A Level Pure Mathematics
Practice Test 1: Coordinate Geometry in the (x, y) Plane

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Time allowed: 2 hours

Section A: Distance and Midpoint Formulas

1. Find the distance between these pairs of points:
   (a) $A(3, 4)$ and $B(7, 1)$
   (b) $C(-2, 5)$ and $D(4, -3)$
   (c) $E(-1, -6)$ and $F(5, 2)$
   (d) $G(0, 0)$ and $H(a, b)$
2. Find the midpoint of the line segment joining:
   (a) $P(2, 7)$ and $Q(8, 3)$
   (b) $R(-4, 1)$ and $S(6, -5)$
   (c) $T(3a, 2b)$ and $U(-a, 4b)$
   (d) The origin and the point $(2p, 3q)$
3. The point $M(5, 2)$ is the midpoint of the line segment AB where $A(1, -4)$.
   (a) Find the coordinates of point B
   (b) Calculate the length of AB
   (c) Find the equation of the perpendicular bisector of AB
4. Points $A(1, 3)$, $B(5, 1)$, and $C(3, 7)$ form a triangle.
   (a) Calculate the length of each side
   (b) Determine what type of triangle ABC is
   (c) Find the coordinates of the circumcenter
   (d) Calculate the area of triangle ABC
5. The points $P(-2, 1)$, $Q(4, 3)$, $R(6, 9)$, and $S(0, 7)$ form a quadrilateral.
   (a) Show that $PQRS$ is a parallelogram
   (b) Find the lengths of the diagonals
   (c) Determine if $PQRS$ is a rectangle
   (d) Calculate the area of parallelogram $PQRS$

Section B: Equations of Straight Lines

6. Find the equation of the straight line:
   (a) With gradient 3 passing through $(2, 5)$
   (b) Passing through $(1, 4)$ and $(3, 10)$
   (c) With x-intercept -2 and y-intercept 6
   (d) Perpendicular to $2x + 3y = 6$ and passing through $(1, -2)$

7. Express these equations in the form $y = mx + c$:
   (a) $3x + 2y = 12$
   (b) $4x - 5y + 10 = 0$
   (c) $x - y + 7 = 0$
   (d) $2x + 3y - 9 = 0$

8. Find the equation of the line that:
   (a) Is parallel to $y = 2x - 3$ and passes through $(4, 1)$
   (b) Is perpendicular to $3x + 4y = 12$ and has y-intercept -2
   (c) Passes through $(2, 3)$ and is parallel to the x-axis
   (d) Passes through $(-1, 5)$ and is perpendicular to the y-axis

9. Two lines have equations $L_1 : 2x - 3y + 6 = 0$ and $L_2 : 4x + y - 8 = 0$.
   (a) Find the point of intersection of L_1 and L_2
   (b) Calculate the acute angle between the lines
   (c) Find the equation of the line through the intersection point perpendicular to L_1
   (d) Determine if the lines are parallel, perpendicular, or neither

10. A triangle has vertices at $A(2, 1)$, $B(6, 4)$, and $C(1, 6)$.
    (a) Find the equation of each side of the triangle
    (b) Find the equation of the altitude from A to side BC
    (c) Find the equation of the median from B to side AC
    (d) Show that the triangle is right-angled

Section C: Angle Between Lines

11. Calculate the acute angle between these pairs of lines:
    (a) $y = 2x + 1$ and $y = -\frac{1}{3}x + 5$
    (b) $x + y = 4$ and $2x - y = 1$
    (c) $3x - 4y + 7 = 0$ and $5x + 2y - 3 = 0$
    (d) $y = \sqrt{3}x + 2$ and $y = \frac{x}{\sqrt{3}} - 1$

12. The line L passes through $(1, 2)$ and makes an angle of $60°$ with the positive x-axis.
    (a) Find the equation of line L
    (b) Find the point where L intersects the line $y = x + 4$
    (c) Calculate the angle between L and the line $y = x + 4$

13. Two lines L_1 and L_2 intersect at the point $(3, -1)$. If L_1 has gradient $\frac{2}{3}$ and the acute angle between the lines is $45°$:

