GCSE Higher Mathematics
Practice Test 4: Number

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Time allowed: 90 minutes

Section A: Powers and Roots

1. Evaluate these expressions:
   (a) 6^2
   (b) 4^{-3}
   (c) 11^0
   (d) $(-6)^2$
   (e) $(-4)^3$
   (f) 8^{-2}

2. Write these as single powers:
   (a) $5^3 \times 5^8$
   (b) $8^7 \div 8^4$
   (c) $(3^5)^2$
   (d) $7^{-4} \times 7^{10}$
   (e) $\frac{9^5}{9^{-6}}$
   (f) $(4^2)^{-5}$

3. Evaluate these fractional indices:
   (a) $49^{\frac{1}{2}}$
   (b) $125^{\frac{1}{3}}$
   (c) $256^{\frac{3}{4}}$
   (d) $32768^{\frac{2}{5}}$
   (e) $343^{\frac{2}{3}}$
   (f) $128^{-\frac{4}{7}}$

4. Simplify these expressions:
   (a) $\sqrt{144}$
   (b) $\sqrt[3]{343}$

(c) $\sqrt[4]{256}$
(d) $\sqrt{0.64}$
(e) $\sqrt[3]{-216}$
(f) $\sqrt[5]{3125}$

5. Express in index form:

(a) $\sqrt{17}$
(b) $\sqrt[3]{11}$
(c) $\frac{1}{\sqrt{7}}$
(d) $\sqrt[4]{w^9}$
(e) $\frac{1}{\sqrt[3]{d^7}}$
(f) $\sqrt{w} \times \sqrt[3]{w}$

Section B: Laws of Indices

6. Simplify these expressions (no calculator):

(a) $5^4 \times 5^{-7} \times 5^8$
(b) $\frac{7^9 \times 7^{-5}}{7^{-4}}$
(c) $(6^2)^{-4} \times 6^{13}$
(d) $\frac{4^{-5} \times 4^{11}}{4^4}$

7. Evaluate these expressions:

(a) $343^{\frac{2}{3}}$
(b) $81^{-\frac{3}{4}}$
(c) $1024^{\frac{3}{5}}$
(d) $729^{-\frac{2}{3}}$
(e) $512^{\frac{1}{3}}$
(f) $3125^{-\frac{4}{5}}$

8. Simplify these expressions:

(a) $x^{\frac{5}{6}} \times x^{\frac{1}{3}}$
(b) $\frac{t^{\frac{8}{3}}}{t^{\frac{1}{5}}}$
(c) $(d^{\frac{1}{5}})^{10}$
(d) $\sqrt{w} \times w^{\frac{1}{6}}$
(e) $\frac{\sqrt[3]{k^7}}{\sqrt{k}}$
(f) $(v^{-\frac{1}{5}})^{-10}$

9. Write these in the form a^n where a and n are rational:

(a) $\sqrt{7} \times 7^5$
(b) $\frac{11^3}{\sqrt[3]{11}}$
(c) $\sqrt[4]{8^9} \times 8^{-\frac{2}{3}}$
(d) $\frac{\sqrt{17}}{\sqrt[3]{17^7}}$

Section C: Surds

10. Simplify these surds:
    (a) $\sqrt{54}$
    (b) $\sqrt{84}$
    (c) $\sqrt{96}$
    (d) $\sqrt{147}$
    (e) $\sqrt{192}$
    (f) $\sqrt{600}$

11. Simplify these expressions:
    (a) $6\sqrt{11} + 9\sqrt{11}$
    (b) $12\sqrt{5} - 5\sqrt{5}$
    (c) $\sqrt{32} + \sqrt{50}$
    (d) $\sqrt{84} - \sqrt{21}$
    (e) $5\sqrt{12} + 7\sqrt{27}$
    (f) $\sqrt{147} - \sqrt{108} + \sqrt{27}$

12. Multiply and simplify:
    (a) $\sqrt{7} \times \sqrt{28}$
    (b) $\sqrt{18} \times \sqrt{72}$
    (c) $6\sqrt{2} \times 4\sqrt{8}$
    (d) $\sqrt{14} \times \sqrt{56}$
    (e) $\sqrt{6} \times \sqrt{24} \times \sqrt{96}$
    (f) $7\sqrt{5} \times 4\sqrt{20}$

