude and direction of force F. [**4 marks**]

(f) Draw a vector diagram showing all forces acting on the particle. [**1 mark**]

Question 2 [**20 marks**] A load of mass 80 kg is suspended by three cables meeting at a point. Cable A is vertical, cable B makes an angle of 35° with the vertical, and cable C makes an angle of 45° with the vertical. All cables lie in the same vertical plane.

(a) Draw a force diagram showing all forces at the junction point. [**3 marks**]

(b) If the tension in cable B is 400 N, write equilibrium equations for horizontal and vertical directions. [**4 marks**]

(c) Calculate the tension in cable C. [**5 marks**]

(d) Find the tension in cable A. [**4 marks**]

(e) Calculate the total weight being supported and verify this matches the given mass. [**3 marks**]

(f) State what would happen to the tensions if angle of cable B were reduced to 20°. [**1 mark**]

Section B: Advanced Inclined Plane Problems [45 marks]

Question 3 [**25 marks**] A block of mass 20 kg rests on an inclined plane at 32° to the horizontal. The coefficient of static friction is 0.7 and the coefficient of kinetic friction is 0.5. A force P is applied horizontally to the block.

(a) Calculate the component of weight parallel to the inclined plane. [**3 marks**]

(b) Find the component of weight perpendicular to the inclined plane. [**2 marks**]

(c) Without force P, determine whether the block will slide down the plane. [**4 marks**]

(d) If force P = 50 N is applied horizontally to push the block up the plane:

(i) Resolve this force into components parallel and perpendicular to the plane. [**4 marks**]

(ii) Calculate the normal reaction force. [**3 marks**]

(iii) Determine whether the block will move up the plane. [**4 marks**]

(e) Find the range of horizontal force P that will keep the block in equilibrium on the plane. [**5 marks**]

Question 4 [**20 marks**] A toboggan of mass 15 kg slides down a snow-covered slope inclined at 18° to the horizontal. The coefficient of kinetic friction between the toboggan and snow is 0.08.

(a) Draw a force diagram for the toboggan. [**3 marks**]

(b) Calculate all force components acting on the toboggan. [**6 marks**]

(c) Find the net force acting down the slope. [**3 marks**]

(d) Calculate the acceleration of the toboggan. [**3 marks**]

(e) If the toboggan starts from rest and slides 25 m down the slope, calculate:

(i) The final velocity. [**2 marks**]

(ii) The time taken. [**3 marks**]

Section C: Complex Connected Particle Systems [50 marks]

Question 5 [**25 marks**] Three particles A, B, and C have masses 6 kg, 4 kg, and 8 kg respectively. Particle A rests on a smooth horizontal table and is connected by a light inextensible string passing over a smooth pulley to particle B, which hangs freely. Particle A is also connected by another string over a second smooth pulley to particle C, which rests on a rough inclined plane at 30° to the horizontal with coefficient of friction 0.4.

(a) Draw separate force diagrams for all three particles. [**6 marks**]

(b) For particle C on the inclined plane, calculate:

(i) The component of weight down the plane. [**2 marks**]

(ii) The normal reaction force. [**2 marks**]

(iii) The maximum friction force available. [**2 marks**]

(c) Determine the direction of motion by analyzing the forces. [**3 marks**]

(d) Write equations of motion for each particle and solve for the acceleration of the system. [**8 marks**]

(e) Calculate the tensions in both strings. **[2 marks]**

Question 6 [25 marks] Two particles P and Q are connected by a light inextensible string of length 3 m passing over a smooth pulley. Particle P (mass 5 kg) can slide on a smooth inclined plane at 40° to the horizontal, while particle Q (mass 3 kg) hangs freely and can move vertically.

(a) Draw force diagrams for both particles. **[4 marks]**

(b) Calculate the component of weight of P acting down the inclined plane. **[3 marks]**

(c) Write the equation of motion for particle P along the inclined plane. **[3 marks]**

(d) Write the equation of motion for particle Q in the vertical direction. **[3 marks]**

(e) Solve these equations to find the acceleration of the system. **[6 marks]**

(f) Calculate the tension in the string. **[3 marks]**

(g) If both particles start from rest, find their velocities after 2 seconds. **[3 marks]**

Section D: Motion with Variable Forces [30 marks]

Question 7 [18 marks] A particle of mass 2.5 kg moves along a straight line under the action of a force $F = (15 + 6t - 2t^2)$ N, where t is time in seconds. At $t = 0$, the particle is at rest.

(a) Calculate the acceleration of the particle at $t = 0$ and $t = 3$ s. **[4 marks]**

(b) Find expressions for the velocity and displacement as functions of time. **[8 marks]**

(c) Calculate the velocity and displacement when $t = 4$ s. **[3 marks]**

(d) At what time does the particle momentarily come to rest again? **[3 marks]**

Question 8 [12 marks] A particle of mass 1.2 kg moves through a medium that exerts a resistance force proportional to velocity. The resistance force is $R = 3v$ N, where v is velocity in m/s. The particle has an initial velocity of 8 m/s.

(a) Write the differential equation of motion for the particle. **[3 marks]**

(b) Solve this equation to find velocity as a function of time. **[5 marks]**

(c) Calculate the time taken for the velocity to reduce to 2 m/s. **[2 marks]**

(d) Find the distance traveled during this time. **[2 marks]**

Section E: Advanced Projectile Motion [35 marks]

Question 9 [20 marks] A projectile is launched from the top of a 60 m tall building at an angle of 38° above the horizontal with an initial speed of 32 m/s.