    (a) Find the two possible gradients for L_2

    (b) Write the equations of both possible lines L_2

    (c) Verify your answers by calculating the angles

14. A line makes angles α and β with the coordinate axes, where $\alpha + \beta = 90°$.

    (a) Express the gradient of the line in terms of α

    (b) If the line passes through $(4, 3)$ and $\alpha = 30°$, find its equation

    (c) Find where this line intersects the coordinate axes

Section D: Equation of a Circle

15. Write the equation of the circle with:

    (a) Center $(0, 0)$ and radius 5

    (b) Center $(3, -2)$ and radius 4

    (c) Center $(-1, 4)$ and passing through the origin

    (d) Diameter with endpoints $(2, 1)$ and $(6, 5)$

16. Express these equations in the form $(x - a)^2 + (y - b)^2 = r^2$ and identify the center and radius:

    (a) $x^2 + y^2 - 6x + 4y - 12 = 0$

    (b) $x^2 + y^2 + 8x - 2y + 8 = 0$

    (c) $x^2 + y^2 - 4x + 6y - 3 = 0$

    (d) $2x^2 + 2y^2 - 8x + 12y - 10 = 0$

17. A circle has center $(2, -3)$ and passes through the point $(5, 1)$.

    (a) Find the equation of the circle

    (b) Determine if the point $(0, 0)$ lies inside, outside, or on the circle

    (c) Find the equation of the tangent to the circle at $(5, 1)$

    (d) Find the length of the chord cut off by the line $y = -3$

18. Two circles have equations $C_1 : x^2 + y^2 = 25$ and $C_2 : (x - 7)^2 + y^2 = 16$.

    (a) Find the centers and radii of both circles

    (b) Determine the relationship between the circles (intersecting, touching, or separate)

    (c) If they intersect, find the points of intersection

    (d) Find the equation of the common chord (if it exists)

19. A circle passes through the points $A(1, 3)$, $B(3, 1)$, and $C(5, 5)$.

    (a) Find the equation of the circle

    (b) Verify that all three points satisfy the equation

    (c) Find the center and radius of the circle

    (d) Determine if the triangle ABC is inscribed in the circle

Section E: Parabolas

20. For the parabola $y^2 = 4ax$:
    (a) Identify the focus and directrix when $a = 3$
    (b) If the focus is at $(2, 0)$, find the value of a and the equation of the directrix
    (c) Find the equation of the parabola with focus $(0, 4)$ and directrix $y = -4$
    (d) Sketch the parabola $y^2 = 12x$, showing the focus and directrix

21. A parabola has vertex at the origin and focus at $(0, 3)$.
    (a) Find the equation of the parabola
    (b) Find the equation of the directrix
    (c) If a point on the parabola has x-coordinate 4, find its y-coordinates
    (d) Find the length of the focal chord that passes through $(1, 6)$

22. The parabola $y = ax^2 + bx + c$ passes through $(0, 3)$, $(1, 2)$, and $(2, 5)$.
    (a) Find the values of a, b, and c
    (b) Express the equation in vertex form
    (c) Find the coordinates of the vertex
    (d) Determine if the parabola opens upward or downward

23. A satellite dish has a parabolic cross-section with equation $x^2 = 8y$.
    (a) Find the focus of the parabola
    (b) If the dish is 2 meters wide at the top, find its depth
    (c) Where should the receiver be placed for optimal signal collection?
    (d) Find the equation of the tangent at the point $(4, 2)$

Section F: Ellipses

24. For the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$:
    (a) When $a = 5$ and $b = 3$, find the coordinates of the foci
    (b) Find the eccentricity when $a = 4$ and $b = 3$
    (c) If the foci are at $(\pm 4, 0)$ and $a = 5$, find b
    (d) Sketch the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$

25. An ellipse has center at the origin, major axis along the x-axis of length 10, and minor axis of length 6.
    (a) Write the equation of the ellipse
    (b) Find the coordinates of the foci
    (c) Calculate the eccentricity
    (d) Find the equations of the directrices

26. The ellipse $\frac{(x-2)^2}{25} + \frac{(y+1)^2}{16} = 1$ has center at $(2, -1)$.
    (a) Identify the lengths of the semi-major and semi-minor axes
    (b) Find the coordinates of the vertices
    (c) Calculate the coordinates of the foci

(d) Find the eccentricity

27. An ellipse passes through the points $(3, 0)$, $(0, 2)$, $(-3, 0)$, and $(0, -2)$.

(a) Find the equation of the ellipse
(b) Determine the lengths of the major and minor axes
(c) Calculate the area of the ellipse (Area $= \pi ab$)
(d) Find the maximum distance from the center to any point on the ellipse

Section G: Hyperbolas

28. For the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:

(a) When $a = 3$ and $b = 4$, find the coordinates of the foci
(b) Find the equations of the asymptotes when $a = 2$ and $b = 5$
(c) If the foci are at $(\pm 5, 0)$ and $a = 3$, find b
(d) Sketch the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$

