

GCSE Higher Mathematics

Practice Test 7: Geometry and Measures

Instructions:

Answer all questions. Show your working clearly.

Calculators may be used unless stated otherwise.

Time allowed: 90 minutes

Section A: Pythagoras' Theorem

- Use Pythagoras' theorem to find the missing lengths:
 - A right-angled triangle with legs 18 cm and 24 cm. Find the hypotenuse.
 - A right-angled triangle with hypotenuse 61 cm and one leg 11 cm. Find the other leg.
 - A right-angled triangle with legs 17.5 cm and 23.5 cm. Find the hypotenuse.
 - A right-angled triangle with hypotenuse 65 cm and one leg 63 cm. Find the other leg.
- Find the distance between these pairs of coordinate points:
 - A(7, 11) and B(19, 26)
 - C(-5, 6) and D(7, -6)
 - E(-11, -7) and F(1, 5)
 - G(0, 17) and H(-24, 9)
- A rectangle has length 36 cm and diagonal 45 cm. Find:
 - The width of the rectangle
 - The area of the rectangle
 - The perimeter of the rectangle
- A ladder 16.5 m long leans against a vertical wall. The foot of the ladder is 9.9 m from the wall.
 - How high up the wall does the ladder reach?
 - If the ladder slips so its foot is now 12 m from the wall, how far does the top slide down?
- Determine whether these triangles are right-angled:
 - Sides 16 cm, 63 cm, 65 cm
 - Sides 17 cm, 144 cm, 145 cm
 - Sides 22 cm, 28 cm, 36 cm
 - Sides 30 cm, 40 cm, 50 cm

Section B: Basic Trigonometry

6. In a right-angled triangle with hypotenuse 37 cm and one leg 35 cm:
- (a) Find the other leg using Pythagoras
 - (b) Calculate $\sin A$, $\cos A$, and $\tan A$ where A is the angle opposite the 35 cm side
 - (c) Find angle A to 1 decimal place
7. Use trigonometry to find the missing sides (give answers to 3 significant figures):
- (a) Right-angled triangle: angle 58° , adjacent side 17 cm, find opposite side
 - (b) Right-angled triangle: angle 36° , hypotenuse 24 cm, find opposite side
 - (c) Right-angled triangle: angle 72° , opposite side 16 cm, find adjacent side
 - (d) Right-angled triangle: angle 51° , adjacent side 14 cm, find hypotenuse
8. Find the missing angles (to 1 decimal place):
- (a) Right-angled triangle: opposite 13 cm, adjacent 21 cm
 - (b) Right-angled triangle: opposite 17.6 cm, hypotenuse 24.8 cm
 - (c) Right-angled triangle: adjacent 12.7 cm, hypotenuse 18.2 cm
 - (d) Right-angled triangle: opposite 32 cm, adjacent 19 cm
9. A 30 m ladder makes an angle of 84° with the ground.
- (a) How high up the wall does it reach?
 - (b) How far is the foot of the ladder from the wall?
 - (c) What angle would the ladder make if its foot was 17 m from the wall?
10. From the top of a 60 m cliff, the angle of depression to a boat is 36° .
- (a) Draw a diagram
 - (b) Find the horizontal distance from the cliff to the boat
 - (c) Find the direct distance from the top of the cliff to the boat

Section C: Advanced Trigonometry - Sine and Cosine Rules

11. Use the sine rule to find the missing parts:
- (a) Triangle ABC: $a = 20$ cm, $A = 59^\circ$, $B = 63^\circ$. Find b and c .
 - (b) Triangle PQR: $p = 27$ cm, $q = 31$ cm, $P = 56^\circ$. Find angle Q .
 - (c) Triangle DEF: $d = 16.9$ cm, $e = 21.7$ cm, $D = 51^\circ$. Find angle E .
12. Use the cosine rule to find the missing parts:
- (a) Triangle ABC: $a = 21$ cm, $b = 18$ cm, $c = 13$ cm. Find angle C .
 - (b) Triangle GHI: $g = 23$ cm, $h = 19$ cm, $I = 66^\circ$. Find side i .
 - (c) Triangle JKL: $j = 15.8$ cm, $k = 19.3$ cm, $l = 12.6$ cm. Find the largest angle.
13. A triangle has sides 23 cm, 25 cm, and 27 cm.
- (a) Use the cosine rule to find the largest angle
 - (b) Use the sine rule to find another angle
 - (c) Find the area using the formula $\text{Area} = \frac{1}{2}ab \sin C$

14. Two ships leave port at the same time. Ship A travels 60 km on a bearing of 105° . Ship B travels 45 km on a bearing of 175° .
- (a) Find the angle between their paths
 - (b) Calculate the distance between the ships
 - (c) Find the bearing of ship B from ship A
15. A parallelogram has sides 20 cm and 26 cm, with an included angle of 82° .
- (a) Find the length of the diagonals
 - (b) Find the area of the parallelogram
 - (c) Find the other angles of the parallelogram

Section D: Circle Theorems

16. Apply circle theorems to find the missing angles:
- (a) Angle at center is 156° . Find the angle at circumference subtending the same arc.
 - (b) Angle at circumference is 44° . Find the angle at center subtending the same arc.
 - (c) Two angles at circumference subtend the same arc. One is 56° . Find the other.
 - (d) Angle in semicircle STU where ST is diameter. Find angle SUV.
17. In a circle with center O:
- (a) Chord PQ subtends angle 82° at circumference point R. Find angle POQ.
 - (b) Tangent BC touches circle at C. If angle $BCO = 49^\circ$, find angle CBO.
 - (c) Two chords EF and GH intersect at B inside the circle. If angles $EBG = 98^\circ$, find angle EBH.
 - (d) Tangents from external point B touch circle at P and Q. If angle $PBQ = 72^\circ$, find angle POQ.
18. Prove these circle theorems (provide written proof):
- (a) Angle in semicircle is 90°
 - (b) Angles in same segment are equal
 - (c) Angle at center is twice angle at circumference
 - (d) Tangent is perpendicular to radius at point of contact
19. A circle has center O and radius 20 cm. Chord PQ has length 32 cm.
- (a) Find the perpendicular distance from O to chord PQ
 - (b) Find angle POQ
 - (c) Find the area of sector POQ
 - (d) Find the area of triangle POQ

