GCSE Higher Mathematics
Practice Test 2: Number

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Time allowed: 90 minutes

Section A: Powers and Roots

1. Evaluate these expressions:

   (a) 3^4

   (b) 2^{-3}

   (c) 7^0

   (d) $(-3)^4$

   (e) $(-2)^5$

   (f) 5^{-2}

2. Write these as single powers:

   (a) $3^4 \times 3^6$

   (b) $5^9 \div 5^3$

   (c) $(7^3)^2$

   (d) $2^{-3} \times 2^8$

   (e) $\frac{4^7}{4^{-3}}$

   (f) $(3^2)^{-4}$

3. Evaluate these fractional indices:

   (a) $25^{\frac{1}{2}}$

   (b) $8^{\frac{1}{3}}$

   (c) $81^{\frac{3}{4}}$

   (d) $243^{\frac{2}{5}}$

   (e) $64^{\frac{2}{3}}$

   (f) $27^{-\frac{2}{3}}$

4. Simplify these expressions:

   (a) $\sqrt{36}$

   (b) $\sqrt[3]{125}$

(c) $\sqrt[4]{16}$

(d) $\sqrt{0.36}$

(e) $\sqrt[3]{-27}$

(f) $\sqrt[5]{243}$

5. Express in index form:

(a) $\sqrt{11}$

(b) $\sqrt[3]{7}$

(c) $\frac{1}{\sqrt{3}}$

(d) $\sqrt[4]{y^5}$

(e) $\frac{1}{\sqrt[3]{b^4}}$

(f) $\sqrt{y} \times \sqrt[3]{y}$

Section B: Laws of Indices

6. Simplify these expressions (no calculator):

(a) $3^2 \times 3^{-4} \times 3^5$

(b) $\frac{2^7 \times 2^{-3}}{2^{-2}}$

(c) $(5^3)^{-2} \times 5^8$

(d) $\frac{7^{-2} \times 7^6}{7^3}$

7. Evaluate these expressions:

(a) $27^{\frac{2}{3}}$

(b) $4^{-\frac{3}{2}}$

(c) $32^{\frac{3}{5}}$

(d) $49^{-\frac{3}{2}}$

(e) $125^{\frac{1}{3}}$

(f) $16^{-\frac{3}{4}}$

8. Simplify these expressions:

(a) $x^{\frac{3}{4}} \times x^{\frac{1}{3}}$

(b) $\frac{z^{\frac{5}{2}}}{z^{\frac{1}{4}}}$

(c) $(b^{\frac{1}{3}})^6$

(d) $\sqrt{y} \times y^{\frac{1}{4}}$

(e) $\frac{\sqrt[3]{n^4}}{\sqrt{n}}$

(f) $(q^{-\frac{1}{3}})^{-6}$

9. Write these in the form a^n where a and n are rational:

(a) $\sqrt{3} \times 3^2$

(b) $\frac{7^3}{\sqrt[3]{7}}$

(c) $\sqrt[4]{2^5} \times 2^{-\frac{1}{3}}$

(d) $\frac{\sqrt{11}}{\sqrt[3]{11^4}}$

Section C: Surds

10. Simplify these surds:

    (a) $\sqrt{32}$
    (b) $\sqrt{75}$
    (c) $\sqrt{48}$
    (d) $\sqrt{128}$
    (e) $\sqrt{180}$
    (f) $\sqrt{450}$

11. Simplify these expressions:

    (a) $4\sqrt{5} + 7\sqrt{5}$
    (b) $9\sqrt{2} - 3\sqrt{2}$
    (c) $\sqrt{12} + \sqrt{27}$
    (d) $\sqrt{75} - \sqrt{48}$
    (e) $3\sqrt{18} + 2\sqrt{50}$
    (f) $\sqrt{98} - \sqrt{72} + \sqrt{8}$

12. Multiply and simplify:

    (a) $\sqrt{5} \times \sqrt{20}$
    (b) $\sqrt{12} \times \sqrt{27}$
    (c) $3\sqrt{7} \times 2\sqrt{14}$
    (d) $\sqrt{8} \times \sqrt{32}$
    (e) $\sqrt{3} \times \sqrt{12} \times \sqrt{48}$
    (f) $4\sqrt{6} \times 3\sqrt{24}$