29. A hyperbola has equation $\frac{x^2}{16} - \frac{y^2}{9} = 1$.

(a) Find the coordinates of the vertices
(b) Calculate the coordinates of the foci
(c) Write the equations of the asymptotes
(d) Find the eccentricity

30. The hyperbola $xy = k$ (rectangular hyperbola):

(a) When $k = 12$, find the coordinates where the hyperbola intersects the line $y = 3$
(b) Find the equation of the tangent to $xy = 8$ at the point $(2, 4)$
(c) Show that the asymptotes of $xy = k$ are the coordinate axes
(d) If a point $(t, \frac{k}{t})$ is on the hyperbola, find the equation of the tangent at this point

31. A hyperbola has center at $(1, -2)$, vertices at $(4, -2)$ and $(-2, -2)$, and passes through $(5, 2)$.

(a) Find the value of a (semi-transverse axis)
(b) Calculate the value of b (semi-conjugate axis)
(c) Write the equation of the hyperbola
(d) Find the coordinates of the foci

Section H: Mixed Conic Sections

32. Identify the type of conic section and find its key properties:

(a) $4x^2 + y^2 = 16$
(b) $x^2 - 4y^2 = 16$
(c) $y^2 = 8x$
(d) $x^2 + y^2 - 6x + 4y - 3 = 0$

33. A conic section has equation $Ax^2 + Cy^2 + Dx + Ey + F = 0$.

(a) State the conditions on A and C for the conic to be a circle
(b) State the conditions for the conic to be an ellipse

(c) State the conditions for the conic to be a hyperbola

(d) What happens when either $A = 0$ or $C = 0$?

34. Find the points of intersection (if any) between:

(a) The line $y = x + 1$ and the circle $x^2 + y^2 = 5$

(b) The line $y = 2x$ and the parabola $y^2 = 8x$

(c) The ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$

(d) The circle $x^2 + y^2 = 25$ and the hyperbola $xy = 12$

35. A tangent line to a conic has specific properties:

(a) Find the equation of the tangent to the circle $x^2 + y^2 = 25$ at the point $(3, 4)$

(b) Find the equation of the tangent to the parabola $y^2 = 12x$ at the point $(3, 6)$

(c) Find the equation of the tangent to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ at the point $(2, \frac{3\sqrt{3}}{2})$

(d) Find the equation of the tangent to the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$ at the point $(5, \frac{16}{3})$

Section I: Applications and Problem Solving

36. A bridge has a parabolic arch with span 40 meters and maximum height 16 meters.

(a) Set up a coordinate system and find the equation of the parabola

(b) Find the height of the arch at distances 5, 10, and 15 meters from the center

(c) If a truck is 3 meters wide and 12 meters tall, can it pass under the bridge?

(d) Find the width of the arch at a height of 10 meters

37. An elliptical mirror has major axis 20 cm and minor axis 12 cm. A light source is placed at one focus.

(a) Find the equation of the ellipse with center at the origin

(b) Calculate the distance between the foci

(c) If the light source is at the right focus, where do all reflected rays pass through?

(d) Find the eccentricity of the ellipse

38. A satellite follows an elliptical orbit around Earth. The closest approach (perigee) is 300 km above Earth's surface, and the farthest point (apogee) is 2000 km above the surface. Earth's radius is 6400 km.

(a) Find the semi-major and semi-minor axes of the orbit

(b) Calculate the eccentricity of the orbit

(c) Where is Earth's center located relative to the ellipse?

(d) Find the equation of the orbital ellipse

39. A cooling tower has a hyperbolic cross-section. At the base (ground level), the radius is 20 meters. At the narrowest point (height 60 meters), the radius is 15 meters. At the top (height 120 meters), the radius is 25 meters.

(a) Set up a coordinate system with the narrowest point at the origin

(b) Find the equation of the hyperbola

(c) Verify that the given points satisfy the equation

(d) Calculate the radius at height 40 meters above the narrowest point

40. Three radio stations at points $A(0,0)$, $B(100,0)$, and $C(50,50)$ (distances in km) can locate a ship using the time differences of radio signals.

    (a) If the signal from A reaches the ship 0.001 seconds before the signal from B, what curve does the ship lie on? (Radio waves travel at 3×10^8 m/s)

    (b) Set up the equation for this curve

    (c) If the ship is also equidistant from stations A and C, find its exact location

    (d) Verify your answer by checking the time differences

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 150

For more resources and practice materials, visit:
stepupmaths.co.uk