(a) Calculate the initial horizontal and vertical components of velocity. **[3 marks]**

(b) Find the maximum height above the ground reached by the projectile. **[5 marks]**

(c) Calculate the time taken to reach maximum height. **[2 marks]**

(d) Determine the total time of flight until the projectile hits the ground. **[4 marks]**

(e) Calculate the horizontal distance from the base of the building where the projectile lands. **[3 marks]**

(f) Find the magnitude and direction of velocity just before impact with the ground. **[3 marks]**

Question 10 [15 marks] A basketball player throws a ball toward a hoop that is 3.0 m high and 8.0 m away horizontally. The ball is released from a height of 2.0 m at an angle of 45° above the horizontal.

(a) Calculate the required initial speed for the ball to pass through the hoop. **[8 marks]**

(b) For this initial speed, find the maximum height reached by the ball. **[3 marks]**

(c) Calculate the time taken for the ball to reach the hoop. **[2 marks]**

(d) Determine the velocity components of the ball as it passes through the hoop. **[2 marks]**

Section F: Limiting Equilibrium and Stability [25 marks]

Question 11 [15 marks] A particle rests on a rough inclined plane. The plane can be tilted to different angles. The coefficient of static friction between the particle and plane is 0.6.

(a) Derive a general expression for the friction force in terms of the angle of the plane to the horizontal. **[3 marks]**

(b) Find the critical angle at which the particle just begins to slide. **[4 marks]**

(c) At this critical angle, calculate the ratio of friction force to normal reaction force. **[2 marks]**

(d) If the angle is increased to 40°, calculate the acceleration of the particle down the plane (coefficient of kinetic friction = 0.45). **[4 marks]**

(e) Find the velocity of the particle after sliding 2 m down the plane at 40°. **[2 marks]**

Question 12 [10 marks] A particle of mass 0.8 kg hangs in equilibrium suspended by two strings. One string makes an angle of 50° with the vertical, and the other makes an angle of 30° with the vertical.

(a) Draw a force diagram for the particle. **[2 marks]**

(b) Calculate the tension in each string. **[6 marks]**

(c) If the 50° string can only support a maximum tension of 6 N, what is the maximum mass that can be supported in this configuration? **[2 marks]**

Physics Data and Formulae

Forces and Equilibrium:

Equilibrium condition: $\sum \vec{F} = 0$ or $\sum F_x = 0, \sum F_y = 0$

Force components: $F_x = F\cos\theta$, $F_y = F\sin\theta$

Resultant: $|\vec{R}| = \sqrt{F_x^2 + F_y^2}$, $\tan\alpha = \frac{F_y}{F_x}$

Newton's Laws:

First Law: $\sum \vec{F} = 0$ (equilibrium or constant velocity)

Second Law: $\sum \vec{F} = m\vec{a}$ or $F = ma$

Third Law: $\vec{F}_{AB} = -\vec{F}_{BA}$

Friction:

Static friction: $f_s \leq \mu_s N$, maximum $f_{s,max} = \mu_s N$

Kinetic friction: $f_k = \mu_k N$
Limiting equilibrium: $f = \mu_s N$

Inclined Planes:
Weight component parallel to plane: $mg\sin\theta$ (down)
Weight component perpendicular to plane: $mg\cos\theta$ (into plane)
Normal reaction: $N = mg\cos\theta$ (no other perpendicular forces)

Kinematics:
$v = u + at$
$s = ut + \frac{1}{2}at^2$
$v^2 = u^2 + 2as$
$s = \frac{u+v}{2}t$

Projectile Motion:
Horizontal motion: $x = u_x t$, $v_x = u_x$ (constant)
Vertical motion: $y = u_y t - \frac{1}{2}gt^2$, $v_y = u_y - gt$
Maximum height: $h_{max} = \frac{u_y^2}{2g}$
Time to maximum height: $t_{max} = \frac{u_y}{g}$
Range (level projection): $R = \frac{u^2 \sin 2\theta}{g}$

Calculus:
If $s = f(t)$, then $v = \frac{ds}{dt}$ and $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$
$\int a\,dt = v + C$ and $\int v\,dt = s + C$

Constants:
Acceleration due to gravity: $g = 9.81$ m/s^2

Trigonometric Values:
$\sin 18 = 0.309$, $\cos 18 = 0.951$, $\tan 18 = 0.325$
$\sin 30 = 0.500$, $\cos 30 = 0.866$, $\tan 30 = 0.577$
$\sin 32 = 0.530$, $\cos 32 = 0.848$, $\tan 32 = 0.625$
$\sin 35 = 0.574$, $\cos 35 = 0.819$, $\tan 35 = 0.700$
$\sin 38 = 0.616$, $\cos 38 = 0.788$, $\tan 38 = 0.781$
$\sin 40 = 0.643$, $\cos 40 = 0.766$, $\tan 40 = 0.839$
$\sin 45 = 0.707$, $\cos 45 = 0.707$, $\tan 45 = 1.000$
$\sin 50 = 0.766$, $\cos 50 = 0.643$, $\tan 50 = 1.192$

END OF TEST

Total marks: 220

Grade boundaries: A* 198, A 176, B 154, C 132, D 110, E 88

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