13. Expand and simplify:

    (a) $(3 + \sqrt{2})(4 - \sqrt{2})$
    (b) $(2 + \sqrt{7})(1 + 3\sqrt{7})$
    (c) $(5 - \sqrt{3})^2$
    (d) $(\sqrt{11} + \sqrt{5})(\sqrt{11} - \sqrt{5})$
    (e) $(3\sqrt{2} + 1)(3\sqrt{2} - 1)$
    (f) $(\sqrt{7} + 3)^2$

14. Rationalize the denominators:

    (a) $\frac{1}{\sqrt{7}}$
    (b) $\frac{5}{\sqrt{11}}$
    (c) $\frac{\sqrt{3}}{\sqrt{12}}$
    (d) $\frac{6}{3\sqrt{2}}$
    (e) $\frac{1}{2+\sqrt{3}}$
    (f) $\frac{3}{1-\sqrt{7}}$

Section D: More Complex Surd Operations

15. Rationalize these denominators:

    (a) $\frac{4}{3+\sqrt{5}}$

    (b) $\frac{7}{2-\sqrt{13}}$

    (c) $\frac{\sqrt{2}}{1+\sqrt{2}}$

    (d) $\frac{3\sqrt{7}}{2+\sqrt{7}}$

    (e) $\frac{1}{\sqrt{8}-\sqrt{3}}$

    (f) $\frac{\sqrt{5}+2}{\sqrt{5}-3}$

16. Simplify these expressions completely:

    (a) $\frac{\sqrt{18}+\sqrt{32}}{\sqrt{2}}$

    (b) $\frac{\sqrt{45}-\sqrt{20}}{\sqrt{5}}$

    (c) $\sqrt{(4+\sqrt{7})(4-\sqrt{7})}$

    (d) $\sqrt{72}-3\sqrt{8}+\sqrt{32}$

    (e) $(\sqrt{3}+\sqrt{12})^2$

    (f) $\frac{\sqrt{63}}{\sqrt{7}}+\frac{\sqrt{28}}{\sqrt{7}}$

17. Prove that:

    (a) $(\sqrt{p}+\sqrt{q})(\sqrt{p}-\sqrt{q})=p-q$

    (b) $\frac{1}{\sqrt{m}+\sqrt{n}}=\frac{\sqrt{m}-\sqrt{n}}{m-n}$

    (c) $(p+q\sqrt{r})^2=p^2+2pq\sqrt{r}+q^2r$

Section E: Standard Form

18. Write these numbers in standard form:

    (a) 456000

    (b) 0.000071

    (c) 2340000000

    (d) 0.0000000087

    (e) 789.3

    (f) 0.00621

19. Write these in ordinary form:

    (a) 5.8×10^5

    (b) 3.47×10^{-5}

    (c) 6.543×10^{-8}

    (d) 2.14×10^{11}

    (e) 7.9×10^{-3}

    (f) 4.56×10^6

20. Calculate, giving answers in standard form:

(a) $(4 \times 10^5) \times (3 \times 10^7)$

(b) $(7 \times 10^{-2}) \times (6 \times 10^8)$

(c) $(8 \times 10^6) \div (4 \times 10^{-2})$

(d) $(6 \times 10^{-5}) \div (2 \times 10^{-8})$

(e) $(5 \times 10^4)^2$

(f) $\sqrt{16 \times 10^{10}}$

21. Calculate these more complex expressions:

(a) $(3.2 \times 10^4) \times (2.5 \times 10^{-6})$

(b) $\frac{8.4 \times 10^7}{2.1 \times 10^{-3}}$

(c) $(4.7 \times 10^{-3}) + (5.6 \times 10^{-4})$

(d) $(6.8 \times 10^6) - (3.4 \times 10^5)$

(e) $\frac{(3.6 \times 10^2) \times (2.5 \times 10^{-3})}{7.5 \times 10^{-2}}$

(f) $(2.25 \times 10^8)^{\frac{1}{2}}$

Section F: Rational Numbers and Operations

22. Calculate these fractions (give answers in simplest form):

(a) $\frac{3}{4} + \frac{7}{12}$

(b) $\frac{5}{8} - \frac{3}{10}$

(c) $\frac{6}{7} \times \frac{14}{9}$

(d) $\frac{8}{15} \div \frac{12}{25}$

(e) $\frac{5}{6} - \frac{2}{9} + \frac{7}{18}$

(f) $\left(\frac{3}{5}\right)^{-2}$

23. Convert these recurring decimals to fractions:

(a) $0.\overline{7}$

(b) $0.\overline{54}$

(c) $0.2\overline{8}$

(d) $0.\overline{285714}$

(e) $1.7\overline{3}$

(f) $0.41\overline{6}$

24. Work out these percentage calculations:

(a) Increase 320 by 25%

(b) Decrease 450 by 12%

(c) Find 17.5% of 640

(d) What percentage is 84 out of 140?