13. Expand and simplify:
    (a) $(5 + \sqrt{7})(2 - \sqrt{7})$
    (b) $(4 + \sqrt{13})(1 + 5\sqrt{13})$
    (c) $(7 - \sqrt{11})^2$
    (d) $(\sqrt{17} + \sqrt{3})(\sqrt{17} - \sqrt{3})$
    (e) $(5\sqrt{7} + 1)(5\sqrt{7} - 1)$
    (f) $(\sqrt{13} + 5)^2$

14. Rationalize the denominators:
    (a) $\frac{1}{\sqrt{13}}$
    (b) $\frac{9}{\sqrt{17}}$
    (c) $\frac{\sqrt{7}}{\sqrt{28}}$
    (d) $\frac{10}{5\sqrt{2}}$
    (e) $\frac{1}{4+\sqrt{7}}$
    (f) $\frac{5}{1-\sqrt{13}}$

Section D: More Complex Surd Operations

15. Rationalize these denominators:

    (a) $\frac{7}{5+\sqrt{11}}$

    (b) $\frac{11}{4-\sqrt{19}}$

    (c) $\frac{\sqrt{7}}{1+\sqrt{7}}$

    (d) $\frac{5\sqrt{13}}{4+\sqrt{13}}$

    (e) $\frac{1}{\sqrt{15}-\sqrt{7}}$

    (f) $\frac{\sqrt{13}+4}{\sqrt{13}-5}$

16. Simplify these expressions completely:

    (a) $\frac{\sqrt{24}+\sqrt{54}}{\sqrt{6}}$

    (b) $\frac{\sqrt{63}-\sqrt{28}}{\sqrt{7}}$

    (c) $\sqrt{(6+\sqrt{13})(6-\sqrt{13})}$

    (d) $\sqrt{96}-5\sqrt{6}+\sqrt{54}$

    (e) $(\sqrt{7}+\sqrt{28})^2$

    (f) $\frac{\sqrt{80}}{\sqrt{5}}+\frac{\sqrt{45}}{\sqrt{5}}$

17. Prove that:

    (a) $(\sqrt{f}+\sqrt{g})(\sqrt{f}-\sqrt{g})=f-g$

    (b) $\frac{1}{\sqrt{e}+\sqrt{h}}=\frac{\sqrt{e}-\sqrt{h}}{e-h}$

    (c) $(e+h\sqrt{j})^2=e^2+2eh\sqrt{j}+h^2j$

Section E: Standard Form

18. Write these numbers in standard form:

    (a) 678000

    (b) 0.000094

    (c) 4560000000

    (d) 0.0000000287

    (e) 945.6

    (f) 0.00947

19. Write these in ordinary form:

    (a) 7.8×10^{3}

    (b) 5.69×10^{-7}

    (c) 8.321×10^{-10}

    (d) 4.36×10^{13}

    (e) 9.5×10^{-5}

    (f) 6.78×10^{8}

20. Calculate, giving answers in standard form:

(a) $(7 \times 10^7) \times (6 \times 10^9)$
(b) $(8 \times 10^{-5}) \times (9 \times 10^{10})$
(c) $(15 \times 10^8) \div (5 \times 10^{-3})$
(d) $(12 \times 10^{-7}) \div (6 \times 10^{-10})$
(e) $(7 \times 10^6)^2$
(f) $\sqrt{36 \times 10^{14}}$

21. Calculate these more complex expressions:

(a) $(5.6 \times 10^6) \times (1.75 \times 10^{-8})$
(b) $\frac{11.2 \times 10^9}{2.8 \times 10^{-6}}$
(c) $(6.3 \times 10^{-5}) + (7.8 \times 10^{-6})$
(d) $(8.1 \times 10^8) - (5.4 \times 10^7)$
(e) $\frac{(5.4 \times 10^4) \times (4.5 \times 10^{-5})}{8.1 \times 10^{-3}}$
(f) $(4.84 \times 10^{12})^{\frac{1}{2}}$

Section F: Rational Numbers and Operations

22. Calculate these fractions (give answers in simplest form):

(a) $\frac{4}{9} + \frac{5}{18}$
(b) $\frac{11}{15} - \frac{7}{20}$
(c) $\frac{12}{13} \times \frac{26}{18}$
(d) $\frac{10}{21} \div \frac{15}{28}$
(e) $\frac{8}{9} - \frac{4}{15} + \frac{7}{45}$
(f) $\left(\frac{5}{8}\right)^{-2}$