Section E: Geometrical Proof and Reasoning

20. Prove that the sum of angles in any triangle is 180° .
21. Prove that the exterior angle of a triangle equals the sum of the two interior opposite angles.
22. In triangle STU, V is the midpoint of TU. Prove that SV is a median and that the three medians of a triangle meet at a single point.

23. IJKL is a parallelogram. Prove that:
- (a) Opposite sides are equal
 - (b) Opposite angles are equal
 - (c) Diagonals bisect each other
24. Prove that if a quadrilateral has both pairs of opposite sides parallel, then it is a parallelogram.
25. In triangle STU, the perpendicular bisectors of sides ST and TU meet at point B. Prove that B is equidistant from S, T, and U.
26. IJKL is a cyclic quadrilateral (vertices lie on a circle). Prove that opposite angles sum to 180° .
27. Prove that the angle bisectors of a triangle meet at a single point (the incenter).

Section F: Surface Area and Volume

28. Calculate the surface area and volume of these shapes:
- (a) Cuboid: length 22 cm, width 17 cm, height 12 cm
 - (b) Cylinder: radius 10 cm, height 24 cm
 - (c) Sphere: radius 14 cm
 - (d) Cone: radius 12 cm, height 30 cm
29. A triangular prism has:
- Triangular cross-section with base 18 cm and height 11 cm
 - Length 32 cm

Calculate:

- (a) The volume
 - (b) The surface area
 - (c) The total edge length
30. A square-based pyramid has:
- Base edge 20 cm
 - Vertical height 15 cm
- Find:
- (a) The volume
 - (b) The slant height
 - (c) The surface area
31. A hemisphere sits on top of a cylinder. Both have radius 9 cm and the cylinder has height 24 cm.
- (a) Find the total volume
 - (b) Find the total surface area
 - (c) If this shape is a water tank, how much water does it hold?
32. A cone has base radius 13 cm and slant height 23 cm.
- (a) Find the vertical height
 - (b) Calculate the volume
 - (c) Calculate the curved surface area
 - (d) Calculate the total surface area

Section G: Complex 3D Problems

33. A rectangular swimming pool is 55 m long, 28 m wide, and 5 m deep.
- (a) Calculate the volume of water needed to fill it
 - (b) Find the surface area of the pool (5 faces - no top)
 - (c) How much does it cost to tile the pool at £75 per m^2 ?
 - (d) If water is added at 2100 liters per minute, how long to fill the pool?
34. A solid consists of a cylinder with radius 15 cm and height 22 cm, with a cone of radius 15 cm and height 20 cm on top.
- (a) Calculate the total volume
 - (b) Calculate the total surface area
 - (c) If the solid is made of metal with density 9.7 g/cm^3 , find its mass
35. A spherical ball has radius 30 cm. A cylindrical hole of radius 11 cm is drilled through the center.
- (a) Calculate the volume of material removed
 - (b) Calculate the remaining volume
 - (c) Calculate the new surface area (including the cylindrical hole)
36. A water tank is a horizontal cylinder with length 12 m and radius 3.0 m.
- (a) Calculate its capacity in liters
 - (b) If it's half full, find the depth of water
 - (c) Calculate the surface area of water

Section H: Problem Solving and Applications

37. A regular nonagon is inscribed in a circle of radius 27 cm.
- (a) Find the side length of the nonagon
 - (b) Calculate the area of the nonagon
 - (c) Find the area between the circle and nonagon
38. From point X, the angle of elevation to the top of a tower is 31° . From point Y, 130 m closer to the tower, the angle of elevation is 56° .
- (a) Draw a diagram
 - (b) Find the height of the tower
 - (c) Find the distances from X and Y to the base of the tower
39. A field is in the shape of a triangle with sides 240 m, 280 m, and 320 m.
- (a) Find the area of the field using Heron's formula
 - (b) Find the area using trigonometry
 - (c) A path 5 m wide runs around the perimeter. Find the area of the path
40. A church spire consists of a square-based pyramid on top of a cuboid. The cuboid is $20 \text{ m} \times 20 \text{ m} \times 32 \text{ m}$ high. The pyramid has height 24 m.
- (a) Calculate the total volume

- (b) Calculate the total surface area
 - (c) Find the angle the pyramid faces make with the horizontal
41. Two circles with radii 24 cm and 17 cm have centers 48 cm apart.
- (a) Do the circles intersect? Justify your answer.
 - (b) Find the length of their common external tangent
 - (c) Calculate the area of overlap if they moved closer together
42. A satellite orbits Earth at height 900 km above the surface. Earth's radius is 6400 km.
- (a) Calculate the satellite's orbital radius
 - (b) Find the maximum distance the satellite can see on Earth's surface
 - (c) Calculate the area of Earth visible from the satellite
43. Prove that in any triangle, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$ where R is the circumradius.
44. A frustum (truncated cone) has top radius 9 cm, bottom radius 24 cm, and height 18 cm.
- (a) Find the height of the complete cone
 - (b) Calculate the volume of the frustum
 - (c) Calculate the curved surface area of the frustum

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 100

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