(e) If 45% of a number is 117, find the number

(f) A price increases from £60 to £72. Find the percentage increase

25. Solve these percentage problems:

(a) After a 30% increase, a price is £169. Find the original price

(b) After a 25% decrease, a quantity is 84. Find the original quantity

(c) The value of a car decreases by 20% each year. If it's worth £15360 now, what was it worth 2 years ago?

(d) An investment grows by 6% per year. After 2 years it's worth £2247.36. Find the initial investment

Section G: Complex Calculations

26. Simplify these mixed expressions:

    (a) $3^{-2} + 2^0 - 4^{-1}$

    (b) $\sqrt{25} \times 27^{\frac{1}{3}} - 3^{-2}$

    (c) $\frac{64^{\frac{2}{3}} - 81^{\frac{3}{4}}}{4^{\frac{1}{2}}}$

    (d) $9^{-\frac{1}{2}} + 36^{\frac{1}{2}} \times 2^{-1}$

27. Calculate exactly (leave surds in your answer):

    (a) $\frac{4}{\sqrt{3}} + \frac{3}{\sqrt{12}}$

    (b) $\sqrt{18} \times \sqrt{32} - \sqrt{72}$

    (c) $\frac{\sqrt{75} + \sqrt{48}}{\sqrt{3}}$

    (d) $(3\sqrt{2} - 2)^2$

28. Work with standard form in context:

    (a) The mass of a proton is 1.67×10^{-27} kg. Find the mass of 6.02×10^{23} protons

    (b) Sound travels at 3.3×10^2 m/s. How far does it travel in one hour (use 1 hour $= 3.6 \times 10^3$ seconds)?

    (c) The diameter of a molecule is approximately 2×10^{-9} m. How many molecules would fit across a distance of 1 mm?

    (d) A supercomputer processes 3.6×10^{10} operations per second. How many operations in 8 minutes?

Section H: Problem Solving

29. Prove that $\sqrt{3}$ is irrational. (Use proof by contradiction: assume $\sqrt{3} = \frac{a}{b}$ where a and b are integers with no common factors)

30. The number ψ satisfies $\psi^2 = \psi - 1$.

    (a) Show that $\psi = \frac{1 - \sqrt{5}}{2}$

    (b) Calculate ψ to 4 decimal places

    (c) Find $\frac{1}{\psi}$ in surd form

31. Rationalize the denominator of $\frac{1}{\sqrt{3} + \sqrt{5} + \sqrt{7}}$. (Hint: First rationalize using $(\sqrt{3} + \sqrt{5}) - \sqrt{7}$)

32. A rectangle has sides of length $(3 + \sqrt{2})$ cm and $(3 - \sqrt{2})$ cm.

    (a) Find the exact area

    (b) Find the exact perimeter

    (c) Show that the area is rational but the perimeter is irrational

33. The population of algae triples every 4 hours. If there are initially 8×10^3 algae:

    (a) How many algae after 16 hours?

    (b) Express your answer in standard form

    (c) After how many hours will there be more than 5×10^6 algae?

34. Show that $\frac{1}{\sqrt{p} + \sqrt{q}} + \frac{1}{\sqrt{p} - \sqrt{q}} = \frac{2\sqrt{p}}{p - q}$

35. A cone has volume $V = \frac{1}{3}\pi r^2 h$. If the volume is 384π cm^3 and $h = 12$ cm:

    (a) Find the radius in surd form

    (b) Find the slant height (use $l^2 = r^2 + h^2$)

    (c) Express both answers exactly

36. The equation $x^2 - 6x + 1 = 0$ has solutions $x = 3 \pm 2\sqrt{2}$.

    (a) Verify this by substitution

    (b) Find $\frac{1}{3+2\sqrt{2}} + \frac{1}{3-2\sqrt{2}}$ without using a calculator

    (c) Hence find the sum of the reciprocals of the roots

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 100

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