23. Convert these recurring decimals to fractions:

(a) $0.\overline{4}$
(b) $0.\overline{63}$
(c) $0.4\overline{7}$
(d) $0.\overline{714285}$
(e) $3.2\overline{8}$
(f) $0.73\overline{6}$

24. Work out these percentage calculations:

(a) Increase 540 by 45%
(b) Decrease 720 by 24%
(c) Find 27.5% of 840
(d) What percentage is 108 out of 144?
(e) If 65% of a number is 169, find the number
(f) A price increases from £90 to £99. Find the percentage increase

25. Solve these percentage problems:

(a) After a 50% increase, a price is £225. Find the original price
(b) After a 45% decrease, a quantity is 99. Find the original quantity
(c) The value of a car decreases by 35% each year. If it's worth £12675 now, what was it worth 2 years ago?
(d) An investment grows by 5% per year. After 2 years it's worth £2205. Find the initial investment

Section G: Complex Calculations

26. Simplify these mixed expressions:

    (a) $5^{-2} + 7^0 - 8^{-1}$

    (b) $\sqrt{49} \times 216^{\frac{1}{3}} - 5^{-2}$

    (c) $\frac{512^{\frac{2}{3}} - 625^{\frac{3}{4}}}{36^{\frac{1}{2}}}$

    (d) $25^{-\frac{1}{2}} + 64^{\frac{1}{2}} \times 4^{-1}$

27. Calculate exactly (leave surds in your answer):

    (a) $\frac{7}{\sqrt{7}} + \frac{5}{\sqrt{28}}$

    (b) $\sqrt{32} \times \sqrt{72} - \sqrt{128}$

    (c) $\frac{\sqrt{112}+\sqrt{63}}{\sqrt{7}}$

    (d) $(5\sqrt{2} - 4)^2$

28. Work with standard form in context:

    (a) The mass of an alpha particle is 6.64×10^{-27} kg. Find the mass of 6.02×10^{23} alpha particles

    (b) Seismic waves travel at 6.5×10^3 m/s. How far do they travel in one hour (use 1 hour = 3.6×10^3 seconds)?

    (c) The thickness of a cell membrane is approximately 7.5×10^{-9} m. How many cell membranes would fit in a stack 3 mm high?

    (d) A parallel processor handles 7.2×10^{12} operations per second. How many operations in 15 minutes?

Section H: Problem Solving

29. Prove that $\sqrt{7}$ is irrational. (Use proof by contradiction: assume $\sqrt{7} = \frac{c}{d}$ where c and d are integers with no common factors)

30. The number β satisfies $\beta^2 = 3\beta - 1$.

    (a) Show that $\beta = \frac{3+\sqrt{5}}{2}$

    (b) Calculate β to 4 decimal places

    (c) Find $\frac{1}{\beta}$ in surd form

31. Rationalize the denominator of $\frac{1}{\sqrt{5}+\sqrt{7}+\sqrt{11}}$. (Hint: First rationalize using $(\sqrt{5} + \sqrt{7}) - \sqrt{11}$)

32. A rectangle has sides of length $(5 + \sqrt{7})$ cm and $(5 - \sqrt{7})$ cm.

    (a) Find the exact area

    (b) Find the exact perimeter

    (c) Show that the area is rational but the perimeter is irrational

33. The population of cells quintuples every 6 hours. If there are initially 4×10^3 cells:

    (a) How many cells after 24 hours?

    (b) Express your answer in standard form

    (c) After how many hours will there be more than 8×10^8 cells?

34. Show that $\frac{1}{\sqrt{f}+\sqrt{g}} + \frac{1}{\sqrt{f}-\sqrt{g}} = \frac{2\sqrt{f}}{f-g}$

35. A sphere has volume $V = \frac{4}{3}\pi r^3$. If the volume is 500π cm^3:

    (a) Find the radius in surd form

    (b) Find the surface area (use $A = 4\pi r^2$)

    (c) Express both answers exactly

36. The equation $x^2 - 10x + 1 = 0$ has solutions $x = 5 \pm 2\sqrt{6}$.

    (a) Verify this by substitution

    (b) Find $\frac{1}{5+2\sqrt{6}} + \frac{1}{5-2\sqrt{6}}$ without using a calculator

    (c) Hence find the sum of the reciprocals of the roots

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 100

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